The Valuation of Stocks and Bonds

A. Valuation of Bonds

Calculating the PV of bonds is straightforward. You simply discount the interest payments and terminal repayment of the debt at the opportunity cost. This is given by the yield on a similar bond. Note that the yield on a bond is the rate of return that would be earned on it if it was held to maturity, i.e. it is the discount rate implicit in the price of the bond. For example, for a two-year bond with annual coupon payments C (the first a year away) and principal of P, it is the value of y such that

$$P_{Bond} = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \frac{P}{(1+y)^2}$$

The return on this bond if it is held to maturity is called the yield-to-maturity. Technically, the yield to maturity is the internal rate of return (IRR) on investing in the bond if you hold it until maturity.

Conceptual question:

What determines (economically) why the yield to maturity varies from one company to the next, or from one time period to the next?

B. Valuation of Stocks

The main difference in calculating the PV's of stocks rather than bonds is the difference in the profiles of earnings. The cash payoff to stocks comes in two forms:

(1) Cash dividends
(2) Capital gains or losses

The price now is given by the present value of the dividend plus the present value of the price expected to obtain in 1 year. Here is some notation:

P_0 - Current price of share
P_1 - Expected price per share in 1 year's time
Div_1 - Expected dividend per share in 1 year's time
r - is the expected return on similar securities (i.e. opportunity cost of capital)

Hence,

$$P_0 = [\, P_1 + Div_1 \,] / (1+r)$$

You could think of this formula as the present value of cash flows to a one-period investor - meaning someone who will sell the stock at the end of the year for the price. It is a bit circular, however, because the formula depends on next year's price.

Another way to think about valuing stock is from the perspective of a 'buy and hold forever' investor (like my mother...). That investor holds the stock and collects dividend checks. In this case you can express the price at the PV of all future dividends:

$$P_0 = Div_1 / (1+r) + Div_2 / (1+r)^2 + ...$$

Both of these two formulas are equivalent, and you can even prove that mathematically if you want (don't need to do this for the test!). The second formula is better, however, because at least we know what we have to do - we have to forecast future dividends. That is a hard thing to do, but at least it's more tangible than trying to forecast next year's price. So, you see, it is hard to value stocks because the game is all about forecasting dividends into the distant future. The dividend stream depends on the firm's ability to generate cash flow in excess of what it needs to cover its expenses and pay interest on debt (and pay taxes).

Bond are simple to value because we know the future cash flows (principal + interest). Stocks are complicated because we do not know the future cash flows. All we can do as investors is construct our best guess (expected value), and use that to build value.

C. A special case of the stock valuation formula

One assumption that is often used to allow stock prices to be calculated easily is that dividends will grow at a constant rate (g). In this case, a stock can be valued like a growing perpetuity, so:

$$P_0 = Div_1 / (r - g)$$

(Remember that r > g for this to make sense!!!)

You have to be very careful using this formula because firms have a life cycle. When they are young, they grow very quickly. Then, when their business matures, they slow down and grow more moderately. Then they sometimes cease to grow or even shrink and go bankrupt. You have to exercise judgment as to whether present growth rates are likely to continue, and if they are not, you must determine what they will fall to and adjust the formula appropriately. Otherwise you may end up with some ridiculous answers. For example, very new firms sometimes grow at 50% per year (think Google). Clearly this can't continue for long. If you tried to use the perpetuity formula to value Google, you would get a nonsense answer.

Good valuation requires great skill in choosing the right figures--it is not simply a matter of plugging in values. An example where a firm goes through two stages of growth is given next.

Example: Suppose stock A has Div_1 = \$10 at t = 1 (or equivalently in period 1). This grows at g_1 = 20% for two years and g_2 = 5% forever thereafter. What is the current price at t = 0 (or in period 0) if the discount rate, r, is 10%?

Hint: As before, construct a time line with the dividends paid in each year to identify when you can use the perpetuity formula, then, apply the formula.

The basic method for valuing stocks is always the same as in this example. First identify the stages of growth. The final stage is where the growth has settled down and you can apply a growing perpetuity. The value of the stock when this final stage of growth starts is usually called the "terminal value" or "horizon value." You find the value of the stock today by evaluating the terminal value, discounting it back to the present and adding it to the discounted stream of dividends occurring in the previous stages.

The other thing to worry about when stocks are valued is when the first dividend is paid. The standard formulas assume the first dividend is paid at date 1. One way of phrasing this is to say a dividend "has just been paid" because that means the next one will be DIV_1 and will be paid one period from now at date 1. If the first dividend is to be paid at date 0 then this can be phrased as a dividend "is just about to be paid." In this case DIV_0 will be paid immediately and needs to be included in the valuation of the stock. Since it is paid immediately it does not need to be discounted back.

There are two ways to compute the dividend growth rate. One way is the use analysts estimates. But how do the analysts do this? The simplest model is based on the idea that the growth of dividends depends on how much of the firm's profits are plowed back into the firm for new investment to support growth.

Here's how it works:

Suppose we have a company with total book value of equity equal to \$100 at the beginning of the year. The firm earned \$10 during the year, so the return on equity (earnings/book-value of equity) equal to 10%. This means that, on average, if the firm invests \$1 in itself, it will earn a 10% return.

Suppose this company pays 50% of its earnings out as dividends, and reinvests (retains) the other 50%. What will happen next year?

Book value of equity at the beginning next year = \$105 (\$100 + \$5 in retained earnings)

Earnings next year = 105x10%=\$10.5

Dividends next year = \$10.5x50%=\$5.25

So, dividends and earnings have both grow by 5% during the year. That's because the firm earns 10% on investments in itself, and it reinvests ½ of its profits.

In general: if ROE and the plowback ratio (percentage of earnings that are retained) are contant, then:

g = ROExPlowback Ratio

where g is the growth rate of dividends.

(NB: The book has a more complicated formula that uses ROE based on end-of-year equity rather than beginning of year. The one here is simpler!)

The formula for valuing stocks as a perpetuity also allows you to break down the total stock return into two pieces - the dividend yield plus the capital gain:

$P_0 = D_1 / (r-g) ==>$

r = total expected return = $D_1 / P_0 + g$

The formula is very obvious if you think about what it means in words: the expected stock return equals the sum of the expected dividend yield (D_1 / P_0) plus the expected capital gain (g, the expected growth rate of the firm).

D. Stock Price and Earnings Per Share

So far we've looked at stock price and dividends. One of the statistics that is often used in the financial press and conversation is the ratio of the stock price to earnings per share (P/E ratio). The motivation example at the beginning of the lecture is an illustration of how it's used: investors are advised to buy stocks with low P/E ratios. In the last part of this lecture we shall develop the relationship between price and earnings per share (EPS). In the last section we looked at price and dividends. The difference between dividends and EPS is that the former is what's actually paid to stockholders, whereas the latter includes retained earnings.

We start by considering a firm that does not retain any earnings at all. We can think of it consisting of a single plant. It does not plow back any earnings and simply produces a constant stream of earnings, which are all paid out in dividends. This firm can be valued using the formula from above, plugging zero in as the growth rate of dividends.

$$P = Div_1 / r$$

For this hypothetical non-growing firm, dividends equals earning per share, so:

$$P = EPS / r$$

What if the firm is not static, but has opportunities to expand the firm in the future? The value of each new investment to shareholders is the net present value of those investments, as we will see in detail later. The sum of all future growth opportunities is called the NPVGO, which stands for **n**et **p**resent **v**alue of **g**rowth **o**pportunities. The value of a stock equals therefore equals the value of the current firm's earnings potential (EPS / r) plus the NPVGO (where we measure this value creation on a per share basis). So:

$$P = EPS / r + NPVGO$$

Or,

$$P / EPS = 1 / r + NPVGO / \text{EPS}$$

Basically the formula tells you to think of the value of a company as the sum of the present value of its assets in place (that's the EPS / r term) and the wealth created by future investments (that's the NPVGO part).