

# How Should A Firm Go Public? A Dynamic Model of the Choice Between Fixed-price Offerings and Auctions in IPOs and Privatizations

Thomas J Chemmanur\*

and

Mark H. Liu\*\*

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\*Carroll School of Management, Boston College, MA02467. Phone: (617) 552 3980. Fax: (617) 552 0431.

E-mail: chemmanu@bc.edu

\*\*Carroll School of Management, Boston College, MA02467. Phone: (617)552 2023. Fax: (617)552 0431.

E-mail: liuhu@bc.edu

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# How Should A Firm Go Public? A Dynamic Model of the Choice Between Fixed-price Offerings and Auctions in IPOs and Privatizations

## Abstract

We develop a theoretical analysis of the choice of firms between fixed-price offerings and uniform-price auctions for selling shares in IPOs and privatizations. We consider a setting in which a firm goes public by selling a fraction of its equity in an IPO market where insiders have private information about intrinsic firm value. Outsiders can, however, produce information at a cost about the firm before bidding for shares. Firm insiders care about the extent of information production by outsiders, since this information will be reflected in the secondary market price, giving a higher secondary market price for higher intrinsic-value firms. We show that auctions and fixed-price offerings have different properties in terms of inducing information production. Thus, in many situations, firms prefer to go public using fixed-price offerings rather than IPO auctions in equilibrium. We relate the equilibrium choice between fixed-price offerings and IPO auctions to various characteristics of the firm going public. Unlike the existing literature, our model is able to explain not only the widely-documented empirical finding that underpricing is lower in IPO auctions than in fixed-price offerings, but also the fact that, despite this, auctions are losing market share around the world. Our model thus suggests a resolution to the above “IPO auction puzzle,” and indicates how current IPO auction mechanisms can be reformed to become more competitive with fixed-price offerings. Our results also provide various other hypotheses for further empirical research.

JEL Classification Code: G30, G32, C72, D44, D82

# 1 Introduction

Many new issues (i.e., initial public offerings, or IPOs) of equity are sold using a fixed-price offering mechanism, where the firm going public sets a fixed-price for the equity, in consultation with the investment bank taking it public.<sup>1</sup> However, many have argued recently, based on results from the economic theory of auctions, that the best way to sell stock in IPOs is to conduct an auction of the shares of the company going public.<sup>2</sup> Indeed, an investment banking firm, W.R. Hambrecht & Co., has been founded with the explicit objective of selling IPO shares using a Dutch (or more precisely, Vickrey) auction. Unfortunately for W.R. Hambrecht, the post-IPO stock price performance of these IPO auctions has been less than stellar, in the sense that the stock of those companies that have followed the auction method have languished badly subsequent to the IPO, and only a few companies have chosen to auction shares in their IPOs in the U.S. Further, the auction method of selling IPOs, far from gaining in popularity and replacing fixed-price offerings, has been losing market share worldwide, and is increasingly being replaced either by the fixed-price or the book-building mechanism even in those countries where it was in place.<sup>3</sup>

The fact that IPO auctions, while theoretically optimal in terms of maximizing proceeds from the IPO (and empirically documented as involving a smaller amount of underpricing), have been losing market share to fixed-price offerings or other non-auction mechanisms has been characterized by several authors as a puzzle.<sup>4</sup> The objective of this paper is to develop a resolution to this “IPO auction puzzle,” based on a theoretical analysis

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<sup>1</sup> The three IPO mechanisms commonly used around the world are fixed-price offerings, bookbuilding, and IPO auctions. Of these, the two non-auction mechanisms (fixed-price offerings and bookbuilding) are currently much more prevalent than IPO auctions. Some examples of countries where fixed-price offerings are (or have been) used are Australia, Belgium, Finland, Germany, Hong Kong, Italy, Japan (pre-April 1, 1989), Korea (post-June 1998), Singapore, Sweden, Switzerland, Taiwan, the U.S. (best-efforts contracts), and the U.K. (offers for sale). While the primary focus of this paper is on firms’ choice between fixed-price offerings and IPO auctions, our analysis will have some implications for the choice between bookbuilding and IPO auctions as well.

<sup>2</sup> The argument that is often made in favor of IPO auctions is often empirical as well as theoretical. See, for example, Ausubel and Cramton (1998): “...in the case of initial public offerings of corporate stock, the magnitude of underpricing under current American practice appears to be vastly larger than necessary. The substantial underpricing is indicative of a badly-performing mechanism for selling new issues...why are IPOs not done instead by an efficient auction?” We show in this paper that, in many situations, fixed-price offerings are better for issuers than IPO auctions, even though such offerings may be underpriced to a greater extent. However, we also characterize situations where auctioning shares in IPOs is indeed optimal.

<sup>3</sup> At one time or another, IPO auctions have been used in Belgium, Brazil, Chile, France, Hong Kong, Israel, Japan, Korea, Portugal, Singapore, Switzerland, Taiwan, and the U.K. They have fallen out of use in many of these countries.

<sup>4</sup> For example, Jenkinson and Ljungqvist (1996) comment: “Auction-like mechanisms such as tenders in the United Kingdom, the Netherlands, and Belgium, or *offres publiques de vente* in France, are generally associated with low levels of underpricing; most Chilean IPOs have also used auctions, and have been modestly underpriced, at least by emerging-markets standards. This is not surprising, given that, unlike fixed-price offers, tenders allow market demand to at least partially influence the issue price. What is curious, though, is that we do not observe a shift towards greater use of auctions.” Derrien and Womack (2003) make similar comments.

rooted in the realities of the IPO market. We argue that there are two problems with the argument that auctions maximize the proceeds from IPOs and therefore are the optimal way of selling shares in IPOs. First, it is based on results from auction theory developed in the context of a monopolist auctioning off various goods in the product market. However, unlike in the case of a monopolist trying to maximize the proceeds from a one-time sale of various items, the objective of a firm in selling shares is *not* to maximize the proceeds from a one-time sale of stock. This is because companies care very much about the price of their stock in the secondary market (one reason why they care about the secondary market price of their stock is that many companies wish to issue more equity two or three years down the road from an IPO; also, if the stock price continues to languish, they can be subject to a takeover at bargain basement prices). Thus, in practice, companies face a dynamic choice: they want to obtain high proceeds from the sale of stock, but they also care about the secondary market price of their stock after the IPO.

The second problem with existing arguments about the optimality of auctions is that they take the information structure of the problem as given. In other words, in much of auction theory, the information that various bidders have about the value of the object being sold is taken as unalterable, and the focus is often on comparing auctions in terms of their ability to extract and aggregate the information available with outsiders into the selling price. However, in many auction situations, bidders can produce information about the true value of the object being sold at a cost. For example, when the government is auctioning off rights to drill for oil or other mineral rights, various participants can spend resources to learn more about the value of the mineral rights (by drilling a test hole in the case of oil rights). In particular, investors in the new issues market can devote time and other resources to learn more about the true value of the firm going public. This is important because different ways of selling various objects have different properties when it comes to inducing information production by outsiders. In particular, we show here that in many situations, a fixed-price offering can induce more investors to learn about the true value of a firm going public compared to an IPO share auction, with implications for the proceeds obtained by the firm and its insiders from these two mechanisms.

Combining the above two ingredients, we show that, in many cases, a company that wishes to maximize a dynamic objective function (i.e., maximize the cash flow to the firm in the long run, rather than the proceeds from a one-time offering of stock) will in fact choose a fixed-price offering rather than an auction. We consider a

setting in which a firm goes public by selling a fraction of its equity in an IPO market characterized by asymmetric information between firm insiders and outsiders. Outsiders, can, however, produce information at a cost before bidding for shares in the IPO. Auctioning off shares in a setting where outsiders can learn more about the company at a cost will maximize the proceeds from a one-shot offering, but may be detrimental to the company's long-run value, since not enough investors will choose to produce information about the company. Insiders care about getting a large number of outsiders to produce information, since this information will be reflected in the secondary market price (thereby leading to a higher secondary market price for truly higher intrinsic-value firms). Thus, in equilibrium, truly higher valued firms will prefer to sell their shares in a fixed-price offering (rather than auctioning them off) because the former is the mechanism that will maximize the long-term value of their firm. Since lower intrinsic value firms will also mimic higher intrinsic value firms by setting the same offer price, this price will be such that it induces the optimal extent of information production by outsiders.

There are two important differences between the initial offer price emerging from an IPO auction and the fixed offer price set by a firm in an IPO.<sup>5</sup> First, the price at which shares are sold in the IPO auction is determined as a result of competition among various informed bidders. This means that the initial offer price in the auction will aggregate, to a significant degree, the information produced by outsiders, unlike in the case of a fixed-price offering, where the offer price is set by the firm. Second, in common value auctions (such as IPO share auctions), bidders, whose information will be correlated with the true value of the firm (and therefore with that of each other), will compete away much of the surplus from each other. Since each bidder expects to be compensated for the cost of producing information, this means that the initial offer price emerging in an IPO auction will be able to support only a smaller number of informed entrants into the auction compared to the number of investors producing information in a fixed-price offering (where the firm can set the offering price to attract the optimal extent of information production by outsiders).

The above intuition is useful in understanding many of our results. First, if a firm is very well known or otherwise faces lower levels of information asymmetry prior to the IPO (so that outsiders' cost of information production is small), then our analysis implies that auctioning its equity is optimal, since the number of infor-

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<sup>5</sup> Throughout this paper, the auction we consider is a  $(k + 1)^{th}$  price auction, where, if the firm going public is selling  $k$  shares, the uniform price paid by all investors is equal to the  $(k + 1)^{th}$  highest bid.

mation producers even in an auction is large enough that the disadvantage of lower information production is offset by the greater price received by higher intrinsic value firms in the IPO. In contrast, if the firm is young, or small, or faces a greater extent of information asymmetry for any other reason (so that outsiders' information production costs are significant), then fixed-price offerings will be the equilibrium choice of the firm, since, in this case, considerations of inducing information production and their impact on the secondary market price become important. Similarly, if the fraction of equity sold by the firm in the IPO is relatively large, then IPO auctions are the equilibrium choice, since, in this case, secondary market considerations are relatively unimportant to firm insiders at the time of the IPO. If, in contrast, the firm is selling only a small fraction of its equity in the IPO (as in the case of many firms going public in the U.S.), then fixed-price offerings are again the equilibrium choice, since, in this case, firm insiders place relatively less weight on maximizing the proceeds from a one-shot equity offering, and more weight on the impact of the IPO mechanism on the secondary market price of its equity.

There is by now a substantial empirical literature comparing the properties of IPOs sold by auction and by fixed-price offerings, in various countries (when both mechanisms are available in the same country) or across countries (see, e.g., Derrien and Womack (2003), Jacquillat (1986), MacDonald and Jacquillat (1974), Jenkinson and Mayer (1988), Kaneko and Pettway (2003), Aggarwal, Leal, and Hernandez (1993), Celis and Maturana (1998), and Kandel, Sarig, and Wohl (1999)). Much of this literature documents that the extent of IPO underpricing is much lower in IPO auctions compared to non-auction IPO mechanisms (i.e., either fixed-price offerings or bookbuilding). Our model can explain this empirical finding. We show that it is optimal for younger, smaller, or more obscure firms, which face a significant extent of information asymmetry in the equity market, to use fixed-price offerings and underprice significantly in their IPO, since they are concerned about inducing the optimal amount of information production. In contrast, older, larger, or more well-known firms, facing a smaller extent of information asymmetry, optimally use IPO auctions, and have a lower extent of underpricing in equilibrium, since they are less concerned about inducing information production by outsiders. Since the bulk of firms going public around the world are smaller, younger, and relatively obscure, our model is able to simultaneously explain the greater popularity of non-auction IPO mechanisms like fixed-price offerings relative to IPO auctions, and the greater extent of underpricing characterizing those mechanisms relative to that in IPO auctions. In addition to explaining these and other regularities documented by the empirical literature, our model also generates as yet

untested predictions useful for further empirical research (see section 5 for a detailed discussion).

The approach we take here differs in two important respects from that in the book-building literature. Following the seminal paper of Benveniste and Spindt (1989), a number of papers in this literature (see, e.g., Benveniste and Wilhelm (1990)) assume that outside institutional investors have information superior to the firm (and its underwriters), and demonstrate that underpricing is part of the optimal mechanism to induce truth-telling by institutional investors about their own valuation of the firm going public.<sup>6</sup> In contrast to the above literature, our assumption here is that it is insiders who have information superior to outsiders about their firm's true value.<sup>7</sup> Our view is that, while outsiders may indeed have private information about their own valuation of a certain firm going public (and therefore about their demand schedule for its equity), it is firm insiders who are most likely to have superior information about the intrinsic (long-term) value of their own firm. However, the two assumptions discussed above are complementary, in the sense that both kinds of information problems may exist simultaneously in practice.

Real-world IPOs have to accomplish two different information flows: First, insiders' private information regarding the intrinsic value of the firm has to be conveyed credibly to outsiders. Second, information regarding outsiders' valuation of the firm (and therefore their demand for the firm's equity) needs to be credibly extracted from these outsiders, so that this information can be used for pricing equity in the IPO. The theoretical book-building literature has focused exclusively on the second information flow, ignoring the first. In contrast, we focus on the first information flow, abstracting away from the second. Both non-auction mechanisms used in practice around the world, namely, fixed-price offerings and bookbuilding, accomplish the above two information flows to a greater or lesser degree, even though bookbuilding has an advantage over fixed-price offerings in terms of accomplishing the second information flow.<sup>8</sup> Consequently, while the focus of this paper is on firms' choice between

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<sup>6</sup> Some other papers in this literature are Maksimovic and Pichler (2001), Parlour and Rajan (2002), and Sherman and Titman (2000). See also Spatt and Srivastava (1991), who show that a posted-price mechanism augmented by preplay communication and participation restrictions, can replicate any optimal auction.

<sup>7</sup> This assumption of firm insiders with private information is consistent with the large literature on IPO underpricing which is not driven by book-building considerations (see, e.g., Allen and Faulhaber (1989), Welch (1989), and Chemmanur (1993)), as well as almost the entire non-IPO literature in corporate finance dealing with private information in various other settings (see, e.g., Myers and Majluf (1984) and Leland and Pyle (1977) on equity issues, Miller and Rock (1985) on dividend policy, or Ross (1977) on capital structure policy). In our setting, information production by outsiders does not give them an informational advantage over firm insiders: it simply brings the precision of their information closer to that of insiders, thereby reducing (and in some cases, eliminating) their informational disadvantage with respect to insiders.

<sup>8</sup> To see that real-world IPOs (whether book-building or fixed-price offerings) place considerable importance on conveying firm-specific information to outsiders, one need only to look at one of the most important activities associated with IPOs, namely, the

fixed-price offerings and IPO auctions, our model also has some implications for the choice between bookbuilding and IPO auctions as well (to the extent that firms using bookbuilding IPOs are also concerned about ensuring that outsiders are able to obtain some of firm-specific information initially available only to insiders).

A second important difference between our approach and that in the book-building literature is that, in the latter, the objective of firm insiders is simply to maximize the proceeds from a one-shot equity offering. This means that, in the book-building literature, underpricing is a cost imposed on the firm because of the presence of informed outsiders so that an important measure of the success of the IPO equity sales mechanism in the above setting is its ability to minimize underpricing. This has important consequences for the ability of papers in this literature to explain the IPO auction puzzle. For example, Sherman (2002) documents IPO mechanisms used around the world, and argues that bookbuilding is superior to IPO auctions because it gives the issuer more discretion in terms of underpricing and allocating shares (and therefore a greater ability to provide incentives to outsiders to collect information about their valuation of the firm's shares) compared to IPO auctions. However, this paper makes the prediction that bookbuilding will lead to greater IPO proceeds (and therefore lower underpricing) than IPO auctions (see proposition 4). Consequently, it cannot provide a resolution to the IPO auction puzzle, since it is unable to explain the greater underpricing observed in practice in the bookbuilding method relative to that in IPO auctions.<sup>9</sup> This is also the case for other papers in this literature: see, e.g., Biais and Faugerson-Crouzet (2002).<sup>10</sup>

In contrast to the above literature, in our setting, the insiders' goal in pricing equity in the IPO is to maximize

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“road-show.” It is well-known that an important objective of roadshows is for insiders (top corporate officers, accompanied by underwriters) to explain details about the firm to institutional investors and other outsiders, thereby making it easier (cheaper) for these outsiders to acquire firm-specific information (“understand the firm and its business”). To give one example, the failure of the bookbuilding IPO of the Medco health pharmacy-benefits subsidiary of Merck was blamed by investors on the company's top officers' inability to explain its business properly to investors during the roadshow. To quote the Wall Street Journal story on this (*WSJ*, April 23, 2003): “Merck bought Medco for \$6.6 billion in 1993. When the company first began to market the Medco IPO in the spring of 2002, many analysts estimated the unit's value at about \$10 billion. But...an inability of its management to explain how exactly Medco makes its money began to push down what investors were willing to pay for the new stock. The IPO was delayed twice before it was scuttled.”

<sup>9</sup> As discussed before, the empirical literature has documented that non-auction IPOs in equity markets around the world yield significantly greater underpricing than IPOs that are auctioned. This implies that, if minimizing underpricing were the objective of the firm, auctioning shares in IPOs would be the right thing to do.

<sup>10</sup> Biais and Faugerson-Crouzet (2002) compare fixed-price offerings, market-clearing uniform-price auctions, and the *Mise en Vente* (an auction procedure used in France) in a setting where outsiders have private information about their demand for the firm's shares and the objective of the firm is to maximize IPO proceeds. In an analysis along the lines of Wilson (1979) and Back and Zender (1993), they argue that uniform-price auctions may not be optimal for selling shares if auction participants are asked to submit their entire demand functions, since bidders can tacitly collude by submitting demand functions such that the clearing price is very low. In contrast, in the *Mise en Vente* (which has some similarities to book-building methods), the price underreacts to demand and thereby unravels tacit collusion on low prices. See also Biais, Bossearts, and Rochet (2002), who argue that uniform-price offerings may indeed be optimal if the underwriter has private information about the demand for IPO shares, institutional investors have private information about share value, and the underwriter and institutional investors are able to collude.

their dynamic objective function, and minimizing underpricing is not the objective (recall that maximizing IPO proceeds is equivalent to minimizing IPO underpricing), though, in some cases, the mechanism that maximizes the insiders' dynamic objective may also happen to be the one that minimizes underpricing. This means that our model is able to explain much more of the empirical evidence comparing auctions with fixed-price offerings, since, in our setting, firms may adopt fixed-price offerings even in situations where they involve greater underpricing. Further, our paper also has implications for the relationship between the IPO mechanism used and the secondary market price following the IPO, since (unlike the other papers cited above) we also model the relationship between the IPO offer price and the secondary market price.<sup>11</sup> Finally, rather than arguing that one or the other method is the always the optimal mechanism, the focus of this paper is on characterizing the situations under which either fixed-price offerings or auctions are optimal for a given firm's IPO or in the privatization of a particular government-owned firm.<sup>12 13 14</sup>

The rest of this paper is structured as follows. Section 2 describes the basic model, while section 3 characterizes the equilibrium of the model and develops results. Section 4 develops two extensions of the basic model: section 4.1 allows the firm to choose between IPO auctions and fixed-price offerings in a setting where the fraction of

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<sup>11</sup> There is some anecdotal evidence that, in practice, the IPO mechanism chosen affects the secondary market price of the firm going public. Consider, for example, the following quote by Reed Foster, Co-founder of Ravenswood Winery, a firm which went public using an IPO auction conducted by the *OpenIPO* website of W.R. Hambrecht & Co: "If you put our secondary market stock price after the IPO to a medical chart, you would only see a horizontal line. We had the EKG of a potato." Other firms which have used the *OpenIPO* website also seem to have had a similar experience in terms of post-IPO secondary market price. The above behavior of the secondary market price is consistent with our model, which implies that a smaller amount of firm-specific information will be reflected in the secondary market price of firms using IPO auctions, leading to lower volatility in the opening secondary market price (see also empirical implication (ii) in section 5).

<sup>12</sup> In contemporaneous work, Bierbaum and Grimm (2002) analyze the problem of selling shares in a divisible good to a large number of buyers when demand is uncertain, in a setting without information production. They compare a fixed-price mechanism and a uniform-price private value auction and show that while truthful bidding is the dominant strategy in the auction, bidders have an incentive to overstate their demand in the fixed-price mechanism. Further, for some parameter values, the fixed-price mechanism outperforms the auction in terms of maximizing one-shot sales proceeds.

<sup>13</sup> Such an analysis of the settings under which each mechanism is optimal has become particularly crucial in the light of the recent innovations in information technology (e.g., the internet) in the U.S. and other countries, making it very easy and inexpensive to conduct on-line auctions of shares (as is evidenced, for instance, by the advent of the "*OpenIPO*" website of W.R. Hambrecht & Co). However, whether such IPO share auctions will indeed become prevalent will clearly depend upon how successful they are in meeting the requirements of firms going public. In section 5, we will also discuss the implications of our analysis for reforms of the IPO auction process.

<sup>14</sup> While our primary goal here is to develop an analysis of the relative merits of auctions and fixed price offerings in selling equity in IPOs, this paper also makes contributions outside financial economics. First, our paper makes a contribution to auction theory by endogenizing the information structure in auctions, unlike much of the auction literature that takes the information available to various participants in an auction as given (exceptions to this include Milgrom (1981), Matthews (1977), Persico (2000), Gair (1997), and Hausch and Li (1993)). Second, our paper makes a contribution to the industrial organization literature on optimal procurement mechanisms by comparing auctions with fixed-price offerings in an environment of information production. Thus, our model has implications for weapons procurement by the U.S. Department of Defense (DOD). Our analysis implies that, if the DOD is not merely concerned about minimizing costs, but also with promoting R&D (and firms perform R&D to compete better in the bidding process), awarding weapons contracts based on a pre-determined fixed payment (rather than auctioning off these contracts) may be optimal in many situations.

equity sold by the firm is endogenous; section 4.2 allows the issuing firm to make the same choice between a fixed-price offering and an IPO auction with an endogenous reservation price (no reservation price is set by the issuer in the IPO auction in the basic model). Section 5 describes the empirical and policy implications of our model. Section 6 concludes the paper. The proofs of all propositions are in the appendix.

## 2 The Basic Model

There are three dates in this model. At time 0, a private firm goes public by selling a proportion  $\alpha \in (0, 1)$  of its equity in an IPO, using either a fixed-price offering or a  $(k + 1)^{th}$ -price auction.<sup>15</sup> <sup>16</sup> Outside investors then decide whether to produce information about the value of the issuing firm, and whether to bid in the firm's IPO. At time 1, the issuing firm's stock is traded in the secondary market. The firm sells the remaining fraction  $1 - \alpha$  of its equity to outsiders in a seasoned equity offering at the price prevailing in the secondary market.<sup>17</sup> At time 2, cash flows are realized and distributed to shareholders.

### 2.1 The Issuing Firm's Private Information and IPO Mechanisms

The issuing firm, which is risk-neutral, may be either good (type  $G$ ) or bad (type  $B$ ). The present value of cash flows of a good firm is  $v = v_G$ , and that of a bad firm is  $v = v_B$ , where  $v_G > v_B$ . We assume that the type G firm has a positive net present value project; the net present value of the type B firm's project may be positive, zero, or negative.<sup>18</sup> For simplicity, we normalize  $v_G$  to 1, and  $v_B$  to 0.<sup>19</sup> While the issuing firm knows its own type,

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<sup>15</sup> We choose to compare the fixed-price offering with the  $(k + 1)^{th}$ -price sealed-bid auction (uniform price auction) because it is by far the most widely used form of IPO auction in practice. For example, this kind of IPO auction is used in Israel, France, and also by an investment banking firm, W. R. Hambrecht, in the U.S. Searching for the optimal auction type is beyond the scope of this paper.

<sup>16</sup> We do not assume a particular motivation for the firm to go public. Thus, the objective of the IPO may either be for the firm to raise new capital for future investment, or for the entrepreneur to diversify his equity holdings in the firm. For a model of the timing and motivation of a firm's going public decision, see Chemmanur and Fulghieri (1999).

<sup>17</sup> The assumption of a seasoned equity offering is made only for concreteness. Welch (1989) documents that in the 1977-1982 period, 288 out of 1028 IPO firms made a total of 395 seasoned offerings, and the average proceeds from the seasoned equity offerings are three times their average IPO proceeds. Even in the absence of a seasoned equity offering, our results go through qualitatively as long as firm insiders place some weight on the secondary market price in their objective function, which seems to be the case in practice. The assumption that insiders care about the secondary market price is also made in Allen and Faulhaber (1989), Chemmanur (1993), and Welch (1989).

<sup>18</sup> While we require the net present value of the type G firm to be positive (so that, if a type G firm is going public to raise capital for its new project, it has the incentive to fund the project), the net present value of the type B firm's project is irrelevant in our setting. Given that a type G firm is selling equity in an IPO, the type B firm (entrepreneur) can benefit from pooling with it and also sell equity, thus maximizing the entrepreneur's payoff (regardless of whether its project is positive or negative NPV).

<sup>19</sup> Further, the normalization of the value of the type G firm to 1 and that of the type B firm to 0 is only meant to keep the mathematical complexity of our model to a minimum: our results remain qualitatively the same even in the absence of the above normalization. For example, it can be shown that the type B firm will choose to pool with the type G in equilibrium even when the type B firm has a positive value ( $v_B > 0$ ): see the discussion in footnote 34 in this regard.

outside investors observe only the prior probability  $\theta$  of a firm being of type G. The equity offered in the IPO is divided into  $k$  shares. We assume that each investor in the IPO is allowed to bid for a maximum of one share.<sup>20</sup>

The issuing firm can choose one of the following two IPO mechanisms: a fixed-price offering or a  $(k + 1)^{th}$  price auction. If the issuing firm chooses a fixed-price offering, it sets an offering price  $F$  per share, and all buyers pay this price. If the total demand is higher than  $k$  shares in the fixed-price offering, there will be rationing of shares, and the  $k$  shares will be allocated to bidders randomly, with each bidder having an equal probability of being allocated one share.<sup>21</sup> In the case when the total demand is less than  $k$  shares, the IPO fails.<sup>22</sup>

In the case when the issuing firm chooses to auction its shares, the shares are allocated as follows. Investors simultaneously submit sealed bids for shares. The  $k$  highest bidders are each allocated one share, and pay a uniform price, which is the  $(k + 1)^{th}$  highest bid (we will refer to this price as the clearing price). If there is a tie, so that there are more than  $k$  bidders above the clearing price, all investors bidding strictly above the clearing price are allocated one share with probability 1, with the remaining shares allocated with equal probability to those who bid at the clearing price. For example, suppose there are 2 shares for auction, and there are 4 bidders, who bid 0.9, 0.8, 0.8, 0.7, respectively. Then the offering price will be 0.8, with the bidder who bids 0.9 being allocated one share, and the two bidders who bid 0.8 having a 50% chance of being allocated one share.<sup>23</sup>

The objective of the issuing firm is to maximize the combined proceeds from the sale of equity in the IPO and in the seasoned equity offering. The issuing firm's choice of IPO mechanism will affect the amount of information production about the firm, which will in turn determine the secondary market price (and therefore the price at which equity can be sold in the seasoned equity offering). In this sense, the IPO mechanism determines both the proceeds from the IPO and the that from the seasoned equity offering. Hence the issuing firm will choose the IPO

<sup>20</sup> Since investors are risk-neutral, if they are allowed to buy at most one share, they will bid for either one share or nothing. So this assumption is equivalent to assuming that every participant is allowed to bid for either one share or nothing. Further, the assumption that each investor is allowed to bid for a maximum of one share is not unduly restrictive, since, for  $k$  small enough, this can translate into a significant fraction of the firm's equity.

<sup>21</sup> Our results are unchanged if we make the assumption that in the event that the demand for shares exceeds supply in a fixed-price offering, all investors will be allocated fractions of shares on a pro rata basis. However, we have chosen to make the assumption of rationing, since this is the allocation rule for IPO shares followed in practice.

<sup>22</sup> We will see later that, in the basic model, the IPO of neither firm type fails in equilibrium when the firm chooses an IPO auction. Even when the firm chooses a fixed-price offering, the type G firm's IPO never fails in equilibrium; only a type B firm's IPO has a positive probability of failure.

<sup>23</sup> Our results will be exactly the same if we make the alternative assumption that in the case of a tie, all those who bid at the clearing price will be allocated equal fractions of a share. In this case, the two investors who bid 0.8 will be allocated 0.5 share each in the above example.

mechanism (and the offering price in the case of a fixed-price offering) which maximizes its combined proceeds.

## 2.2 The Investors' Information Production Technology

There are a large number of risk-neutral investors in the market, who do not know the true type of the firm, but have a prior belief that the firm is of type  $G$  with probability  $\theta$ , and of type  $B$  with the complementary probability, i.e.,

$$Pr(v = 1) = \theta, \quad Pr(v = 0) = 1 - \theta. \quad (1)$$

In addition to the equity of the issuing firm, there is a risk-free asset in the economy, the net return on which is normalized to 0.

After the issuing firm chooses its IPO mechanism (auction versus fixed-price offering, and the offering price in the latter case), outside investors make one of the following three choices based on their prior valuation of the firm and other parameters (e.g., information production cost  $C$ ): engage in uninformed bidding, produce information about the firm and then decide how to bid, or ignore the IPO and invest in the riskless asset. In the case of an IPO auction, we can show that uninformed bidding is always dominated by informed bidding.<sup>24</sup> In the case of a fixed-price offering, if sufficiently precise information is available to investors at a low enough cost, and the offering price is not too low, informed bidding always dominates uninformed bidding. In order to focus on issues related to information production, we assume that the model parameters are such that only informed bidding occurs in the fixed-price offering as well.<sup>25</sup>

If an investor chooses to produce information about the issuing firm, he has to pay an information production cost  $C$ , and will receive a signal about the type of the issuing firm. We assume that each information producer receives a signal, which can be high ( $H$ ), medium ( $M$ ), or low ( $L$ ), with the following probabilities:

$$Prob(S_i = H|v = 1) = Prob(S_i = L|v = 0) = p; \quad Prob(S_i = M|v = 1) = Prob(S_i = M|v = 0) = 1 - p, \quad (2)$$

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<sup>24</sup> We will show later that a bidder who produces information and receives a signal  $M$  (which does not allow the investor to distinguish between type  $G$  and type  $B$  firms) has an expected payoff of zero. The payoff to an investor who bids without producing information is always less than that to a bidder who produces information and receives a signal  $M$ . This is because, in an equilibrium where there are  $n$  bidders, an investor who bids after receiving a signal  $M$  is bidding against only  $(n-1)$  investors who have a potentially more informative signal than him. In contrast, an investor who bids without producing information is bidding against  $n$  investors who have a potentially more informative signal. Thus, an uninformed bidder faces more adverse selection than an investor who receives a signal  $M$  and bids.

<sup>25</sup> This assumption translates into a parametric restriction (available to interested readers) on the fraction of equity sold in the IPO,  $\alpha$ , and the cost of information production to outsiders.

where  $p \in (0, 1)$  is the probability that a signal reveals the true value of the issuing firm. The signals received by different information producers are independent of each other. After receiving the above private signal, each information producer decides whether to bid for one share or not (in the case of a fixed-price offering) or how much to bid (in the case of an IPO auction), using Bayes' rule where appropriate.

### 3 Market Equilibrium

*Definition of Equilibrium:* An equilibrium consists of (i) a choice of IPO mechanism by the issuing firm (conditional on its type) at time 0 (between fixed-price offering and IPO auction), and the offering price in the case of a fixed-price offering; (ii) a system of beliefs formed by investors about the type of the issuing firm after observing the issuer's IPO choice; (iii) a choice made by each investor whether to produce information after seeing the choice of the issuing firm in the IPO; (iv) a decision of whether to bid for one share or not (in the case of a fixed-price offering) or how much to bid (in the case of an IPO auction) made by each information producer after observing a private signal about the type of the firm; (v) a price at which the stock of the issuing firm is traded in the secondary market. The above set of prices, choices, and beliefs must be such that: (a) the choice of each party maximizes his objective, given the choices and beliefs of others and the expected secondary market price; (b) the beliefs of all parties are consistent with the equilibrium choices of others; further, along the equilibrium path, these beliefs are formed using Bayes' rule; (c) the number of investors producing information is the largest integer such that all information producers obtain a non-negative expected payoff from information production;<sup>26</sup> (d) the secondary market price of the firm's shares is such that no investor can profit from trading after observing the price; (e) any deviation from his equilibrium strategy by any party is met by beliefs by other parties which yield the deviating party a lower payoff compared to that obtained in equilibrium.

Given the rich strategy space for firms and investors in our model, we can think of three broad categories of equilibria that may exist (depending on parameter values): (i) Separating equilibria, where the type G and type B firms behave differently in equilibrium, thus revealing their types; (ii) Pooling equilibria, where the type B firm

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<sup>26</sup> The value of  $n$  at which the expected payoff from producing information equals the cost of doing so in an IPO auction (so that (4) holds as an equality) may not be an integer. However, the number of information producers has to be an integer so that various functions in our model are well defined. Therefore, for tractability, we will require the equilibrium number of information producers in the IPO auction as well as in fixed-price offerings to be the largest integer value for which all information producers obtain a nonnegative expected payoff.

always attempts to mimic the equilibrium choice made by the type G firm; (iii) Partial pooling equilibria, where the type B firm mimics the type G firm with some probability, while separating with the remaining probability. However, for a wide range of model parameter values (which we study here), it can be shown that neither separating nor partial pooling equilibria exist in our setting (for the reasons discussed in footnote 34). Therefore, we focus only on pooling equilibria, where the type B firm always pools with the type G.

To facilitate exposition, we discuss the model in reverse order. We discuss the equilibrium in the secondary market before going on to the equilibrium in the IPO market.

### 3.1 Equilibrium in the Secondary Market

We assume that there is no restriction on how many shares an investor can buy or short in the secondary market. Note that the equilibrium secondary market price is dependent on the actual IPO mechanism (fixed-price offering versus auction) only to the extent that this affects the number of information producers about the firm. In other words, if the number of information producers is the same under the two IPO mechanisms, the expected secondary market price will be the same. At time 0, when the firm chooses between a fixed-price offering and an IPO auction, the actual realization of the signals obtained by outsiders is not known to insiders. Therefore, it is the expected secondary market price that enters the insiders' objective function. Proposition 1 gives the expected equilibrium secondary market price for a type G firm, a type B firm, and for firms across types, as a function of the number of information producers,  $n$ .

**Proposition 1 (*Equilibrium Price in the Secondary Market*)**

- (i) *The price in the secondary market aggregates all the information obtained by outsiders in the IPO.*
- (ii) *The expected secondary market price of a type G firm, conditional on insider's information at time 0, is given by  $1 - (1 - \theta)(1 - p)^n$ . This price is increasing in the number of information producers,  $n$ .*
- (iii) *The expected secondary market price of a type B firm, conditional on insider's information at time 0, is given by  $\theta(1 - p)^n$ . This price is decreasing in the number of information producers,  $n$ .*
- (iv) *The expected secondary market price of the issuing firm (across types) is  $\theta$ , which is independent of the number of information producers,  $n$ .*

The secondary market price will reflect all the information obtained by participants in the IPO. The reason is as follows. Since there is no limit on how many shares an investor could buy or short in the secondary market, and investors are risk-neutral, if an investor finds that the secondary market price is inconsistent with the signal he receives, he will keep trading until the private information is reflected in the price.

Given the information production technology of investors, the information (private signals) held by information producers could be one of the following three cases: (i) at least one signal is  $H$ ; (ii) at least one signal is  $L$ ; (iii) all signals are  $M$ . In case (i), the secondary market price must be 1. Since at least one information producer observes a signal  $H$  in the IPO, he knows that the firm is of type G. If the secondary market price is less than 1, he has an incentive to demand more shares and drive the price up. Similarly, in case (ii) the secondary market price will be 0, since, otherwise, there is an incentive for the investor who observes  $L$  to short shares. In case (iii), when all information producers receive a signal of  $M$ , the secondary market price will reflect this information and equal  $\theta$ .<sup>27</sup>

Since the price system here is fully revealing (i.e., the secondary market price incorporates all the information produced by outsiders), outsiders do not have the incentive to engage in information production at time 1. This is because, while the costs of information production are privately incurred, the benefits no longer accrue to individual outsiders. To illustrate, consider the case where the secondary market price is  $\theta$  (i.e., as in case (iii) discussed above).<sup>28</sup> Suppose an investor incurs the information production cost  $C$  at time 1, and obtains a signal  $H$ . In order to profit from this information, he has to buy equity at this time. However, no other investor will be willing to sell him any shares at a price  $\theta$ , since investors can infer his information from his demand function. A symmetric argument applies if the investor has a signal  $L$ . Thus no investor has the incentive to produce information in the secondary market.<sup>29 30</sup>

Part (ii) gives the expected secondary market price for the type G firm, and shows that it is increasing with the number of information producers in the IPO. Part (iii) gives the expected secondary market price for the type B firm, and shows that it is decreasing with the number of information producers in the IPO. Part (iv)

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<sup>27</sup> The formal proof that the secondary market price is  $\theta$  when all the signals are  $M$  is given in the proof of Proposition 1 in the appendix.

<sup>28</sup> If the secondary market price is 1 or 0, the true type of the firm is already revealed, so that there is no need for further information production.

<sup>29</sup> In practice, the price system may be only partially revealing (perhaps due to additional uncertainty in the economy not modeled here). The equilibrium in the secondary market may then be a noisy rational expectations equilibrium. The intuition behind our model holds even in this case, since we merely require that outsiders' incentives to produce information diminish after the start of trading in the equity in the secondary market. See Grossman (1976), Hellwig (1980), and Diamond and Verrecchia (1981) for a discussion of the reduction in investors' incentives to produce information under alternative assumptions about the degree of noise in prices.

<sup>30</sup> Consistent with this, there is considerable evidence that a large majority of small firms attract very little analyst coverage subsequent to their IPOs. Further, Rajan and Servaes (1997) document that the extent of analyst coverage following the IPO is increasing in IPO underpricing.

demonstrates that the average secondary market price of the firm across types is the prior average value of the issuing firm, and independent of the number of information producers in the IPO. Since information production only separates the pool, it does not change the average secondary market value of the issuing firm.

### 3.2 Equilibrium in the IPO Market

We now discuss the equilibrium in the IPO market. We first discuss the case where the firm chooses to auction its shares in the IPO, and then discuss the case where the firm chooses to use a fixed-price offering.

#### 3.2.1 The Case Where the Issuing Firm Chooses an IPO Auction

In the case where the firm chooses to auction its shares in the IPO, the issuing firm does not need to set a price: the offering price is determined by the bids submitted by investors. Each outsider decides whether or not to enter the auction (produce information) based on his prior probability  $\theta$  of the firm being of type G (and other IPO parameters). If he chooses to produce information, each investor observes a private signal and bids according to it. Below, we characterize the situation where the type G firm and the type B firm pool together by choosing to auction their shares in the IPO (we will show later that this is indeed what happens in equilibrium). Each investor observes a private signal through the information production technology discussed before, and bids based on his updated value of the firm. The following proposition characterizes the equilibrium bidding strategies of investors in an IPO auction.

**Proposition 2** (*Equilibrium in an IPO Auction*)

- (i) *When the issuing firm uses an IPO auction and there are  $n \geq k + 1$  information producers, the equilibrium bidding strategy is as follows. Every bidder bids  $\frac{\alpha}{k}$  when he observes  $H$ , 0 when he observes  $L$ , and a random withdrawal  $b$  from the interval  $(0, \frac{\alpha}{k}]$  with cdf  $M(b; n)$  when he observes  $M$ , where  $M(b; n)$  is characterized by the following equation:*

$$\begin{aligned} & \theta \left( \frac{\alpha}{k} - b \right) [p + (1-p)(1-M(b))]^{k-1} [(1-p)M(b)]^{n-k-1} \\ = & (1-\theta)b[(1-p)(1-M(b))]^{k-1} [p + (1-p)M(b)]^{n-k-1}. \end{aligned} \quad (3)$$

- (ii) *The equilibrium number of information producers is the largest integer such that:*

$$\theta p \int_0^{\alpha/k} \left( \frac{\alpha}{k} - x \right) \binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1} dx \geq C, \quad (4)$$

where  $m(x)$  is the probability density function associated with  $M(x)$ , i.e.,  $m(x) = \frac{dM(x)}{dx}$ , and  $\binom{n-1}{1}$ ,  $\binom{n-2}{k-1}$  are binomial probabilities.

The true value of each share is either  $\frac{\alpha}{k}$  (type G firm) or 0 (type B firm). When an investor observes a signal  $H$ , he knows that the firm is of type G and will bid the true value of the firm, which is  $\frac{\alpha}{k}$ . Similarly, investors who observe  $L$  will infer that the firm is of type B and will bid 0. However, it is not an equilibrium for all investors who observe  $M$  to bid the expected value of the issuing firm conditional on a signal  $M$ , which is  $\theta$ , since they will face a “winner’s curse”: they will have a greater probability of receiving shares when the firm is of type B. Therefore, it can be shown that all investors who observe  $M$  will bid a random draw from the distribution  $M(b; n)$  in equilibrium.

The left hand side of equation (4) is the payoff to one bidder, say, bidder  $i$ . To understand this formula, first note that the payoff to bidder  $i$  is zero when he observes a signal of  $M$  or  $L$  (the proof is given in the appendix). The payoff is still 0 if he receives a signal  $H$  and the clearing price in the auction is  $\frac{\alpha}{k}$  (this happens when at least  $k + 1$  investors observe the signal  $H$ ).  $\theta p$  is the probability that bidder  $i$  observes a signal  $H$ . When the clearing price is  $x \in (0, \frac{\alpha}{k})$ , his payoff is  $(\frac{\alpha}{k} - x)$ , and  $\binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1}$  is the pdf of the clearing price being  $x$  conditional on bidder  $i$  observing  $H$  and the clearing price being less than  $\frac{\alpha}{k}$ . Thus, the expected payoff to each information producer is given by the L.H.S. of equation (4). It can be shown that, as more investors produce information, the expected payoff from producing information decreases (in other words, the L.H.S. of (4) is a decreasing function of  $n$ ). Thus, in equilibrium, the number of information producers will be the largest integer such that the expected payoff to each information producer is non-negative.

**Proposition 3 (Proceeds to the Issuing Firm from an IPO Auction):**

(i) When the issuing firm is of type G, its expected proceeds from the IPO auction is

$$E[IR^G(n)] = [1 - \sum_{j=0}^k \binom{n}{j} p^j (1-p)^{n-j}] \frac{\alpha}{k} k + k \sum_{j=0}^k \binom{n}{j} p^j (1-p)^{n-j} \int_0^{\alpha/k} x \binom{n-j}{1} m(x) \binom{n-j-1}{k-j} (1-M(x))^{k-j} M(x)^{n-k-1} dx; \quad (5)$$

and when the issuing firm is of type B, its expected proceeds from the IPO auction is

$$E[IR^B(n)] = k \int_0^{\alpha/k} x \binom{n}{1} (1-p)m(x) \binom{n-1}{k} [(1-p)(1-M(x))]^k [p + (1-p)M(x)]^{n-k-1} dx. \quad (6)$$

(ii) In the IPO auction, if the issuing firm is of type G, the expected secondary market price is higher than the expected offering price; if the issuing firm is of type B, the expected secondary market price is lower than the expected offering price. In other words, the secondary market price is more informative than the IPO price when an IPO auction is used.

When the issuing firm is of type G, there are two possibilities. With probability  $1 - \sum_{j=0}^k \binom{n}{j} p^j (1-p)^{n-j}$ , the number of information producers  $j$  who observe a signal H (and therefore bid  $\frac{\alpha}{k}$ ) will be greater than  $k$ . The offering price in this case will be  $\frac{\alpha}{k}$  per share. With probability  $\sum_{j=0}^k \binom{n}{j} p^j (1-p)^{n-j}$ , the number of information producers  $j$  who observe a signal H (and therefore bid  $\frac{\alpha}{k}$ ) will be less than or equal to  $k$ . In the latter case, the other  $(n-j)$  information producers will observe  $M$  and bid a withdrawal from the distribution  $M(b; n)$ , so that the offering price per share will be the  $(k+1-j)$ th highest signal from  $(n-j)$  signals withdrawn from  $M(b; n)$ . Thus the expected proceeds to the type G firm in the IPO is as specified in equation (5).

In order to understand equation (6), note that if the issuing firm is of type B, the proceeds from the IPO auction is 0 if at most  $k$  bidders observe the signal  $M$ . The probability density of the clearing price in the auction being  $x \in (0, \frac{\alpha}{k})$  is given by  $\binom{n}{1} (1-p)m(x) \binom{n-1}{k} [(1-p)(1-M(x))]^k [p + (1-p)M(x)]^{n-k-1}$ . The integration in equation (6) gives the expected clearing price conditional on the issuing firm being of type B. Since there are  $k$  shares, the expected proceeds from the IPO to the type B firm is given by the right hand side of equation (6).

Part (ii) of Proposition 3 demonstrates that the IPO auction price is less informative than the secondary market price. In the IPO auction, investors are not able to fully exploit their information signal, given that each investor can place a bid in the auction only once (in other words, no bid revision is allowed after observing the clearing price set in the auction). Thus, while there is some aggregation of information in the IPO auction, this information aggregation is incomplete, in the sense that the clearing price in IPO auction does not fully reflect the information available with all investors.<sup>31</sup> In contrast, secondary market price fully aggregates all information available with outsiders. As discussed in section 3.1, in the secondary market, all investors can fully exploit their private information by taking advantage of any deviation of the secondary market price from the expected value of each share conditional on their private information (over several rounds of trading, if necessary). In the following, we say that an issue is underpriced if the average initial offering price (across types) is lower than the average secondary market price (consistent with the empirical literature).

**Proposition 4 (*Underpricing in an IPO Auction*):** *There is underpricing on average if the issuing firm uses an IPO auction. The average percentage underpricing is given by  $\frac{n_a C}{\theta \alpha - n_a C}$ , where  $n_a$  is the equilibrium number of information producers in the IPO auction.*

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<sup>31</sup> The fact that the clearing price in an auction does not, in general, fully aggregate the information available with all participants (since the bids are submitted simultaneously, and bidders must make their bids in ignorance of the clearing price) has been noted (in a different context) by Milgrom (1981).

It is costly for bidders to produce information about the issuing firm, so that in equilibrium their payoff from bidding in the IPO auction must exactly cover the information production cost. In equilibrium, the aggregate information production cost of investors is  $n_a C$ , which is borne by the issuing firm through underpricing. The expected secondary market value of the equity sold in the IPO is  $\theta\alpha$ , so that the percentage underpricing is  $\frac{n_a C}{\theta\alpha - n_a C}$ .

### 3.2.2 The Case Where the Issuing Firm Chooses a Fixed-price Offering

We now characterize the situation where both the type G and the type B firm pool by issuing equity in the IPO using a fixed-price offering. In this case, the type G firm will set the IPO offering price  $F$  to maximize its combined proceeds from the IPO and the seasoned equity offering, taking into account the effect of the number of information producers in the IPO on the expected secondary market (seasoned equity offering) price. Below, we characterize the situation where the type G and the type B firm pool together using a fixed-price offering (we will show later that this is indeed what happens in equilibrium). In this case, the type G firm faces a trade-off when setting the optimal offering price  $F$ . On the one hand, a higher offering price means greater proceeds from the IPO; on the other hand, a higher offering price means less information producers in the IPO, and hence lower proceeds from the seasoned equity offering.

The objective of the type G firm is to

$$\begin{aligned} \underset{F}{Max} \quad & kF + (1 - \alpha)[1 - (1 - \theta)(1 - p)^n], \\ \text{s.t.} \quad & \pi(F, n) \geq C. \end{aligned} \tag{7}$$

The following proposition characterizes the equilibrium bidding strategies of investors in the fixed-price offering.

**Proposition 5 (Equilibrium in a Fixed-price Offering):**

- (i) Suppose there are  $n \geq k + 1$  information producers in the IPO, and the offering price satisfies  $F \leq \frac{\theta(1-p)}{\theta(1-p)+1-\theta} \frac{\alpha}{k}$ , then the equilibrium bidding strategies for information producers are: bid for one share if the signal is  $H$  or  $M$ , and do not bid if the signal is  $L$ .
- (ii) The equilibrium number of information producers in a fixed-price offering,  $n_f$ , is the largest integer such that:

$$\theta\left(\frac{\alpha}{k} - F\right)\frac{k}{n} - (1 - \theta)F \sum_{j=k-1}^{n-1} \binom{n-1}{j} (1-p)^j p^{n-1-j} \frac{k}{j+1} \geq C. \tag{8}$$

(iii) *The equilibrium number of information producers in a fixed-price offering is decreasing in the offering price, i.e.,  $\frac{\partial n_f}{\partial F} < 0$ .*

Part (i) demonstrates that when the offering price is not too high, every information producer will bid for one share when the signal is H or M, and not bid if the signal is L. In equilibrium, we can see that if the issuing firm is of type G, all  $n$  information producers will bid, since everyone's signal will be either  $H$  or  $M$ , and both will lead the information producer to bid for one share. Therefore, the type G firm's IPO never fails in equilibrium. However, if the issuing firm is of type B, it is possible that a large number of information producers receive the signal L and not bid, so that less than  $k$  investors bid for shares in the IPO. In this case, the possibility of IPO failure arises. We assume that if there are less than  $k$  information producers bidding for shares, the IPO fails and the type B firm will be liquidated in the secondary market at its true (full information) value of 0.<sup>32</sup> Since all the choices are made by type G firms, and type B firms only mimic, the possibility of type B firms' IPO failure does not affect the equilibrium in our model.<sup>33</sup>

The payoff to each information producer in the above equilibrium is as follows. When the issuing firm is of type G, all  $n$  information producers will bid, so that each information producer will be allocated one share with probability  $\frac{k}{n}$ . The payoff to each investor who is allocated one share will then be equal to  $(\frac{\alpha}{k} - F)$ . When the issuing firm is of type B, the total number of bidders depends on how many information producers have received the signal  $M$ . In this case, each information producer can receive either the signal either  $M$  or  $L$ . When he receives a signal  $L$ , he will not bid. When he receives  $M$ , his probability of being allocated one share depends on the number of information producers receiving a signal  $M$  among the remaining  $(n-1)$  information producers. We denote that number by  $j$ . Thus, each information producer will be allocated one share with probability  $\frac{k}{j+1}$

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<sup>32</sup> This assumption is appropriate. Since only the type B firm's IPO can fail in equilibrium, any IPO failure will reveal that the issuing firm is of type B.

<sup>33</sup> It should be noted that expected secondary market price across types will be still  $\theta$  even after accounting for the possibility of IPO failure by a type B firm. The reasoning is as follows. If the issuing firm is of type B, its true type will be revealed in the secondary market if at least one participant observes a signal  $L$  in the IPO so that its secondary market price will be 0. If however, no investor observes L (i.e., all investors observe M), the type B firm will trade at a price  $\theta$  in the secondary market. In case where the number of investors observing a signal M is less than  $k$  (so that the number of bidders in the IPO is also less than  $k$ , and the IPO fails), there will be at least  $n-k+1$  participants observing L, so that the type B firm again trades at a price of 0 in the secondary market. On the other hand, if the issuing firm is of type G, there are two possibilities. If at least one investor observes a signal H, the secondary market price of the firm will be its true value, i.e., 1. If all investors observe a signal M, the secondary market price will be  $\theta$ . As we show in Proposition 1(iv), the expected firm value across these various scenarios is  $\theta$ , which will be the expected secondary market price.

if  $j \geq k - 1$ ; otherwise the IPO fails. So the payoff to each information producer is:

$$\pi(F, n) = \theta \left( \frac{\alpha}{k} - F \right) \frac{k}{n} - (1 - \theta) F \sum_{j=k-1}^{n-1} \binom{n-1}{j} (1-p)^j p^{n-1-j} \frac{k}{j+1}. \quad (9)$$

It can be shown that, as more investors produce information, the expected payoff from producing information decreases (in other words, the left hand side of (8) is a decreasing function of  $n$ ). Thus, in equilibrium, the payoff to each information producer only covers the cost of doing so. In other words, the equilibrium number of information producers is determined by (8). Further, for a given number of information producers, the payoff to each information producer is higher when the offer price is lower. Therefore, the equilibrium number of information producers is decreasing in the offering price.

**Proposition 6 (*Underpricing in a Fixed-price Offering*):**

- (i)  $F < \theta \frac{\alpha}{k}$  represents an underpricing equilibrium.
- (ii) There is always an underpricing equilibrium for  $\alpha \leq \bar{\alpha}$ .
- (iii) The smaller the fraction of shares the firm sells in the IPO, the greater the degree of underpricing, i.e.,  $\frac{\theta(\alpha/k) - F}{F}$  is decreasing in  $\alpha$ .

Note that the average secondary market price of equity (of the entire firm) is always  $\theta$ , so that the average secondary market price of each share is  $\theta \frac{\alpha}{k}$ . When the offering price is lower than the expected secondary market price, the issue is underpriced. Part (ii) of the above proposition demonstrates that, in the equilibrium defined in Proposition 5, the IPO is underpriced on average. The reason is that the issuing firm needs to compensate investors for information production through underpricing. Part (iii) demonstrates that when the firm sells a smaller fraction of its shares in the IPO, it is optimal to underprice more. This is because, in this case, a greater fraction of the combined proceeds to the firm arise from the seasoned equity offering, so that the cost of underpricing is smaller, while the benefit from doing so (in terms of obtaining a higher secondary market price and therefore greater proceeds from the seasoned equity offering) is larger.

### 3.3 The Choice between IPO Auctions and Fixed-price Offerings for Going Public

We now discuss the overall equilibrium of the model, including the firm's choice of IPO mechanism between fixed-price offerings and IPO auctions.

**Proposition 7 (*Overall Equilibrium*):**

- (i) In equilibrium, the type G firm chooses the mechanism which maximizes its expected combined proceeds from the IPO and the seasoned equity offering. In the case where it chooses a fixed-price offering, the details of the offering are as characterized in section 3.2.2. In the case where the firm chooses to auction its shares, the details of the offering are as characterized in section 3.2.1.
- (ii) In equilibrium, the type B firm pools with the type G firm by choosing the same offering mechanism and offering price (in the case of a fixed-price offering).
- (iii) The equilibrium beliefs of investors at time 0 are such that they assign a probability  $\theta$  of any issuing firm choosing an equilibrium action being of type G. The equilibrium bidding strategies of investors in response to a firm choosing a fixed-price offering or an auction are given by sections 3.2.2 and 3.2.1 respectively. If investors observe a firm choosing an out-of-equilibrium strategy, they assign a probability 0 to that firm being of type G.
- (iv) The equilibrium in the secondary market in this case is as characterized in Proposition 1.

At time 0, the type G firm has to choose an IPO mechanism which maximizes the expected value of its combined proceeds from the IPO and the seasoned equity offering. The type B firm will choose to mimic the type G firm in equilibrium. This is because if it chooses a different IPO mechanism (including an offering price in the case of a fixed-price offering) than the type G firm, it will be revealed to be of type B, thereby obtaining a lower expected proceeds than that obtained from mimicking the type G firm.<sup>34</sup> Consistent with this equilibrium strategy of the two types of firms, outsiders assign a probability  $\theta$  to any firm following the equilibrium strategy being of type G.

The following proposition gives the issuing firm's choice between fixed-price offerings and IPO auctions for different values of the information production cost,  $C$ .

**Proposition 8 (Choice between Fixed-price Offerings and IPO Auctions as a Function of  $C$ ):** *If the information production cost  $C$  is less than a certain upper bound  $C_{\max}$ , then:*

- (i) *When the information production cost is low ( $C \leq C'$ ), the firm will choose an IPO auction.*
- (ii) *When the information production cost is high ( $C \geq C''$ ), the firm will choose a fixed-price offering in its IPO.<sup>35</sup>*

The intuition behind this proposition is the following. When the information production cost is low, there will be enough participation (and hence information production) in the IPO no matter which mechanism is used.

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<sup>34</sup> As discussed before, if the type B firm attempts to mimic the type G, there is a positive probability that its IPO will fail (i.e., it fails to sell all its shares in its IPO, and its true firm type is revealed). On the other hand, since the outsiders' information is noisy, the type B firm has a positive probability of successfully mimicking the type G (and obtaining the higher offer price resulting from doing so). Clearly, in the absence of any exogenous costs associated with IPO failure, the type B firm is always better off mimicking the type G rather than revealing its type by choosing a different IPO mechanism or by setting a different offer price (for fixed-price offerings). Given that the type B has the incentive and ability to costlessly mimic the type G in our setting, equilibria where the two firm types separate, or pool only partially, do not exist for the range of parameter values studied here. Note that the above reasoning is not driven by the normalization of the value of the type G firm to 1 or of the type B firm to 0 (numerical examples demonstrating that the type G and type B firms will pool even when  $v_B > 0$  is available to interested readers upon request).

<sup>35</sup>  $C'$ ,  $C''$ , and  $C_{\max}$  are defined in the appendix.

From section 2 we know that a large amount of information production in the IPO means that the secondary market price will be close to the fully revealing price, so that the proceeds to the type G firm will be close to its true value. Furthermore, in this case, the offering price in an IPO auction is also very informative since there is a sufficient amount of information production, yielding the type G firm very high proceeds. In contrast, when a fixed-price offering is used in this situation, both types of firms will get the same proceeds from the IPO, which is not in the best interest of the type G firm. Thus, the total proceeds to the type G firm will be higher by using an IPO auction when the information production cost is low.

However, when the information production cost is high, the number of information producers will be small if the issuing firm uses an IPO auction. This means that neither the offering price nor the secondary market price will be informative, and the type G firm's proceeds will be close to the average value (across types) of the issuing firm. However, by using a fixed-price offering, the type G firm can optimally underprice its shares in the IPO and induce the right amount of information production, so that the secondary market price will be much more revealing compared to the situation where the firm uses an IPO auction. Thus, when the information production cost is high, the type G firm can generate significantly higher proceeds from the secondary market by using a fixed-price offering rather than an IPO auction, and have only slightly lower proceeds from the IPO. In summary, the total proceeds to the type G firm will be higher by using a fixed-price offering when the information production cost is high, making this the equilibrium choice.

Figure 1 illustrates the above insights. Notice that for low values of  $C$ , an IPO auction is the equilibrium choice of the firm; for higher values of  $C$ , a fixed-price offering is the equilibrium choice. The following proposition gives the choice between a fixed-price offering and an IPO auction as a function of the fraction of equity sold in the IPO,  $\alpha$ .

**Proposition 9** (*Choice between Fixed-price Offerings and IPO Auctions as a Function of  $\alpha$* ): *If  $\alpha < \tilde{\alpha}$ , and the information production cost is relatively low ( $C \leq \tilde{C}$ ), then:*

- (i) *When the firm sells a small fraction of its equity in the IPO ( $\alpha < \underline{\alpha}^a$ ), it will choose a fixed-price offering.*
- (ii) *When the firm sells a large fraction of its equity in the IPO ( $\alpha > \alpha'$ ), it will choose an IPO auction.*

The number of information producers is monotonically increasing with the fraction of equity sold in the IPO,  $\alpha$ , in the case of an IPO auction. This means that, when the firm sells a small fraction of its equity in the IPO,

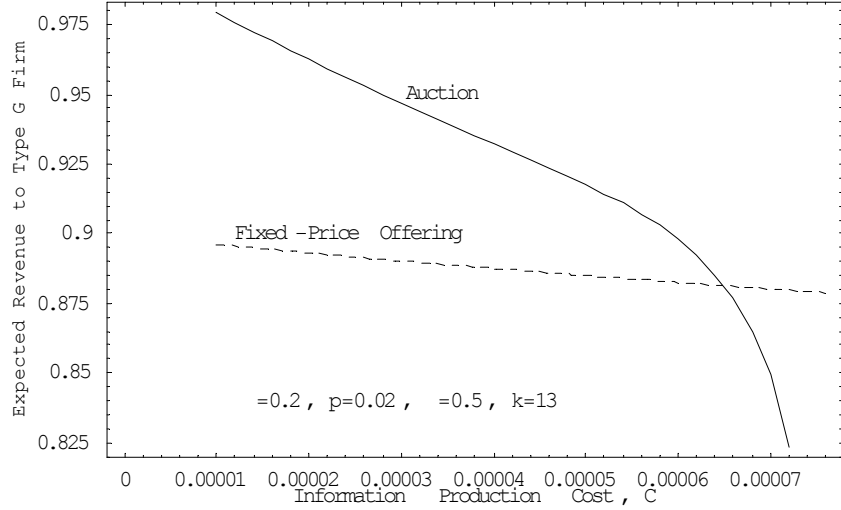


Figure 1: Choice of IPO Mechanism as A Function of C

the number of information producers will be small in the case of an IPO auction, and neither the IPO price nor the secondary market price will be very revealing. In contrast, if the issuing firm uses a fixed-price offering, it can underprice its equity in the IPO and induce a larger amount of information production. The secondary market price will then be more revealing, so that the secondary market proceeds to the type G firm will be larger. Further, when the type G firm sells only a small fraction of its equity in the IPO, the combined proceeds is dominated by the proceeds from the seasoned equity issue. Therefore, the type G firm's combined proceeds is maximized when it uses a fixed-price offering rather than IPO auction, making this the equilibrium choice.

In contrast, when the issuing firm sells a large fraction of its equity in the IPO, there will be a large number of information producers even when the firm uses an IPO auction. This means that both the IPO price and the secondary market price will be high for the type G firm. In this case, the IPO auction dominates, since the fixed-price offering does not confer any additional advantage to the type G firm over an IPO auction (but with the disadvantage of a lower IPO price). Figure 2 illustrates the above intuition. We can see that when the issuing firm sells a small fraction of its equity in the IPO ( $\alpha < 0.32$ ), it chooses a fixed-price offering; when it sells a large fraction of its equity in the IPO ( $\alpha > 0.32$ ), it chooses an IPO auction.

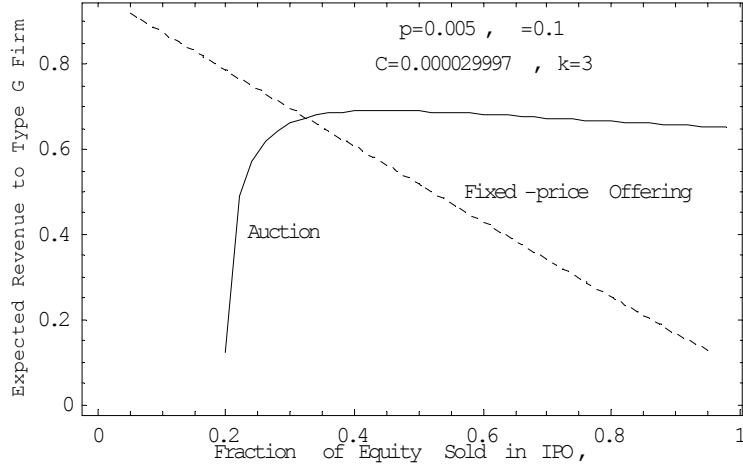


Figure 2: Choice of IPO Mechanisms as a Function of  $\alpha$

Table 1: Comparison of Underpricing in Fixed-Price Offerings and IPO Auctions

$\alpha$	Optimal IPO Mechanism	Underpricing (in Percentage)
0.06	fixed-price offering	39.14%
0.08	fixed-price offering	26.58%
0.1	fixed-price offering	20.05%
0.12	fixed-price offering	16.08%
0.14	fixed-price offering	13.39%
0.16	fixed-price offering	11.46%
0.18	fixed-price offering	10%
0.2	fixed-price offering	8.85%
0.22	fixed-price offering	7.93%
0.24	IPO auction	6.18%
0.26	IPO auction	6.97%
0.28	IPO auction	7.41%
0.3	IPO auction	7.65%
0.32	IPO auction	7.75%
0.34	IPO auction	7.78%
0.36	IPO auction	7.76%
0.38	IPO auction	7.69%
0.4	IPO auction	7.61%

### 3.4 Comparison of Underpricing in Fixed-price Offerings and in IPO Auctions: Numerical Examples

In this section, we use numerical examples to compare the degree of underpricing in fixed-price offerings and in IPO auctions. We assume that the equity in the IPO is divided into  $k = 13$  shares. Investors believe with probability  $\theta = 0.75$  that the issuing firm is of type G. If an investor chooses to produce information, he has a probability  $p = 0.005$  of observing the true value of the firm. The information production cost is  $C = 0.0000144164$ .<sup>36</sup> The fraction of equity sold in the IPO ranges from 6% to 40%.

Table 1 gives the optimal IPO mechanism and the percentage underpricing as a function of the fraction of equity sold in the IPO,  $\alpha$ . We can see that when the firm sells a small fraction of equity in the IPO (i.e.,  $\alpha \in [6\%, 22\%]$ ), it is optimal to use fixed-price offerings, and when it sells a large fraction of equity in the IPO (i.e.,  $\alpha \in [24\%, 40\%]$ ), it is optimal to use IPO auctions. This illustrates our results in Proposition 9. Further, we can see that when the firm uses fixed-price offerings, the degree of underpricing is decreasing with  $\alpha$ . For example, when  $\alpha = 6\%$ , the underpricing is 39.14%; when  $\alpha = 10\%$ , underpricing decreases to 20.05%; when  $\alpha$  increases to 20%, the underpricing decreases to 8.85%. This illustrates our results in Proposition 6.

From the above examples, we can see that the degree of underpricing is not monotonic in  $\alpha$  when an IPO auction is used. When  $\alpha$  increases from 24% to 34%, underpricing increases monotonically from 6.18% to 7.78%. However, when  $\alpha$  increases from 34% to 40%, underpricing decreases monotonically from 7.78% to 7.61%. We know that the total number of information producers is increasing in  $\alpha$ , hence the dollar amount of underpricing (“money left on the table”) is also increasing with  $\alpha$  (since the underpricing is to compensate information production costs of outsiders, more information production is associated with a greater dollar amount of underpricing). However, the average value of shares sold in the IPO (which is always  $\theta\alpha$  from Proposition 1) also increases as  $\alpha$  increases. So the impact of an increase in  $\alpha$  on percentage underpricing is ambiguous: when  $\alpha$  is relatively small, the number of information producers depends crucially on the fraction of equity sold in the IPO and increases very fast; for large values of  $\alpha$ , the number of information producers increases at a lower rate with  $\alpha$ . Thus the percentage underpricing is first increasing in  $\alpha$ , and then decreasing in  $\alpha$ , in the case of IPO auctions.

We can also see from the above example that the average underpricing conditional on the firm choosing a

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<sup>36</sup> The specific value of  $C$  assumed here ensures that the equilibrium number of information producers is an integer.

fixed-price offering (for  $\alpha \in [6\%, 22\%]$ ) is 17.05%, while that conditional on the firm choosing an IPO auction (for  $\alpha \in [24\%, 40\%]$ ) is only 7.42%. Thus, the above numerical examples are consistent with the situation documented by the empirical literature in various countries, where IPO auctions have been shown to have significantly lower underpricing compared to non-auction IPO mechanisms like fixed-price offerings (not only in the context of privately owned firms going public, but also in the privatizations of government owned firms in countries like the U.K.).

## 4 Extensions to the Basic Model

In this section, we extend the basic model in two different directions, by relaxing two of its assumptions (one at a time). In the first subsection, we relax the assumption that the fraction of equity sold in the IPO,  $\alpha$ , is exogenous. In the second subsection, we relax the assumption that the issuing firm does not set any reservation price in the IPO auction.<sup>37</sup>

### 4.1 Fixed-price Offerings versus IPO Auctions When the Fraction of Equity Offered is Endogenous

In the basic model, we assumed that the fraction of equity sold in the IPO,  $\alpha$ , is exogenous. In reality, the issuing firm may have some degree of freedom in choosing how much equity to sell in the IPO. In this subsection, we explore this possibility by assuming that the issuing firm can endogenously choose the fraction of equity sold in the IPO, subject to the constraint that at least a certain fraction  $\alpha_{\min}$  has to be sold (all other assumptions remain the same as in the basic model). In this case, the problem facing the issuing firm is two-dimensional. It has to choose: (a) an IPO mechanism: either a fixed-price offering (along with an optimal offering price) or an IPO auction; and (b) the optimal fraction of equity to offer in the IPO.

Figures 3, 4, and 5 illustrate the firm's equilibrium choice of offering mechanism for various values of  $C$ , as well as its equilibrium choice of  $\alpha$ . When  $C$  is low, the type G firm will choose the IPO auction and sell the smallest possible fraction,  $\alpha_{\min}$ , as illustrated in figure 3. This is because there will be enough information production even when the firm sells the minimal fraction  $\alpha_{\min}$ . Since the IPO price is not as informative as the secondary

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<sup>37</sup> Due to space considerations, we will focus primarily on the intuition behind various results in this section. A formal characterization of these results with proofs are available in the working paper version of this article.

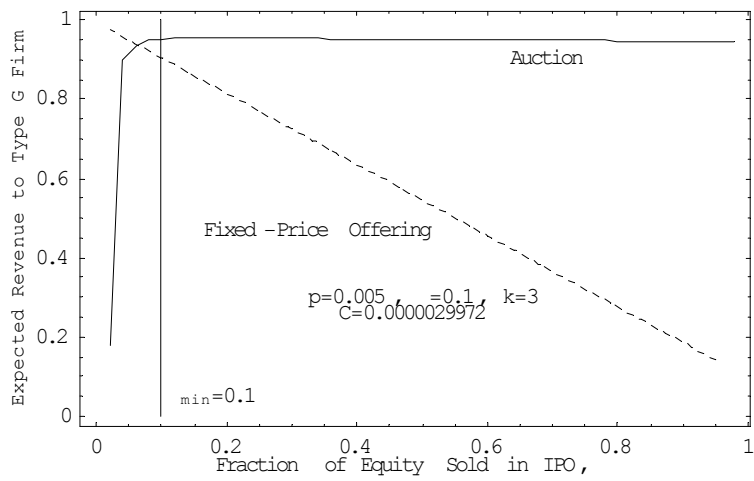


Figure 3: Endogenous Fraction of Equity Sold,  $\alpha$ , When  $C$  Is Low

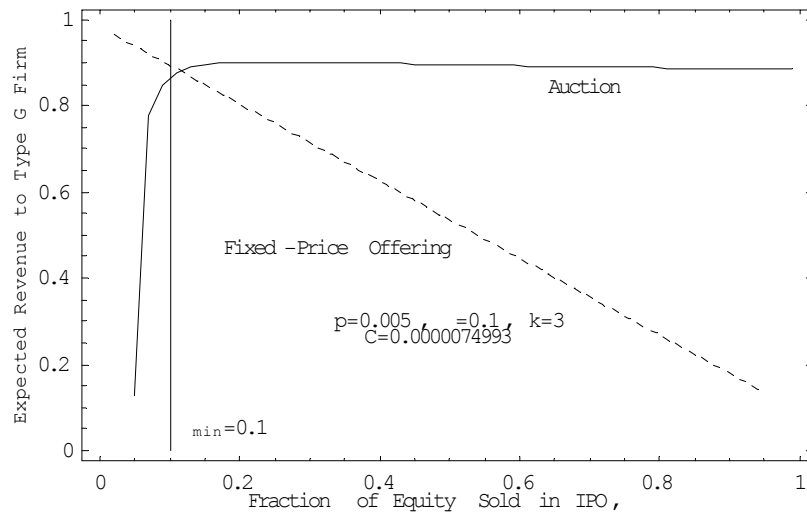


Figure 4: Endogenous Fraction of Equity Sold,  $\alpha$ , When  $C$  Is Moderate

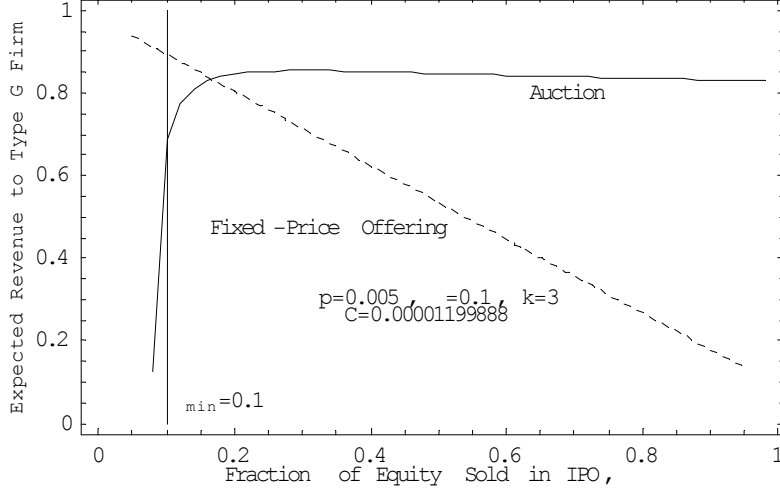


Figure 5: Endogenous Fraction of Equity Sold,  $\alpha$ , When  $C$  Is High

market price (for a type G firm, the expected offering price is always lower than the expected secondary market price), the type G firm tries to sell as little as possible in the IPO in this case.

When the information production cost is moderate, the type G firm will choose to auction its shares and sell a fraction more than  $\alpha_{\min}$ , as illustrated in figure 4. Since, in this case, the number of information producers in the IPO crucially depends on the fraction of equity sold, the type G firm faces a trade-off when choosing the optimal fraction of equity to offer. On the one hand, a large fraction sold means more information producers in the IPO, so that both the IPO price and the secondary market price will be higher. On the other hand, since the expected IPO price is always lower than the expected secondary market price for the type G firm, a larger fraction of equity sold in the IPO means a lower expected value of the combined proceeds to the type G firm. Depending upon this trade-off, for a wide variety of parameter values, the type G firm will choose an optimal  $\alpha$  larger than  $\alpha_{\min}$  when  $C$  is moderate (as is the case in figure 4). In the figure, we assume  $p = 0.005$ ,  $\theta = 0.1$ ,  $k = 3$ , and  $C = 7.4993 \times 10^{-6}$ . The optimal fraction of equity sold in the IPO is  $\alpha = 0.26$ , which is higher than the minimum required fraction,  $\alpha_{\min} = 0.1$ .

When the information production cost is high, the type G firm will choose a fixed-price offering, and again sell the minimum fraction  $\alpha_{\min}$ , as illustrated by figure 5. The intuition behind the type G firm choosing a fixed-price offering rather than IPO auction for large values of  $C$  is the same as that discussed under the basic model. The type G firm faces a trade-off when choosing the fraction of equity to sell: it can sell a larger fraction in the

IPO and underprice less, or sell a smaller fraction and underprice more in order to induce the same amount of information production. It can be shown that, for a wide variety of parameter values, it is optimal for the type G firm to sell a smaller fraction and underprice more, as in figure 5.

## 4.2 IPO Auction with an Endogenous Reservation Price

In the basic model, we assume that the issuing firm does not set any reservation price in the IPO auction. In this subsection, we relax this assumption by allowing the issuing firm to choose a reservation price  $r$  optimally in the IPO auction (all other assumptions remain the same as in the basic model).

The introduction of a reservation price will affect three things in an IPO auction. First, by setting a reservation price, the issuing firm can protect itself from selling at a very low price in the IPO. It makes sure that the offering price (if the IPO succeeds) is higher than the reservation price. Second, there is a possibility of IPO failure when the issuing firm sets a high reservation price. When most bidders choose to make a low bid, there may be less than  $k+1$  bidders bidding above  $r$ , so that the IPO auction fails. We assume that if the IPO auction fails, the issuing firm is able to obtain financing from an alternative source (say, a private placement of equity). We denote the cash flow to the issuing firm from this alternative source by  $R^{fail}$ , which is common to type G and type B firms.<sup>38</sup> Third, when there is a reservation price in the IPO auction, the information producers' payoff will be less, since they have no chance of obtaining shares at a price lower than the reservation price, so that there will be a smaller number of information producers in the IPO. In summary, the benefit to the issuing firm of having a reservation price is that it can extract some of the surplus obtained by the bidders in the firm's IPO. The cost of having a reservation price is that it may reduce the number of information producers about the firm and increase the probability of IPO failure. The equilibrium reservation price set by the issuing firm emerges from the above trade-off.

As in the basic model, the type B firm mimics the equilibrium strategy of the type G firm, so that the two types set the same reservation price in equilibrium. Further, even when the issuing firm is free to set a reservation price in the IPO auction, we obtain a result similar to proposition 9, i.e., the type G firm will choose a fixed-price

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<sup>38</sup> In the basic model we assume that the proceeds to the type B firm is 0 in the event of IPO failure. In contrast, here we assume that both types get  $R^{fail}$ . The reason is that, in the basic model, only the IPO of the type B firm may fail, so that IPO failure will reveal its true type. Here the IPOs of both types may fail, so that IPO failure does not reveal firm type. Our results remain qualitatively unchanged if we assume that  $R^{fail}$  is different for the two types of firms, and equal to zero for the type B firm.

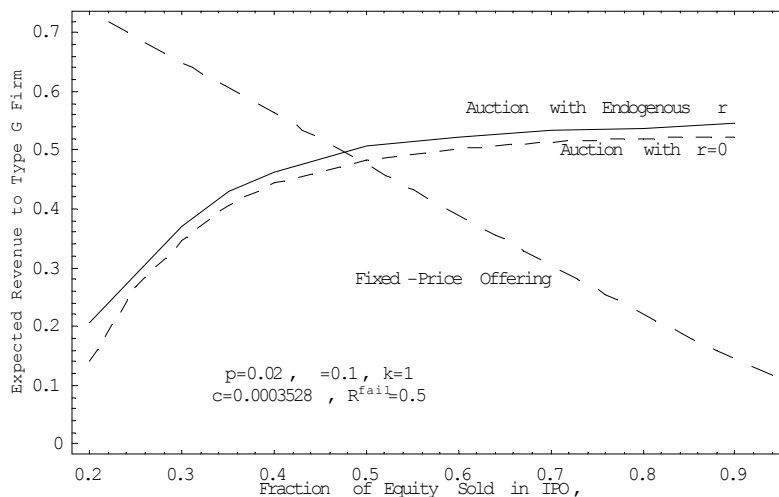


Figure 6: Comparison of the Three Mechanisms as a Function of  $\alpha$

offering when it sells a small fraction of equity in the IPO, and it will choose an IPO auction when it sells a large fraction. The intuition here is as follows. Having a reservation price in an IPO auction can only deter information production. Therefore, when the firm sells a small fraction of its equity, the information production is even lower if the IPO auction is used and there is a positive reservation price. If, however, the issuing firm sells a large fraction of its equity in the IPO, an IPO auction dominates a fixed-price offering for the same reasons as discussed in the basic model.<sup>39</sup> In this case, setting a strictly positive reservation price yields greater proceeds to the issuer compared to an IPO auction with no reservation price. Figure 6 presents the relationship between  $\alpha$ , the fraction of equity sold in the IPO, and the choice between fixed-price offerings and IPO auctions with an endogenous reservation price (the solid line gives the expected proceeds to the issuer in an IPO auction with an endogenous reservation price while the dotted line gives the proceeds in an IPO auction without a reservation price). It can be seen that the firm chooses a fixed-price offering for low values of  $\alpha$ , while it chooses an IPO auction for high values of  $\alpha$ .

As in the basic model, firms will continue to choose fixed-price offerings when the information production cost is high, and IPO auctions when the information production cost is low, even when it is free to set a reservation price. When the information production cost is high, we already know from the basic model that a fixed-price

<sup>39</sup> Recall that, since the issuer has the freedom to set the reservation price optimally (including setting  $r=0$ , which is equivalent to setting no reservation price, as in the basic model), an auction with an endogenous reservation price will also dominate a fixed-price offering in all settings where an IPO auction without a reservation price dominates the fixed-price offering.

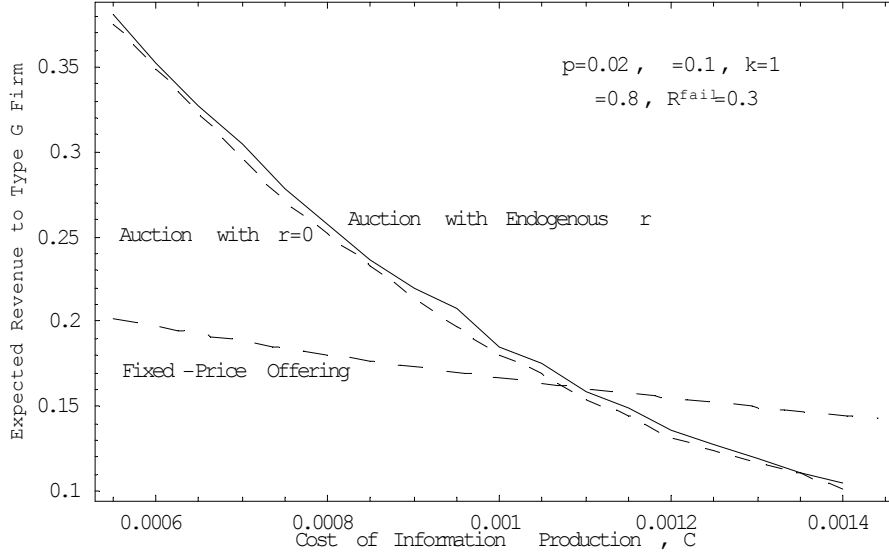


Figure 7: Comparison of the Three Mechanisms as a Function of C

offering dominates an IPO auction without a reservation price. When the firm has the freedom to set a reservation price, it will not help in this case, since a reservation price will only restrict information production. When the information production cost is low, the IPO auction dominates a fixed-price offering. In this case, the introduction of a reservation price will not hurt the IPO auction, since the issuing firm can always choose to set the reservation price equal to zero. Thus an IPO auction with an endogenous reservation price dominates a fixed-price offering when the information production cost is low, as illustrated in figure 7.

The higher the reservation price, the higher the payoff to a type G firm when the IPO succeeds. On the other hand, the higher the reservation price, the higher the probability of IPO failure. The optimal reservation price emerges from this trade-off. Thus, the optimal reservation price is increasing in  $R^{fail}$ , the amount the firm can raise from other sources in the event of IPO failure: the higher the value of  $R^{fail}$ , the lower the cost of IPO failure, resulting in a higher equilibrium reservation price.

## 5 Empirical and Policy Implications

We highlight some of the empirical and policy implications of our model below.

(i) *The relationship between firm and IPO characteristics and the optimal mechanism for going public:* First, our model predicts that if a firm is young, or small, or faces a greater extent of information asymmetry for some

other reason (so that outsiders' information production costs are significant), then fixed-price offerings will be the equilibrium choice of the firm, since, in this case, considerations of inducing information production and their impact on the secondary market price become important. In contrast, if a firm is older, or larger, or has a well-known (reputable) product, or faces a lower level of information asymmetry for some other reason (so that outsiders' cost of evaluating the firm is smaller), then our analysis implies that it will choose an IPO auction. Second, our model predicts that, *ceteris paribus*, firms selling smaller fractions of equity in the IPO will choose fixed-price offerings, while those selling larger fractions of their equity will choose IPO auctions.

(ii) *The relationship between offering mechanism and IPO underpricing:* Our model predicts that IPO auctions will exhibit a significantly lower mean and variance of underpricing compared to fixed-price offerings. This is due to the fact that the offering price in an IPO auction aggregates the information produced by outsiders to a significant degree, so that this offering price is greater for higher intrinsic-value firms (and lower for lower-intrinsic-value firms) in IPO auctions than in fixed-price offerings. At the same time, there is less information production in IPO auctions compared to fixed-price offerings (where the offering price is set by insiders to induce the optimal degree of information production), so that a lower amount of information is reflected in the opening price in the secondary market in this case. Since the impact of increased information production is to increase the separation between higher and lower intrinsic-value firms in the secondary market, the price jump (either upward or downward) from the IPO to the secondary market is therefore smaller for IPO auctions than for fixed-price offerings, leading to both a lower mean (see section 3.4 for an illustration) and a lower variance of underpricing in IPO auctions. The British privatization study of Jenkinson and Mayer (1988) indicates that, in the U.K., the extent of underpricing was much lower in the auction sample than in the fixed-price sample. A comparison of IPO underpricing from Japanese IPO auctions (Pettway and Kaneko, 1996) with underpricing in Japanese fixed-price offerings around the same period (Jenkinson, 1990) also indicates that underpricing in IPO auctions is significantly lower (see also Loughran, Ritter, and Rydqvist (1994), and Ritter (2003)).<sup>40</sup> IPOs where shares are auctioned have lower underpricing compared to those using fixed-price offerings in Taiwan as well (see Lin

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<sup>40</sup> Japanese IPOs auctions were discriminatory rather than uniform-price auctions. However, since our results are driven by the lack of flexibility in IPO auctions (regardless of whether the auction is uniform-price or discriminatory) in inducing information production by outsiders, our prediction that underpricing will be lower in IPO auctions compared to fixed-price offerings can be expected to hold qualitatively even when the IPO auction used is discriminatory.

and Sheu (1997), Liaw, Liu, and Wei (2000), and Ritter (2003)). A comparison of the initial returns from shares bought in Chilean IPO auctions (Aggarwal et al, 1993) with the developing-market average for non-auction IPOs around the same period (Jenkinson and Ljungqvist, 1996) also leads to similar conclusions in the Chilean and developing-market setting.

*(iii) The relationship between offering mechanism and the average number of bidders in the IPO:* Our model predicts that the number of bidders in fixed-price offerings will be significantly larger than that in IPO auctions. Recall that, in our setting, firms set the offering price in a fixed-price offering so as to induce a greater extent of information production in the IPO, resulting in a larger number of bidders in fixed-price offerings.

*(iv) A Resolution of the IPO Auction Puzzle:* Unlike the existing literature, our model is able to explain why auctions are losing market share to fixed-price offerings, while simultaneously predicting that fixed-price offerings will exhibit a greater extent of underpricing compared to IPO auctions. If, in practice, firm insiders' objective is not merely to maximize the proceeds from a one-shot equity offering, it is indeed optimal for younger and smaller firms, and those selling smaller fractions of equity to go public using fixed-price offerings.<sup>41</sup> Since a large majority of firms going public in the U.S. as well as in most other countries fall into this category, it is hardly surprising that IPO auctions are not gaining market-share in these countries. Our analysis indicates that one setting where IPO auctions may indeed be optimal is in the privatization of large, well-run government firms (where the government may be selling off its entire equity in the IPO) or in the IPOs of older and larger firms (for instance, firms going public again after being taken-private in an LBO).<sup>42</sup>

*(v) The benefit from selling equity in tranches and the optimal fraction of equity to sell in the IPO:* First, our analysis indicates the benefits of selling equity in tranches, since the firm may be able to obtain a significantly higher share price in later sales of equity. Second, our results from section 4.1 indicate that, when the firm needs to raise only a small amount of capital from the IPO (so that the minimum amount of equity the firm needs to sell in the IPO is small), the fraction of equity to be sold in the IPO may have to be decided jointly with the

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<sup>41</sup> The fact that, on average, firms sell only about one third of their equity in IPOs in the U.S. seems to indicate that insiders may indeed not be focused on maximizing proceeds from a one-shot equity offering here.

<sup>42</sup> The privatization of large, well-known government-owned firms seems to be one setting where IPO auctions seem to have been used extensively to date (e.g., in privatizations in Britain and several other countries). In terms of firms going public in the U.S., our analysis indicates that IPO auctions have been tried by precisely the wrong kind of firms, namely, small, lesser known firms (possibly because IPO auctions appealed to these firms purely from the point of view of providing savings in investment banking fees); older, larger, or better-known firms might have made use of IPO auctions with considerably more success (though most firms going public clearly do not belong to this category).

choice of offering mechanism. Thus, when the outsiders' cost of information production is large, the firm may choose to sell the smallest fraction of equity possible, using a fixed-price offering. At the other extreme, when the outsiders' cost of information production is very small, the firm may also choose to sell the smallest fraction of equity possible, but using an IPO auction. At moderate levels of the cost of information production, however, the firm may choose to sell a fraction of equity larger than the minimum it needs to sell from the point of view of raising capital alone using either an IPO auction (for smaller  $C$  values in this range) or a fixed-price offering (for larger  $C$  values in this range). This latter result obtains because the number of information producers tends to increase with the fraction of equity sold in IPOs.<sup>43</sup>

*(vi) Determination of the reservation price in IPO auctions:* Our analysis of section 4.2 indicates that firms which have little or no immediate capital requirements, or have a low opportunity cost of postponing their IPOs (either because they have alternative sources of capital, or because the stock issue was not meant to raise significant capital, as in the case of many privatizations) will set a higher reservation price compared to those which do have significant capital requirements or a significant opportunity cost of postponing their IPO.

*(vii) Reforming the procedures used in existing IPO auctions:* Our analysis indicates that existing IPO auction procedures may be reformed in several directions to make IPO auctions more competitive with fixed-price offerings. First, our analysis indicates that firms going public may benefit from offering the IPO at a discount to the clearing-price in the IPO auction.<sup>44</sup> <sup>45</sup> Second, this discount may be adjusted to account for the characteristics of firms going public: for instance, a greater discount may be offered in the IPOs of younger, smaller, or lesser-known firms. The idea here is to encourage greater information production by outsiders, over and above that "naturally occurring" in auctions.<sup>46</sup>

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<sup>43</sup> The determination of optimal tranche size was an often-debated problem in the privatizations of government firms in many countries. Our analysis provides two pieces of guidance in this case. First, it indicates that, even when the government has no reason to hold on to large fractions of equity after the IPO, it is optimal from the point of revenue maximization to sell the equity in tranches. Second, even when there is no minimum capital to be raised from an IPO or privatization, considerations of inducing information production dictate that the government (or firm going public) sell at least a certain minimum fraction of equity.

<sup>44</sup> The IPO auctions in France do seem to set the offer price 2 to 5% below the auction-clearing price (see MacDonald and Jacquillat (1974) or Biais and Faugeron-Crouzet (2000)).

<sup>45</sup> For reasons unrelated to ours, the analysis of Biais and Faugeron-Crouzet (2002) and Parlour and Rajan (2002) also suggest that it may be optimal for the issuer to set the offer price in IPOs below the market-clearing price. While in Biais and Faugeron-Crouzet (2002) this is motivated by a need to unravel tacit collusion among investors, in Parlour and Rajan (2002) the motivation is to mitigate the winner's curse faced by uninformed investors.

<sup>46</sup> Of course, investors may anticipate such a discount and bid more aggressively in the IPO auction, thus partially eliminating the information production benefits of the discount. However, as long as such aggressive bidding does not completely eliminate the additional surplus provided by this discount to those investors who win shares in the IPO auction, the equilibrium number of

## 6 Conclusion

We have developed a theoretical analysis of the choice of firms between fixed-price offerings and uniform-price auctions for selling shares in IPOs and privatizations. We considered a setting in which a firm goes public by selling a fraction of its equity in an IPO market where insiders have private information about intrinsic firm value, but where outsiders could produce information at a cost about true firm value before bidding for shares. We showed that, while auctioning off shares in such a setting may maximize the proceeds from a one-shot offering, it does not maximize long-run firm value, since not enough investors will choose to produce information about the firm in equilibrium. Insiders care about inducing the optimal degree of information production by outsiders, since this information will be reflected in the secondary market price, giving a higher secondary market price for truly higher intrinsic-value firms. Thus, we demonstrated that, in many situations, firms will prefer to go public using fixed-price offerings rather than IPO auctions in equilibrium, since such offerings allow the firm to induce the optimal extent of information production. We related the equilibrium choice between fixed-price offerings and auctions to various characteristics of the firm going public and that of its IPO. Unlike the existing literature, our model is able to explain not only the widely-documented empirical finding that the extent of underpricing is lower in IPO auctions than in fixed-price offerings, but also the fact that, despite this, auctions are losing market share around the world. Our model thus provides a resolution of the “IPO auction puzzle,” and suggests how current IPO auction mechanisms may be reformed to become more competitive with fixed-price offerings. Our results also provide various other hypotheses for further empirical research.

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information producers will be greater in IPO auctions when such a discount is provided. One way to implement this discount is to place an upper limit on the IPO share price (with the IPO price set at the lower of the market clearing price in the auction and this upper limit), with all bidders above this upper limit having an equal chance of getting a share allocation. It can be shown that the number of information producers under an IPO auction with a discount implemented in the above manner will be strictly greater than that in an IPO auction without a discount.

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Appendix:

**A Proofs of Propositions:**

## Appendix: Proofs of Propositions

Before presenting the proofs of various propositions, we define some functions and point out some of their properties, which will be used later in the formal proofs.

$n_{\min}$ : The function  $I(n)$  obtains its maximum value at this point, and  $\frac{\partial I(n)}{\partial n} < 0$  for all  $n \geq n_{\min}$ .

$\underline{\alpha}^f$ : The minimum fraction of equity the firm must sell to make sure that at least  $k + 1$  investors produce information in the IPO when fixed-price offering is used.

$\underline{\alpha}^a$ : The minimum fraction of equity the firm must sell to make sure that at least  $k + 1$  investors produce information in the IPO when auction is used. Note that we always have  $\underline{\alpha}^a > \underline{\alpha}^f$ , since when using fixed-price offering, the issuing firm can always set the offering price low to make sure that there will be enough entry.

$C_{\max}$ : The maximum information production cost which makes sure that auction is feasible when  $\alpha = 1$ , i.e.,

$$C_{\max} = p(1-p)\theta(1-\theta)\frac{1}{k}I(n_{\min}). \quad (\text{A.1})$$

$\tilde{\alpha}$ : The maximum  $\alpha$  to make sure that the equilibrium used in fixed-price offering exists for all  $C \in (0, C_{\max})$ , i.e.,

$$\tilde{\alpha} \equiv \min\{\bar{\alpha}(C) : C \in (0, C_{\max})\}, \quad (\text{A.2})$$

which makes sure that for any  $C \leq C_{\max}$ , and  $\alpha \leq \tilde{\alpha}$ , the NE exists.

**Proof of Proposition 1:** We first prove that the secondary market price will be  $\theta$  if all  $n$  signals are  $M$ . We use  $M^n$  to denote the event that all  $n$  signals are  $M$ . Then we have

$$\begin{aligned} SP(M^n) &= E[v|M^n] = 1 * \Pr(v = 1|M^n) + 0 * \Pr(v = 0|M^n) = \Pr(v = 1|M^n) \\ &= \frac{\Pr(v = 1)\Pr(M^n|v = 1)}{\Pr(v = 1)\Pr(M^n|v = 1) + \Pr(v = 0)\Pr(M^n|v = 0)} = \frac{\theta(1-p)^n}{\theta(1-p)^n + (1-\theta)(1-p)^n} = \theta, \end{aligned} \quad (\text{A.3})$$

which completes the proof.

If the firm is of type G, with probability  $(1-p)^n$  all participants will receive the signal  $M$ , and the secondary market price is  $\theta$ ; with probability  $1 - (1-p)^n$  at least one participant will receive a signal  $H$ , and the secondary market price is 1. So the expected secondary market price for a high value firm is:

$$E[SP^G(n)] = \theta(1-p)^n + (1 - (1-p)^n) \times 1 = 1 - (1-\theta)(1-p)^n, \quad (\text{A.4})$$

which is increasing in  $n$  since  $\partial E[SP^G(n)]/\partial n = -(1-\theta)(1-p)^n \ln(1-p) > 0$ . Similarly, the expected secondary market price for a low value firm is:

$$E[SP^B(n)] = \theta(1-p)^n + (1 - (1-p)^n) \times 0 = \theta(1-p)^n, \quad (\text{A.5})$$

which is decreasing in  $n$  since  $\partial E[SP^B(n)]/\partial n = \theta(1-p)^n \ln(1-p) < 0$ . The expected secondary market price for the firm across types is

$$E[SP(n)] = \theta E[P_2^h] + (1-\theta)E[P_2^l]. \quad (\text{A.6})$$

Plugging equations (A.4) and (A.5) into (A.6), we have

$$E[SP(n)] = \theta, \quad (\text{A.7})$$

which is independent of  $n$ .

**Proof of Proposition 2:** Suppose the other  $n-1$  bidders, 2, 3, ...  $n-1$ , use the strategy defined in the proposition, if we can prove that it is also optimal for bidder 1 to use this strategy, then the strategy is a

symmetric NE. When bidder 1 observes H, he knows that the true value of each share is  $\frac{\alpha}{k}$ . Suppose  $i$  bidders receive signal H other than bidder 1, where  $i$  could range from 0 to  $n-1$ .

Case 1:  $i < k$ . The  $k^{th}$  highest bid other than 1 will be a bid from a participant with signal  $M$ , hence the clearing price is less than  $\frac{\alpha}{k}$ , let's call it  $m_k$ . If bidder 1 bids  $\frac{\alpha}{k}$ , he will win one unit and pay a price of  $m_k$ , the payoff is positive; if he bids a value in  $[0, m_k)$ , he can never win the object and the payoff is zero; if he bids a value in  $(m_k, 1)$ , the payoff is the same as bidding  $\frac{\alpha}{k}$ . We can see that bidding  $\frac{\alpha}{k}$  is weakly optimal in this case.

Case 2:  $i \geq k$ . The  $k^{th}$  highest bid among investors  $2, 3, \dots, n$  is  $\frac{\alpha}{k}$ . If investor 1 bids  $\frac{\alpha}{k}$ , he will win one unit with probability  $\frac{k}{i+1} < 1$ , and pay a price of  $\frac{\alpha}{k}$ , the payoff is zero; if he bids a value in  $[0, \frac{\alpha}{k})$ , he can never win the object and the payoff is zero. We can see that bidding  $\frac{\alpha}{k}$  is also weakly optimal in this case.

When a participant observes signal L, he knows that  $v = 0$ . Suppose that investors  $2, 3, \dots, n$  use the equilibrium bidding strategy. If investor 1 bids 0, the payoff to him is always zero. If he bids above 0, there is some chance that he will receive one unit at a price higher than 0. If he bids less than 0, his bid will never be filled hence the payoff is 0. So it is optimal to bid 0 on L when all other investors use the equilibrium strategy.

We now characterize the bidding strategy of the investors who receive signal M. Define  $M(b)$  as the cdf of a random variable with support on the interval  $(\underline{b}, \bar{b}]$ , and  $m(b)$  is the corresponding pdf. Suppose bidders  $2, 3, \dots, n$  use the following strategy: withdraw a bid from the distribution  $M(b)$  after receiving a signal M. If investor 1 observes M, and if he submits a bid  $b \in (\underline{b}, \bar{b}]$ , the expected payoff to him is

$$\begin{aligned} \pi(b) = & \theta \int_{\underline{b}}^b \left(\frac{\alpha}{k} - x\right) \binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1} dx \\ & - (1-\theta) \int_{\underline{b}}^b x \binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [(1-p)(1-M(x))]^{k-1} [p + (1-p)M(x)]^{n-k-1} dx. \end{aligned}$$

To understand the above formula, note that if we define  $x$  as the  $k^{th}$  highest bid among investors  $2, 3, \dots, n$ , investor 1 will win one unit at price  $x$  iff  $b > x$ . When the firm is of type G,  $\binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1}$  is the pdf that the  $k^{th}$  highest bid among investors  $2, 3, \dots, n$  is  $x$ , and bidder 1's payoff is  $\frac{\alpha}{k} - x$  if  $x < b$  and 0 otherwise. When  $v = 0$ ,  $\binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [(1-p)(1-M(x))]^{k-1} [p + (1-p)M(x)]^{n-k-1}$  is the pdf that the  $k^{th}$  highest bid among investors  $2, 3, \dots, n$  is  $x$ , and bidder 1's payoff is  $-x$  if  $x < b$  and 0 otherwise.

If investor 1 is willing to randomize in the range  $(\underline{b}, \bar{b}]$ , the payoff must be the same to him, otherwise he will choose to bid whichever amount gives him the highest payoff. This means  $\partial\pi(b)/\partial b = 0$ , which leads directly to Equation (3). Since  $M(b) \rightarrow 0$  when  $b \rightarrow 0$  and  $M(b) = 1$  when  $b = \frac{\alpha}{k}$ , we have

$$\underline{b} = 0 \text{ and } \bar{b} = \frac{\alpha}{k}. \quad (\text{A.8})$$

Further note that  $M(b)$  is strictly positive and strictly increasing over the interval  $(0, \frac{\alpha}{k}]$ , which qualifies  $M(b)$  as a cdf.

Each participant's expected payoff is

$$E[\pi(n)] = Pr(s_i = H)E[\pi(n)|H] + Pr(s_i = M)E[\pi(n)|M] + Pr(s_i = L)E[\pi(n)|L]. \quad (\text{A.9})$$

It is obvious that  $E[\pi(n)|L] = 0$ . The expected profit for a participant when he observes M is also 0. To see this, plug equation (3) into  $\pi(b)$ . Then, we have  $\pi(b) = 0$  for any  $b$ , which implies  $E[\pi(n)|M] = 0$ . The expected profit when bidder 1 observes H is

$$E[\pi(n)|H] = \int_0^{\frac{\alpha}{k}} \left(\frac{\alpha}{k} - x\right) \binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1} dx, \quad (\text{A.10})$$

where  $x$  is the  $k^{th}$  highest bid among the bids from bidder  $2, 3, \dots, n$ , and  $\binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1}$  is the pdf of  $x$  conditional on the firm being of type G and  $x < \frac{\alpha}{k}$ ;  $(\frac{\alpha}{k} - x)$  is the payoff to bidder 1 in this case. So the expected profit of each participant is

$$\begin{aligned} E[\pi(n)] &= Pr(H)E[\pi(n)|H] + 0 \\ &= \theta p \int_0^{\frac{\alpha}{k}} \left(\frac{\alpha}{k} - x\right) \binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1} dx. \end{aligned} \quad (\text{A.11})$$

In equilibrium, the number of information producers is such that the profit from information production is no less than the information production cost  $C$ , but at the same time, there is no incentive for more investors to produce information. This proves part (ii) of the proposition.

**Proof of Proposition 3:** When the issuing firm is of type G, with probability  $1 - \sum_{j=0}^k \binom{n}{j} p^j (1-p)^{n-j}$ , there will be more than  $k$  participants who observe H and bid  $\frac{\alpha}{k}$ . The clearing price will be  $\frac{\alpha}{k}$  in this case. With probability  $\binom{n}{j} p^j (1-p)^{n-j}$ , there will be  $j \in \{0, 1, \dots, k\}$  participants who observe H and bid  $\frac{\alpha}{k}$ . In this case, the other  $n-j$  participants will observe  $M$  and will bid a withdrawal from the distribution  $M(b; n)$ , so the offering price per share will be the  $(k+1-j)$ th highest signal from  $n-j$  signals withdrawn from  $M(b; n)$ . Thus the expected proceeds to the type G firm in IPO market is given by equation (5). If we change the integration into an integration over  $M$ , we have

$$E[IR^G(n)] = \alpha - \alpha \int_0^1 \frac{(1-\theta)[(1-p)(1-M)]^{k-1} [p + (1-p)M]^{n-k-1} \binom{n-1}{1} \binom{n-1}{k} (p + (1-p)(1-M))^k (1-p)^{n-k} M^{n-k-1}}{\theta[p + (1-p)(1-M)]^{k-1} [(1-p)M]^{n-k-1} + (1-\theta)[(1-p)(1-M)]^{k-1} [p + (1-p)M]^{n-k-1}} dM. \quad (\text{A.12})$$

When the firm is of type B, if the  $(k+1)$ th highest bid is 0, then the proceeds is just 0. Suppose the  $(k+1)$ th highest bid is  $x \in (0, \frac{\alpha}{k})$ , the pdf of  $x$  is  $\binom{n}{1} (1-p)m(x) \binom{n-1}{k} [(1-p)(1-M(x))]^k [p + (1-p)M(x)]^{n-k-1}$ , and the expected proceeds to the issuing firm is given by equation (6). This completes the proof of part (i).

From equation (A.12), we can see that

$$E[IR^G(n)] < \alpha - \alpha \int_0^1 (1-\theta) \binom{n}{1} \binom{n-1}{k} (1-p)^n (1-M)^k M^{n-k-1} dM. \quad (\text{A.13})$$

Use the fact that  $\int_0^1 \binom{n}{1} \binom{n-1}{k} (1-M)^k M^{n-k-1} dM = 1$ , we have  $E[IR^G(n)] < \alpha[1 - (1-\theta)(1-p)^n]$ . Note that  $E[IR^G(n)]$  gives only the proceeds from selling a fraction  $\alpha$  of the firm, so that the value of the whole firm (based on the IPO equity price) will be less than  $[1 - (1-\theta)(1-p)^n]$ , which is the expected secondary market value of the whole firm when it is of type G.

Similarly, if we express equation (6) as an integration of  $M$ , we have

$$\begin{aligned} R_a^B(n) &= \alpha \int_0^1 \frac{\theta[p + (1-p)(1-M)]^{k-1} [(1-p)M]^{n-k-1} \binom{n-1}{1} \binom{n-1}{k} (1-p)^{k+1} (1-M)^k (p + (1-p)M)^{n-k-1}}{\theta[p + (1-p)(1-M)]^{k-1} [(1-p)M]^{n-k-1} + (1-\theta)[(1-p)(1-M)]^{k-1} [p + (1-p)M]^{n-k-1}} dM \\ &> \alpha \theta (1-p)^n \int_0^1 \binom{n}{1} \binom{n-1}{k} (1-M)^k M^{n-k-1} dM = \alpha \theta (1-p)^n. \end{aligned}$$

Hence the value of the whole firm is higher than  $\theta(1-p)^n$ , which is the expected secondary market price of the firm when it is of type B. This completes the proof of part (ii).

**Proof of Proposition 4:** First note that the value of the fraction of the firm offered in the IPO goes to two sources: part of it goes to the issuing firm, which is the proceeds to the issuing firm in the IPO; part of it goes to the bidders, i.e.,

$$\theta E[IR^G(n)] + (1-\theta)E[IR^B(n)] + n_a \pi(n_a) = \theta \alpha. \quad (\text{A.14})$$

Note that in equilibrium we have  $\pi(n_a) = C$ , so the average offering price is given by

$$\theta E[IR^G(n)] + (1-\theta)E[IR^B(n)] = \theta \alpha - n_a C. \quad (\text{A.15})$$

In Proposition 1 we have proved that the average secondary market price (of the whole firm) is  $\theta$ , so a fraction  $\alpha$  of the firm will have an average price of  $\theta \alpha$  in the secondary market. Hence the underpricing is given by

$$\theta \alpha - (\theta E[IR^G(n)] + (1-\theta)E[IR^B(n)]) = n_a C. \quad (\text{A.16})$$

The percentage underpricing is thereby  $\frac{n_a C}{\theta \alpha - n_a C}$ .

**Proof of Proposition 5:** Suppose bidders 2, 3, ..., n use the specified bidding strategy. When bidder 1 observes L, he knows that the firm is a type B firm. The expected payoff from bidding is  $(0-F) \sum_{j=k-1}^{n-1} \binom{n-1}{j} (1-p)^j p^{n-1-j} \frac{k}{j+1} < 0$ , so he will not bid. When bidder 1 observes H, the payoff from bidding is  $(\frac{\alpha}{k} - F) \frac{k}{n} > 0$ , so he will bid. When bidder 1 observes M, the payoff from bidding is  $\theta(\frac{\alpha}{k} - F) \frac{k}{n} - (1-\theta)F \frac{k}{n(1-p)} \sum_{i=k}^n \binom{n}{i} (1-p)^i p^{n-i} > \theta(\frac{\alpha}{k} - F) \frac{k}{n} - (1-\theta)F \frac{k}{n(1-p)}$ , which is nonnegative if  $F \leq \frac{\theta(1-p)}{\theta(1-p)+1-\theta} \frac{\alpha}{k}$ , so it is optimal for bidder 1 to bid if he observes M. This means that it is also optimal for bidder 1 to play the specified strategy.

In Proposition 6, we proved that  $\frac{\theta \alpha / k - F}{F}$  is decreasing in  $\alpha$ , which is equivalent to  $F/\alpha$  is increasing in  $\alpha$ . Since what we need is  $F \leq \frac{\theta(1-p)}{\theta(1-p)+1-\theta} \frac{\alpha}{k}$ , or equivalently,  $F/\alpha \leq \frac{\theta(1-p)}{\theta(1-p)+1-\theta} \frac{1}{k}$ , which can be satisfied for a low value of  $\alpha$ .

When the offering price is set at F, the expected payoff to each participant as a function of total number of participants, n, is

$$\pi(n, F) = \theta \left( \frac{\alpha}{k} - F \right) \frac{k}{n} - (1-\theta)(1-p)F \sum_{j=k-1}^{n-1} \binom{n-1}{j} (1-p)^j p^{n-1-j} \frac{k}{j+1}. \quad (\text{A.17})$$

Since  $\sum_{j=k-1}^{n-1} \binom{n-1}{j} (1-p)^j p^{n-1-j} \frac{k}{j+1} = \frac{k}{n(1-p)} \sum_{i=k}^n \binom{n}{i} (1-p)^i p^{n-i} \simeq \frac{k}{n(1-p)}$ , we have  $\pi(n, F) = (\frac{\theta \alpha}{k} - F) \frac{k}{n}$ .<sup>47</sup> We can see that the expected payoff to each participant is a decreasing function of total number of participants, and the equilibrium number of participants is determined by the following equation  $(\frac{\theta \alpha}{k} - F) \frac{k}{n} = C$ . By the implicit function theorem we have

$$\frac{\partial n_f}{\partial F} = - \frac{-k/n}{-(\frac{\theta \alpha}{k} - F)k/n^2} = \frac{-n}{(\frac{\theta \alpha}{k} - F)} < 0, \quad (\text{A.18})$$

which completes the proof.

**Proof of Proposition 6:** The average secondary market price of the whole firm is  $\theta$ , as we proved in Proposition 1. Since a fraction  $\alpha$  of the firm is sold in the IPO, and that fraction is divided into k shares, the average secondary market price of each share is  $\alpha\theta/k$ . If the offering price of each share,  $F < \alpha\theta/k$ , it is an underpricing equilibrium.

We proved in Proposition 5 that for  $\alpha \leq \bar{\alpha}$ , it is optimal for the type G firm to set  $F \leq \frac{\theta(1-p)}{\theta(1-p)+1-\theta} \frac{\alpha}{k} < \alpha\theta/k$ , which is an underpricing equilibrium.

In order to prove  $\frac{\theta \alpha / k - F}{F}$  is decreasing in  $\alpha$ , it is sufficient to prove that  $\frac{F}{\theta \alpha}$  is increasing in  $\alpha$ . In Proposition 5 we proved that the number of participants in the fixed-price offering is  $(\frac{\theta \alpha}{k} - F) \frac{k}{n} = C$ , or  $\theta \alpha = kF + nC$ . The objective of the type G firm is to

$$\underset{F}{Max} \quad kF + (1-\alpha)[1 - (1-\theta)(1-p)^n] \quad (\text{A.19})$$

$$s.t. \quad \theta \alpha = kF + nC.$$

Solving the above optimization problem, we find that

$$F = \frac{\theta \alpha}{k} - \frac{C \ln\{(1-\theta)(1-\alpha) \ln[1/(1-p)]\} - \ln C}{\ln[1/(1-p)]}. \quad (\text{A.20})$$

This means  $\frac{F}{\theta \alpha} = \frac{1}{k} - \frac{C \ln\{(1-\theta)(1-\alpha) \ln[1/(1-p)]\} - \ln C}{\theta \alpha \ln[1/(1-p)]}$ , which is indeed increasing in  $\alpha$ .

**Proof of Proposition 7:** In section 3.2.1 and 3.2.2 we assume that the type G firm chooses the IPO mechanism to maximize its total proceeds, so part (i) is automatically satisfied. We have to show that it is indeed

<sup>47</sup> We make this approximation since  $\sum_{i=k}^n C_n^i (1-p)^i p^{n-i}$  is very close to 1. For example, when  $k = 13$ , and  $n = 14$  (we assume  $n \geq k + 1$  in either auction or fixed-price offering), we found that  $\sum_{i=k}^n C_n^i (1-p)^i p^{n-i} = 0.9916$ . For  $n > 14$ , it is even closer to 1: for example, when  $n = 15$ ,  $\sum_{i=k}^n C_n^i (1-p)^i p^{n-i} = 0.9995842$ , and for  $n=16$ ,  $\sum_{i=k}^n C_n^i (1-p)^i p^{n-i} = 0.9999835$ . We find that the error is only of a similar magnitude for other values of k.

optimal for the type B firm to mimic the type G firm. From equation (6) we know that the proceeds to the type B firm from the IPO auction is positive. In a fixed-price offering, the proceeds to the type B firm is  $F$  when the IPO succeeds, and 0 when the IPO fails, so the expected proceeds is also positive. In contrast, if the type B firm chooses not to mimic the type G firm, the market believes that it is a type B firm with probability 1 and the proceeds will be 0. So it is optimal for the type B firm to mimic the type G firm.

Since in equilibrium, both types of the issuing firm pool together, the specified equilibrium beliefs are consistent with equilibrium strategies of all parties. Finally, the secondary market price is also consistent with equilibrium strategies and beliefs of all parties from the proof of Proposition 1.

**Proof of Proposition 8:** The total proceeds to the type G firm when fixed-price offering is used is

$$E[R_f^G(\alpha)] = kF + (1 - \alpha)[1 - (1 - \theta)(1 - p)^n]. \quad (\text{A.21})$$

Since  $F \leq \frac{\theta(1-p)}{\theta(1-p)+1-\theta} \frac{\alpha}{k} < \theta\alpha/k$ , and  $1 - (1 - \theta)(1 - p)^n < 1$ , we have  $E[R_f^G(\alpha)] < 1 - (1 - \theta)\alpha$ . From Proposition 3 we know that there exists a critical value  $C'$  such that for all  $C \leq C'$ ,  $E[R_a^G(\alpha)] \geq 1 - (1 - \theta)\alpha$ , so that  $E[R_a^G(\alpha)] > E[R_f^G(\alpha)]$ .

Define  $C''$  as the maximum  $C$  such that there are at least  $k+1$  information producers in the IPO auction for a given  $\alpha$ :  $C'' = p(1 - p)\theta(1 - \theta)\frac{\alpha}{k}I(n_{\min})$ . Then for  $C > C''$ , the issuing firm will choose a fixed-price offering, since there will not be enough information producers if auction is used.

**Proof of Proposition 9:** We proved in Proposition 8 that for any  $\alpha \leq \tilde{\alpha}$ , there exist  $C'(\alpha)$  such that for all  $C \leq C'(\alpha)$ , auction dominates. For any  $a' < \tilde{a}$ , we define  $\tilde{C} = \min\{C'(\alpha) : \alpha \in [a', \tilde{a}]\}$ , so that, as long as  $C \leq \tilde{C}$ , auction dominates for  $\alpha \in [a', \tilde{a}]$ . For  $\alpha < \underline{\alpha}^a$ , there will be less than  $k + 1$  participants if auction is used, therefore the fixed-price offering is the equilibrium choice.