

3. DUE WEDNESDAY, SEPT. 26

Exercise 3.1. Exercises 8, 17, 20, and 23 of §1.7.

Exercise 3.2. Exercise 1 of §4.1.

Exercise 3.3. Let G be a group and X a G -set. Pick two elements $x, y \in X$.

- (a) Show that if $x \in O_y$ then $O_x = O_y$.
- (b) Show that either $O_x = O_y$ or $O_x \cap O_y = \emptyset$.

Hint: make use of the equivalence relation of Proposition 2.1.8.

Exercise 3.4. Let $X \subset D_8$ be the set containing the four reflections, and let D_8 act on X by conjugation.

- (a) Compute the orbit and stabilizer of every element of X . Is this action transitive?
- (b) As $|X| = 4$, this action determines a homomorphism $D_8 \rightarrow S_4$. Compute the kernel and image of this homomorphism.

Exercise 3.5. Let Q_8 denote the quaternion group of order 8.

- (a) Let Q_8 act on itself by conjugation. Compute the orbit and stabilizer of every element, and the kernel of the associated homomorphism $Q_8 \rightarrow S_8$.
- (b) The left regular action of Q_8 on itself determines an injective homomorphism $Q_8 \rightarrow S_8$. Determine the image.

Exercise 3.6. Prove that every element of S_n can be written as a product of transpositions.