Review of Present Values

A. Present Values

One of the most important ideas in finance is the present value. If I put \$1 in the bank today at 10% interest, then in one year I'll have \$1.10. Interest rates allow you to compare money in your pocket today (\$1 in this example) with money in your pocket in the future (\$1.10 in this example). So, we can say that the present value (PV) of \$1.10 at the end of the year is \$1.

$$PV = \frac{1.10}{1+0.10} = \$1$$

In general, $\$C_1$ in one year from now if the interest rate (also called the <u>discount rate</u>) is r equals:

$$PV = \frac{C_1}{1+r}$$

If I put \$1 in the bank today at a 10% interest rate, in two years I will have $\$1\text{x}1.1^2=\$1.21$. So, the present value of \$1.21 received in two years if the interest rate is 10% is \$1.

$$PV = \frac{1.21}{(1+0.1)^2} = \$1$$

In general, $\$C_2$ in two years if the interest rate is r equals:

$$PV = \frac{C_2}{(1+r)^2}$$

Even more generally, if $\$C_t$ is to be paid in t years, and if the interest rate is r equals:

$$PV = \frac{C_t}{(1+r)^t}$$

Question: What happens to the PV if the interest rate goes up?

B. PV for a flow of payments

As we will see, often you will have to compute the PV of a stream of payments C_1, C_2, ... C_T. We can represent this on a time line.

Date	0	1	2	...	T
Cash Payment		C_1	C_2	...	C_T

The mathematical expression for the PV of the stream of payments equals the following:

$$PV = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_T}{(1+r)^T}$$

NB: This formula is very important!

C. Perpetuities

A perpetuity is a special kind of stream of cash flows with two features: First, you receive the same payment every year, and second, the payments go on FOREVER.

Date	0	1	2	...	T	...
Cash Payment		C	C	...	C	...

We can write the PV of the perpetuity by applying the formula from above:

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots$$

Multiply this by (1+r) to get:

$$(1+r)PV = C + \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots$$

If we now subtract the first equation from the second, we get:

$$rPV = C \Rightarrow PV = \frac{C}{r}$$

For example, if you are going to receive $100 every year forever starting next year and the interest rate is 10%, the PV equals $100/0.1=$1000.

This makes sense because you could generate a flow of income equal to $100 per year forever by investing $1000 (today) into a bank account paying 10% interest.

Two rules for using the perpetuity formula:

Rule 1: r>0, the interest rate must be greater than zero
Rule 2: the first payment starts at the END of the period!

D. Growing perpetuities

A growing perpetuity is a like a regular perpetuity, except that the payments grow each year at a constant rate. Here's what the time line looks like:

Date	0	1	2	...	T	...
Cash Payment		C	C(1+g)	...	$C(1+g)^{T-1}$	...

The PV of a growing perpetuity is:

$$PV = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \ldots$$

We can use a similar trick; multiply the whole thing by (1+r)/(1+g)

$$PV \times \frac{1+r}{1+g} = \frac{C}{(1+g)} + \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \ldots$$

Now, subtract the top expression from the bottom to get:

$$PV \times \frac{1+r}{1+g} - PV = \frac{C}{1+g}$$

After some simple (but tedious) algebra, this expression simplifies to:

$$PV = \frac{C}{(r-g)}$$

Rules for using the growing perpetuity formula:

Rule 1: r>g, the interest rate must be greater than the growth rate
Rule 2: the first payment starts at the END of the period!

I will not ask you to derive the perpetuity formulas on a test, but you must know how to use them!

Growing perpetuities show up in real life. For example, we will use it in the valuation of common stock.

E. Annuities

An annuity is like a perpetuity that dies after a while. That is, an annuity is a constant cash flow over a fixed interval of time, then zero after that. A silly example of an annuity is what you will get when you win the lottery. Here is how the cash flows look:

Date	0	1	2	...	T
Cash Payment		C	C	...	C

The PV of an annuity is as follows:

$$PV = \frac{C}{r} - \frac{C}{r(1+r)^T} = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^T} \right]$$

The term in square brackets is called an <u>annuity factor.</u> The annuity factor depends on how long the annuity lasts (T) and the interest rate.

Annuities are important in real life. For example, when you are older and you buy a house, you will get a mortgage and make constant series of payments over the life of the mortgage. Thus, you can compute the PV of your (future) mortgage using the annuity formula.

Question: Take a look at the car ad at the end of these notes. The price is $18,995. You can also buy this car by paying $589.46 per month for 36 months. How is this monthly payment calculated? Should you pay in cash or with credit? (We will go thru this in class.)

Email the Dealer

Herb Chambers Toyota Scion of Auburn
888-590-3859

Enter Your Message Here

First Name:*

Last Name:*

Email:*

Phone:* ☑ Day ☐ Evening

( ) -

City:

State: Choose

ZIP Code:*

Do you have a trade-in? ○ Yes ◉ No

☐ I am interested in saving 20% when selling my car on Cars.com.

Send Email

* Required

Cars.com Privacy Statement

Estimate Loan Payments

Vehicle Price:	$ 18995
Down Payment:	$ 0
Trade-In Value:	$ 0
Sales Tax:	0 %
Interest Rate:	7.34 %
Term (Months):	36
Monthly Payment:	$ 589.46

Calculate Payments

Get Financing First

Our lending partners will make buying a vehicle easy, regardless of your credit.

○ **Credit Problems?**
Let carloan.com connect you with local dealer financing.

○ **Pre-Qualify**
Apply securely in 30 seconds.

Apply Today

See Your Credit Score for $0!
Get a free credit report from Experian.

Current Interest Rates

New (48 mo.)	**7.00**%
Used (36 mo.)	**7.40**%

Check local rates from Bankrate.com

Your Estimated Payment Summary*

View More Calculators

Vehicle Price:	Total Cost:	Down Payment:	Sales Tax:
$18,995	$21,221	$0	0%

APR	24 mo.	36 mo.	48 mo.	60 mo.	72 mo.
5.5%	$838	$574	$442	$363	$310
6.0%	$842	$578	$446	$367	$315
6.5%	$846	$582	$450	$372	$319
7.0%	$850	$587	$455	$376	$324
7.34%	$853	**$589**	$458	$379	$327
8.0%	$859	$595	$464	$385	$333
8.5%	$863	$600	$468	$390	$338
9.0%	$868	$604	$473	$394	$342
9.5%	$872	$608	$477	$399	$347

■ Acquisition fees, destination charges, tag, title, and other fees and incentives are not included in this calculation, which is an estimate only.

■ The default interest rate is based on a 48-month loan for a new vehicle.

* Monthly payment estimates are for informational purposes only and do not represent a financing offer from the seller of this vehicle.

Back to Search Results | New Search

Next Car

The information on vehicles provided in this service is supplied by the seller or other third parties; cars.com is not responsible for the accuracy of such information. Cars.com provides this service and materials without representations or warranties of any kind, either expressed or implied. All prices and specifications are subject to change without notice. Prices may not include additional fees such as government fees and taxes, title and registration fees, finance charges, dealer document preparation fees, processing fees, and emission testing and compliance charges. Click here for more details.

Ask.cars GET EXPERT ANSWERS | MOTHER PROOF MOM REVIEWS | KickingTires LATEST NEW-CAR NEWS

Home | Contact Us | Site Index | About Cars.com | Employment Opportunities | Become a Cars.com Dealer

Cars.com Mobile | Búsqueda en Español de Carros Usados

Search Cars.com

Powered by: Google