

8. DUE WEDNESDAY, NOV. 7

*Exercise 8.1.* From Dummit and Foote §4.5 do problems 19, 20, 21, 22, 23, and 24.

*Exercise 8.2.* Let  $G$  be a group of order 20. How many elements of order 5 does  $G$  have?

*Exercise 8.3.* Let  $G$  be a group of order  $p^2q$  with  $p, q$  distinct primes. Show that  $G$  is not simple. Hint: treat separately the cases  $p < q$  and  $q < p$ .

*Exercise 8.4.* Let  $G$  be a group of order  $pqr$  where  $p < q < r$  are primes. Show that  $G$  has a normal Sylow subgroup.

*Exercise 8.5.* Let  $P$  be a  $p$ -Sylow subgroup of a group  $G$ , and let  $N \triangleleft G$  be a normal subgroup of  $G$ .

- (a) Show that  $PN/N$  is a  $p$ -Sylow subgroup of  $G/N$ .
- (b) Show that  $P \cap N$  is a  $p$ -Sylow subgroup of  $N$ .

*Exercise 8.6.* Let  $G$  be a simple group with  $|G| < 60$ . Show that  $G$  is cyclic of prime order.