IRR Notes - Answer to Questions

Examples: Suppose the current interest rate is 8% and the yield curve is flat. Compute the change in the price of a 3-year, 6% coupon bond if interest rates rise to 9%. Do the same for a 10-year, 6% coupon bond.

Three-Year Bond:

Current price (per $100 face value)

$P = 6 / 0.08 [1 - 1/1.08^3] + 100 / 1.08^3$ = 94.85

Price at 9%

$P = 6 / 0.09 [1 - 1/1.09^3] + 100 / 1.09^3$ = 92.41 (falls by 2.57%)

(I am using the annuity formula to get the PV of the coupon payments.)

For 10-year bond:

Current price (per $100 face value)

$P = 6 / 0.08 [1 - 1/1.08^{10}] + 100 / 1.08^{10}$ = 86.58

Price at 9%

$P = 6 / 0.09 [1 - 1/1.09^{10}] + 100 / 1.09^{10}$ = 80.75 (falls by 6.7%)

Now, see what happens when you increase rates further, from 9% to 10%. If you did it right, you should find that the changes in the prices of the two bonds are smaller than they were in the first part of the example. This illustrates convexity:

For 10-year bond:

Price at 10%

$P = 6 / 0.10 [1 - 1/1.10^{10}] + 100 / 1.10^{10}$ = 75.42 (falls by another 6.6%)

Example: Compute Macaulay Duration for the 6% coupon bond (assume annual payments) with 3 years until maturity when interest rates are 8% (use a spreadsheet).

Modified Duration:

D*(3 years, 8%) = 2.62 years

D*(10 years, 8%) = 7.05 years

(McCaulay is just modified times 1.08)

see: http://www2.bc.edu/~strahan/irr_example.xls

Example: Compute D* for the 6% coupon bond that matures in 3-years when interest rates are 8%. Now, compute the change in the value of this bond when rates move to 9%. How does this approximate change compare with the actual change computed earlier?

Approx. Change = -2.62x94.85x(0.01) = -2.485

This is a bigger decline than the actual one (-2.44) due to convexity.

Example: Go back an re-compute the effect of a change in rates from 8% to 9% for the 6% coupon bond with 3-years to maturity using the convexity term. Convince yourself that you get a better approximation this way. (Hint: I would suggest using a spreadsheet for this.)

Convexity = 9.51 (see: http://www2.bc.edu/~strahan/irr_example.xls)

Approx. Change = $-2.62 \times 94.85 \times (0.01) + (1/2) \times 9.51 \times 94.85 \times (0.01)^2 = -2.44$

So, we are now very close to the actual change.

Hedging Equity

Example. Let's see how this works. Look at a simple bank. Here is its market-value balance sheet:

Assets	
6% coupon bond with maturity of 3 years	1000
Liabilities	
1-year CDs	900
Net Worth	100

Now, we are assuming that this balance sheet is based on market value! Also, to make the math easy, assume that the 1-year CD operates like a zero-coupon bond. Interest rates are 8%, so this CD will pay $900x1.08 in 1 year.

First, compute the modified durations of the assets and liabilities. Next, compute the exact change in net worth for a 1% increase in interest rates. Then, using the duration of net worth, compute the approximate change stemming from a 1% increase in interest rates.

$D^*(A) = 2.62$ years (from before)
$D^*(L) = 1/1.08 = 0.926$

So,

$\Delta NW = -(2.62 - 0.926*0.9)*1000*(.01) = -\$17.866$

Exact change in equity:

From above, assets fall by 2.57%, or $25.7 (from above)
Liabilities fall by: 900 - 900x(1.08/1.09) = 8.26
So, the exact change in equity is:

$\Delta NW = -25.7 - (-8.26) = -\$17.44$

Since the approximate decline in equity is larger than the actual, we know we have positive convexity.

Example: Let's continue the example from above. Suppose you want to eliminate the interest rate risk of net worth using a 3-month futures contract on Treasury bond with 10 years to maturity and annual coupon rate of 6%. How many futures contracts do you need to hold?

To hedge out the duration risk, we need to construct a position in futures that will increase in value by \$17.866 when rates rise from 8% to 9%. (We use the duration approximation for both the futures and the net worth.)

The price of the underlying 10-year, 6% coupon Treasury, from before, is \$86.58 (per \$100 of face value).

So, the futures price is:

$$F = 86.58(1.08)^{0.25} = 88.26$$

(Remember that the exponent here corresponds to the expiration of the futures - 3 months time in this example.)

The modified duration equals the duration of the underlying, from above, 7.05 years.

Now, we need to decide how many contracts (N_F) to enter into to hedge the risk of net worth:

$$\Delta F = 17.866 = -88.26(N_F)(7.05)(0.01)$$

==>

$$N_F = -17.866 / [(88.26)(7.05)(0.01)] = -2.87 \text{ contracts}$$

So, we need to take a short position (due to the negative sign) in 2.87 futures contracts with the 10-year, 6% coupon bond as the underlying. The value of this position *rises* (since we are short) as interest rates rise, so we hedge our net worth position, which *falls* as rates rise.

Example. Let's do the same hedge using a swap. We will consider a 5-year, pay-fixed, receive-floating rate swap. (This will be the correct type of swap because we want the swap to increase in value when rates rise. Note that I set this up backwards last night on the blackboard, which is why the notional amount came out negative.) The floating rate is assumed to reset at the end of each year, and there is assumed to be one payment at the end of each year, as in the notes (see page 12). A pay-fixed, receive floating rate swap can be de-constructed into a long position in the floating-rate side financed by a short position in the fixed-rate side. So:

$$\Delta S \approx -(D^*_{variable} - D^*_{fixed}) \times NP \times \Delta r$$

Since the floating rate resets at the end of the first year, this means the floating rate side acts like a zero coupon bond that matures in 1 year. Thus, the floating rate side has Macaulay duration of 1 year. So:

$D^*_{variable}$ = 1/1.08 = 0.926 years (this is the asset side since we received float)

The fixed rate side has the same *Modified* Duration as a 5-year bond with 8% coupon rate:

D^*_{fixed} = 3.99 years (this is the liability side since we pay fixed)

See irr_example.xls on the website.

So, we can now solve for the NP (that notional principle, or size of the swap) that will hedge our net worth:

17.866 = -(0.926-3.99)(NP)(0.01) ==>

NP = 17.866 / (3.99-0.926)(0.01) = $583.09

So, we want to enter into a pay-fixed, receive floating rate swap with notional principal of $583.09 million to hedge our net worth. Again, intuitively, this swap position will increase in value when interest rate rise (since we are receiving floating rate), so it offsets the risk in our net worth.