Notes on Comparative Advantage

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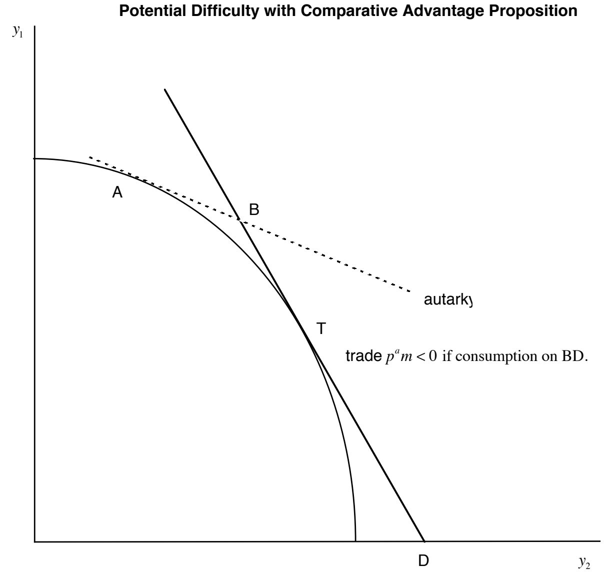
Dixit and Norman, ch.1, presents the theory of comparative advantage in a context imposing the axioms of revealed preference at the aggregate level. This is for simplicity only. General formulations of utility functions and income distribution allow for violations of the axioms of revealed preference at the aggregate level. This violation makes possible multiple equilibria. (An interesting 1950's paper of Harry Johnson provides an illustration.) Nevertheless, all trading equilibria of such a trading world will obey the theory of comparative advantage under the assumptions of convex structure and perfectly competitive markets.

Let the vector of outputs y be such as to maximize the value of national income, py at price vector p, where the vector inner product is understood.. This is true for example of the competitive neoclassical economy. Then due to balanced trade, the trade equilibrium aggregate consumption vector x lies somewhere on a budget plane outside the autarkic production set. The diagram illustrates the potential difficulty with the comparative advantage proposition.

Potential Difficulty with Comparative Advantage Proposition



Comparative advantage characterizes the trade pattern if the value of the trade equilibrium aggregate trade bundle at the autarkic prices is greater than the value of the autarkic aggregate trade bundle at autarkic prices. This latter condition can be shown to be met due to household expenditure minimization and production efficiency (profit maximization).

By profit (national income) maximization, $p^a y \leq p^a y^a$. By individual household expenditure minimization, $p^a x \geq \Sigma_h e^h(p^a, u^h)$. Then

$$p^a(x - y) \geq \Sigma_h e^h(p^a, u^h) - p^a y^a. \tag{1}$$

The right hand side of (1) is equal to $\Sigma_h e^h(p^a, u^h) - \Sigma_h e^h(p^a, u^{ha})$. If the right hand side is weakly positive, we have proven comparative advantage, since by balanced trade $p(x-y)=0$ and hence if the right hand side of the inequality is positive, $(p-p^a)(x-y)\geq 0$.

It remains to show that $\Sigma_h e^h(p^a,u^h)-\Sigma_h e^h(p^a,u^{ha})\geq 0$. This follows from the gains from trade proof extended to the 'on average' sense needed for the aggregate economy. For each individual household, $e^h(p,u^{ha})\leq px^{ha}$. This implies that

$$\sum_h e^h(p,u^{ha})\leq px^a=py^a\leq py=\sum_h e^h(p,u^h). \tag{2}$$

This relationship holds for any price vector p and hence in particular for $p=p^a$. Taking the two endpoints of (2), we have shown that the right hand side above is indeed positive.

Notice that the expenditure functions are not restricted; non-normality and Giffen goods for some agents are admissible and no restriction on income distribution is used at all.

Notice also that the 'on average' gains from trade proof used above does not imply gains for each household.