

Designing Innovative Securities in Response to Market Imperfections: A Theory of Mandatory Convertibles

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Abstract

The objective of this paper is to develop a theoretical analysis of how innovative securities are designed in response to various market imperfections, using the specific example of "mandatory convertibles," which are securities that automatically ("mandatorily") convert to common stock on a pre-specified date. We consider a firm facing a financial market characterized by asymmetric information and significant costs in the event of financial distress. The firm can raise capital either by issuing mandatory convertibles, or by issuing more conventional securities like straight debt, ordinary convertibles, or equity. We show that, in equilibrium, the firm issues straight debt or ordinary convertibles if the extent of asymmetric information facing it is large, but the probability of being in financial distress is relatively small; it issues mandatory convertibles if it faces a smaller extent of asymmetric information but a greater probability of financial distress. Our model provides a rationale for the three commonly observed features of mandatory convertibles: mandatory conversion, capped (or limited) capital appreciation, and a higher dividend yield compared to common stock. We also characterize the equilibrium design of mandatory convertibles.

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1 Introduction

The objective of this paper is to develop a theoretical analysis of how innovative securities are designed in response to various market imperfections, using the specific example of “mandatory convertibles,” which are securities that automatically (“mandatorily”) convert to common stock on a pre-specified date. It is well known that the financial markets are characterized by several market imperfections which impose significant costs on firms seeking to raise external financing. One of the most important of these is asymmetric information, which characterizes a situation where firm insiders know more about intrinsic firm value compared to outsiders: see, e.g., Myers and Majluf (1984), who studies how asymmetric information imposes costs on firms issuing equity. Myers and Majluf (1984) has shown that the costs associated with asymmetric information are most significant for equity and securities with payoff structures similar to equity and that such costs are the least for fixed income securities. On the other hand, issuing fixed income securities such as straight debt require the firm to incur costs arising from another market imperfection, namely costs of financial distress (see, e.g., Ross (1977)). In this paper, we develop a theory of mandatory convertibles and demonstrate that this important financial innovation has arisen primarily as an attempt by firms to reduce the total costs arising from both asymmetric information and financial distress simultaneously.

Mandatory convertibles are equity-linked hybrid securities such as PERCS (Preferred Equity Redemption Cumulative Stock) or DECS (Debt Exchangeable for Common Stock, or Dividend Enhanced Convertible Securities), which automatically (“mandatorily”) convert to common stock on

a pre-specified date. Starting from small beginnings in 1988, such mandatory convertibles have become extremely popular in recent times: \$5 billion worth of mandatory convertibles were issued in 1996 (a quarter of the convertible market); in 2001, about \$20 billion worth of mandatory convertibles were issued (about 18% of the convertible market). Mandatory convertibles have been designed with a variety of payoff structures, and carry different names depending on their payoff structure and the investment bank underwriting their issue: examples are Morgan Stanley's PERCS and PEPS, Merrill Lynch's PRIDES, Salomon Brothers' DECS, and Goldman Sach's ACES. They have been issued by a number of companies, large and small, to raise capital: these include Texas Instruments, General Motors, Citicorp, Sears, Kaiser Aluminium, Reynolds Metals, American Express, First Chicago, Boise Cascade, and All State. Two prominent issuers were AT&T and Motorola, which raised \$900 million and \$1.2 billion, respectively, in 2001 by selling mandatory convertibles. See table 1 for illustrative examples of the different mandatory convertibles designed by different investment banks on behalf of various firms.

Even though there are differences among the above mentioned variations of mandatory convertibles in their payoff structures as well as in some other provisions, certain basic features are common to all of them. Three such features are as follows. First, as discussed above, conversion to equity is mandatory at the maturity of the convertible (as against conversion to equity at the option of the security holder in the case of ordinary convertibles). Second, mandatory convertibles have either a capped or limited appreciation potential compared to the underlying common stock. Third, the dividend yield on a mandatory convertible is typically higher than that on the underlying common stock. In section 2, we use two real world examples of mandatory convertible issues to illustrate the above three important features of mandatory convertibles.

The increasing popularity of mandatory convertibles over the last decade as an instrument for

raising capital by firms prompts us to raise several questions. When should a firm issue mandatory convertibles to raise capital, rather than issuing ordinary convertibles, or even more conventional securities such as straight debt? What explains the prevalence of the three fundamental features discussed above in almost all mandatory convertibles? How should a mandatory convertible be designed in terms of the mix of various features (e.g., the optimal cap, the number of shares of equity into which the mandatory convertible should be exchanged in the event of conversion, the dividend yield on the mandatory convertible)? Unfortunately, there has been no theoretical analysis so far in the literature which enables us to answer such questions. The objective of this paper is to develop a theoretical analysis of mandatory convertibles which allows us to answer the above and related questions.

Our analysis rests on two assumptions based on certain stylized facts about the mandatory convertibles market (and the securities market in general). First, firms are concerned about the misvaluation of their securities in the capital market, and would like to issue securities which would yield them the required amount of capital with the minimum dissipation in the long-term value of the equity held by the current shareholders. Second, firms are also concerned about their probability of being in financial distress (bankruptcy), and incurring financial distress costs. Thus, we consider a setting of asymmetric information, where firm insiders have more information about the intrinsic value of their firm compared to potential outside investors. In such a setting, higher intrinsic valued firms have an incentive to distinguish themselves from lower intrinsic valued firms in order to obtain their true value in the securities market. One way to accomplish this is to issue securities such as straight debt or ordinary (callable) convertibles, which have the possibility of forcing the firm into financial distress: since, for the same amount of debt issued, lower intrinsic valued firms have a higher chance of going into financial distress compared to higher intrinsic valued

firms, the former would not wish to mimic such a strategy, enabling higher intrinsic valued firms to separate themselves from lower intrinsic valued firms and obtain their true valuation in the securities market.

Such signaling strategies, however, have their own pitfalls. In a world with uncertainty, higher valued firms themselves have a positive probability of being in financial distress, and when the costs related to financial distress are significant, the cost of issuing straight debt or ordinary convertibles to distinguish themselves may exceed the valuation benefits from doing so (recall that there is a significant risk of financial distress in the case of ordinary convertibles, since conversion is at the option of convertible holders alone).¹ In such a situation, firms have an incentive to turn to mandatory convertibles. Since conversion to equity is mandatory in the case of these securities, firms do not have to be concerned about incurring financial distress costs if such securities are issued instead of straight debt or ordinary callable convertibles.² At the same time, mandatory convertibles enable the firm to minimize the extent of undervaluation of the firm's securities: we show that, while some undervaluation of intrinsically higher-valued firms is unavoidable if mandatory convertibles are issued, such undervaluation is lower than would be the case if the firm issued other securities (such as equity) which also do not increase the chance of the firm going into financial distress. Thus, whether a firm chooses, in equilibrium, to issue mandatory convertibles, or more conventional securities like straight debt, ordinary callable convertibles, or equity, depends on the

¹ The callability feature of convertibles does not eliminate this danger of the convertibles remaining as a fixed income security and the firm incurring financial distress costs. Calling these convertibles in order to force conversion will be optimal for the firm only if the share price is high enough, in which case there is no danger of financial distress in the first place. In other words, the callability feature of convertibles only serves to expedite conversion by convertible holders in the range of share prices where it is optimal for them to convert to equity in the first place; it cannot force conversion if the stock price is low.

² This advantage of mandatory convertibles in avoiding the costs associated with financial distress has been noted by practitioners. For example, see the magazine story entitled "Tech Companies Have a New Currency, and Its Mandatory" (Red Herring, January 2002). We quote: "Because they are guaranteed to convert to equity, mandatories come without the potential yield and redemption hassles for their issuers that other bonds carry. "If the stock drops, you don't get stuck with a bond that you have to continue to service" says F. Barry Nelson, senior vice president and portfolio manager of Advent Capital Management, which has \$900 million invested in convertibles."

magnitude of the above costs and benefits of issuing these different securities.

In the above setting, we develop a variety of results relevant to a firm's choice of mandatory convertibles as a means of raising capital. First, we develop predictions regarding the kind of firms which issue mandatory convertibles rather than more conventional securities, and the situations in which such firms will issue mandatory convertibles. In particular, our model predicts that, when faced with a choice between ordinary and mandatory convertibles, firms facing a larger extent of asymmetric information, but a relatively smaller probability of financial distress will choose to issue ordinary convertibles, while those facing a smaller extent of asymmetric information, but a larger financial distress probability will issue mandatory convertibles. Thus, a larger firm, which is already highly leveraged (or facing a financial downturn) will choose mandatory convertibles over ordinary convertibles, while a smaller firm, which is relatively debt free will make the reverse choice. Second, we develop a rationale for the prevalence of the three common features of mandatory convertibles discussed above, namely, mandatory conversion, capped (or limited) capital appreciation, and higher dividend yield relative to equity. Third, we characterize the optimal configuration of the above three features as well as the optimal exchange ratio (fraction of a firm's equity the mandatory convertible issue should convert into) for an issue of mandatory convertibles.

It is not our view here that asymmetric information and financial distress costs are the only two factors driving the issuance of mandatory convertibles. As Miller (1986) has noted, a number of financial innovations over the last twenty years have been driven by considerations of minimizing taxes: mandatory convertibles are no exception. Many mandatory convertible securities (e.g., PEPS and FELINE PRIDES) offer tax advantages: e.g., deductibility of the dividend paid, similar to the coupon paid on corporate debt. However, it is worth noting that many of the original mandatory convertible issues were not tax advantaged (i.e., the dividend paid was not tax deductible), so

that it is unlikely that the financial innovation of mandatory convertibles issues was prompted purely as a means of minimizing taxes. Rather, it seems to be the case that, while originally driven by other considerations, tax advantaged structures were added to make these securities more attractive to issuers. Another motivation driving the issuance of mandatory convertibles are legal restrictions on liquidating securities faced by large shareholders in some firms. These large shareholders issue mandatory convertibles which are convertible into the equity of their portfolio firms, thus immediately monetizing their holdings in their portfolio firms without having to sell these holdings immediately. Finally, another motivation driving the issuance of some mandatory convertibles may be “clientele” effects, i.e., driven by the desire of issuing firms to take advantage of institutional investors’ desire for higher dividend paying securities. In summary, similar to other securities like debt and equity, the issuance of mandatory convertibles may also be potentially driven by other market imperfections in addition to the ones we analyze here: we have chosen to focus here only on asymmetric information and financial distress costs as two of the most important of these, abstracting away from other considerations for the sake of analytical tractability.³

The existing literature on mandatory convertibles is quite small. Arzac (1997) provides a good description of some mandatory convertibles such as PERCS and DECS, with some discussion of the valuation of these based on the option pricing methodology.⁴ As mentioned before, there have been no theoretical models of the choice of firms between mandatory convertibles and other securities in the literature so far, and almost no empirical literature. Thus, the theoretical literature closest to this paper is the literature on the issue of ordinary convertibles in an environment of asymmetric

³ Note that, even if we explicitly include any tax advantages of issuing mandatory convertibles in our theoretical analysis, the equilibria studied here will continue to exist, though the parameter regions in which various equilibria arise will be modified. In other words, our qualitative results will hold even in this case.

⁴ There are also a few other practitioner oriented discussions and pedagogical cases on mandatory convertibles. Excellent examples include the HBS cases on Avon Products PERCS (Tiemann, 1989), Telmex PRIDES (Seasholes and Froot, 1996), Times Mirror PEPS (Tufano and Poetzscher, 1996), and Cox Communications FELINE PRIDES (Chacko and Tufano, 2000).

information: see, e.g., Constantinides and Grundy (1989), Brennan (1986), and Stein (1992).⁵ In particular, the rationale for issuing ordinary convertibles in our setting is similar to that in Stein (1992), though ordinary convertibles are not the focus of this paper.⁶ It is worth emphasizing that the rationale for issuing mandatory convertibles that we present here is completely new, and not derived from explanations for the issuance of any other security (including ordinary convertibles) that has been presented so far in the literature. Our paper is also broadly related to the literature on designing securities to minimize the dissipative costs imposed by market imperfections, such as Gale and Hellwig (1985), Allen and Gale (1988), or Nachman and Noe (1994). Finally, it is also indirectly related to the large literature on raising external financing under asymmetric information: see, e.g., Myers and Majluf (1984), Giammarino and Lewis (1988), or Noe (1988).

The rest of this paper is organized as follows. Section 2 provides two examples of mandatory convertibles. Section 3 describes the model. Section 4 characterizes the equilibria of the model and develops results. Section 5 characterizes the equilibrium design of mandatory convertibles. Section 6 describes the testable implications of the model, and section 7 concludes. The proofs of all propositions are confined to the appendix.

⁵ Brennan (1986) suggests a convertible security ("a reverting consol bond") which is completely insensitive to asymmetric information. The proposed security converts to the firm's equity at the end of a specified period of time at a conversion price dependent upon the prevailing price of the issuers' common stock at that time. However, Brennan's security relies crucially on the information asymmetry between firm insiders and outsiders resolving completely before the conversion date of the security (since the price at which the bond is converted needs to reflect the true value of the issuing firm) in order for the security to be insensitive to information asymmetry, a requirement unlikely to be satisfied in practice. In contrast, in our setting, issuing mandatory convertibles minimizes the valuation effects of long-lived private information (which continues to exist at the time of conversion), by limiting the upside payoff of the security (thus minimizing the difference in payoffs across mandatory convertibles issued by firms of different intrinsic values).

⁶ Some other papers provide rationales for issuing ordinary convertibles which are not based on asymmetric information: see, e.g., Green (1984) and Mayers (1998).

2 Two Examples of Mandatory Convertibles

In this section, we illustrate the three most important features of mandatory convertibles using two examples. The first example illustrates an issue of PERCS. In September 1991, K-Mart Corporation issued \$1.012 billion worth of PERCS at \$44.00 (K-Mart stock was also selling at this price on the day of issue). The PERCS paid a dividend of 7.75%, while K-Mart's common stock was paying a dividend of only 4% at this time. Each unit of PERCS was mandatorily convertible to one share of K-Mart common stock on September 15, 1994, subject to a cap of \$57.20: i.e., if the share price of K-Mart exceeded \$57.20, each unit of PERCS would receive only a fraction of a share worth a total of \$57.20. Figure 1(a) gives the payoff at maturity (excluding dividends) of the K-Mart PERCS, as a function of its underlying stock price.

The second example illustrates an issue of PEPS (Premium Equity Participating Securities). In June 2000, Valero Energy Corporation issued \$150 million worth of PEPS at \$25 per unit (which was the price of 0.85837 shares of its common stock, which was then selling at \$29.125 per share). The PEPS paid a quarterly dividend of 7.75%, while the dividend on the underlying common stock was only 2.75%. The PEPS were mandatorily convertible to shares of common stock on August 18th, 2003, with the number of shares per PEPS unit given to investors upon conversion depending on the price of the companies' common stock: if the price of the common stock was \$29.125 or below (so that 0.85837 shares would be worth \$25 or below), then each PEPS unit would receive only 0.85837 shares of common stock, giving them a payoff of \$25 or below. If the common was between \$29.125 and \$34.95, then PEPS holders would receive a variable number of shares such that their total value would remain at \$25 (in other words, \$25 was the "cap" value of the PEPS). If, however, the common stock price exceeded a "threshold appreciation price" of \$34.95 on the mandatory conversion date, each PEPS holder would receive 0.71531 shares of common stock.

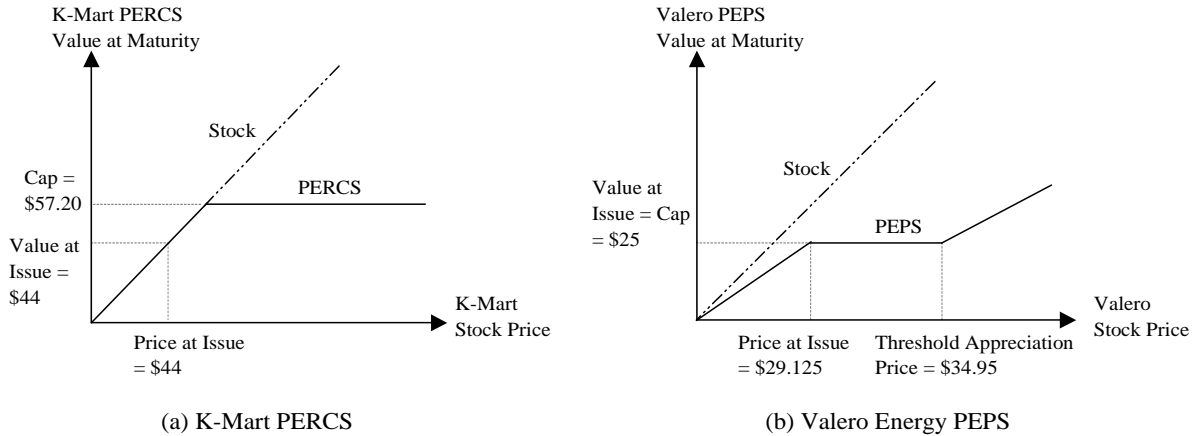


Figure 1: Payoff at Maturity (Excluding Dividends) of Two Mandatory Convertibles

Figure 1(b) gives the payoff at maturity (excluding dividends) of the Valero PEPS as a function of its underlying stock price.

Notice that, while the K-Mart PERCS value was completely capped at a price of \$57.20, the Valero PEPS holders would receive a fraction of the appreciation of the underlying stock beyond the threshold appreciation price of \$34.95. On the other hand, while holders of PERCS received 100% of the appreciation of the common stock between the stock price on the date of issue (\$44.00) and the cap price of \$57.20, the Valero PEPS holders did not receive any appreciation on their investment until the stock price exceeded the threshold appreciation price of \$34.95. In other words, the PEPS holder did not share in the first 20% of the appreciation in the underlying common stock (between the stock price of \$29.125 at issue and the threshold appreciation price of \$34.95). However, notice that both the PERCS and the PEPS issues share the three basic features, common to all mandatory convertibles, that we discussed above, namely, mandatory conversion, capped (either completely, as in the case of PERCS, or partially, as in the case of PEPS) appreciation potential, and dividend yield significantly in excess of the underlying common stock.⁷

⁷ Both PERCS and PEPS offerings were underwritten by Morgan Stanley. Most other mandatory convertibles,

3 The Model

The model has three dates (time 0, 1, and 2). Consider a risk-neutral entrepreneur owning an all-equity firm. To begin with, we assume that the entrepreneur owns all the equity in the firm: for simplicity, we normalize the number of shares of equity at time 0 to be one. The firm needs to raise an amount I externally to finance a new positive net present value project. We assume that the firm has no other ongoing projects, so that the cash flows received by the firm are the same as those generated by the new project. We normalize the risk-free rate of return to be zero, and assume that investors are risk-neutral as well.

3.1 Cash Flow and Information Structure

There are three types of firms: good (type G hereafter), medium (type M hereafter), or bad (type B hereafter). The cash flows from the new investment are realized at time 2. Each firm receives a gross cash flow of x_H (the high cash flow) or x_L (the low cash flow) at this date, $x_H > I > x_L$. The differences between the three types of firms are characterized by their probabilities of receiving the high and low cash flows at time 2. Further, at time 1, these firms “deteriorate” with a certain probability. In particular, the type k firm deteriorates with a probability ϕ_k , where $k \in \{G, M, B\}$. In the event of deterioration, all firm types realize the low cash flow x_L with probability 1. In the event there is no deterioration at time 1, the type k firm has a probability $1 - \delta_k$ of receiving the high cash flow x_H and δ_k of receiving the low cash flow x_L . For analytical simplicity, we assume

including those underwritten by investment banks other than Morgan Stanley, have a payoff structure similar to PERCS and PEPS (though these mandatory convertibles often differ from PERCS and PEPS in terms of many institutional arrangements). Thus, ACES (Automatically Convertible Equity Securities), PRIDES (Preferred Redemption Increased Dividend Equity Securities), FELINE PRIDES (Flexible Equity-Linked Exchangeable PRIDES), DECS, SAILS (Stock Appreciation Income Linked Securities), MARCS (Mandatory Adjustable Redeemable Convertible Securities), and TAPS (Threshold Appreciation Price Securities) are examples of mandatory convertibles with a payoff structure similar to PEPS. CHIPS (Common-linked Higher Income Participating Debt Securities), EYES (Enhanced Yield Equity Securities), TARGETS (Targeted Growth Enhanced Term Securities), and YES (Yield Enhanced Stock) are examples of securities which perform like PERCS. See Morgan Stanley (1998) and Nelken (2000) for a more detailed listing.

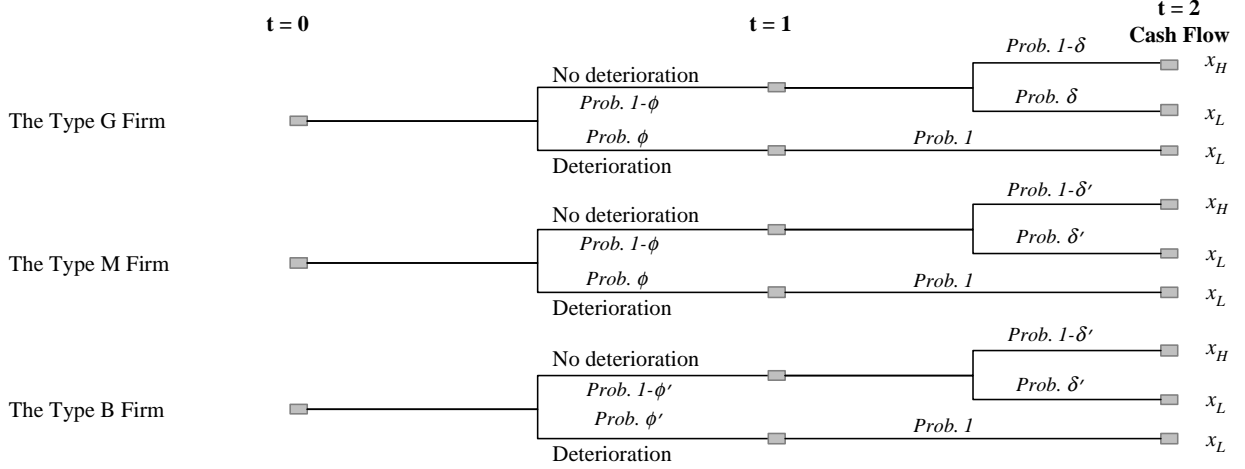


Figure 2: Cash Flow Structure for the Type G, Type M, and Type B Firms

throughout our analysis that $\phi_G = \phi_M \equiv \phi$, $\phi_B \equiv \phi'$, $\delta_G \equiv \delta$, and $\delta_M = \delta_B \equiv \delta'$, where $\delta < \delta'$ and $\phi < \phi'$. Thus, the difference between the type G and the type M firms is that, conditional on no deterioration, the type G has a lower probability of receiving x_L at time 2 than the type M. The difference between the type M and the type B firms is that the type M has a lower probability of deteriorating at time 1 than the type B.

The cash flow structure of the three types of firms is depicted in figure 2. Note that the ex ante (time 0) probability of the type B firm receiving a low cash flow, $\phi' + (1 - \phi')\delta'$, is greater than that of the type M firm, $\phi + (1 - \phi)\delta'$, which in turn is greater than that of the type G firm, $\phi + (1 - \phi)\delta$. Denote the true value of the type k firm $V_k = \phi_k x_L + (1 - \phi_k)[(1 - \delta_k)x_H + \delta_k x_L]$, where $k \in \{G, M, B\}$. Then, $V_G > V_M > V_B$. We assume that all three types of firm have positive net present value projects, i.e., $V_B \geq I$.

Firm types are private information to the entrepreneur at time 0, with outsiders having only a prior probability distribution over firm types. The outsiders' prior of any given firm being of type G, M, or B are γ_G , γ_M , and γ_B , respectively, $\gamma_G + \gamma_M + \gamma_B = 1$. At time 1, however, outsiders

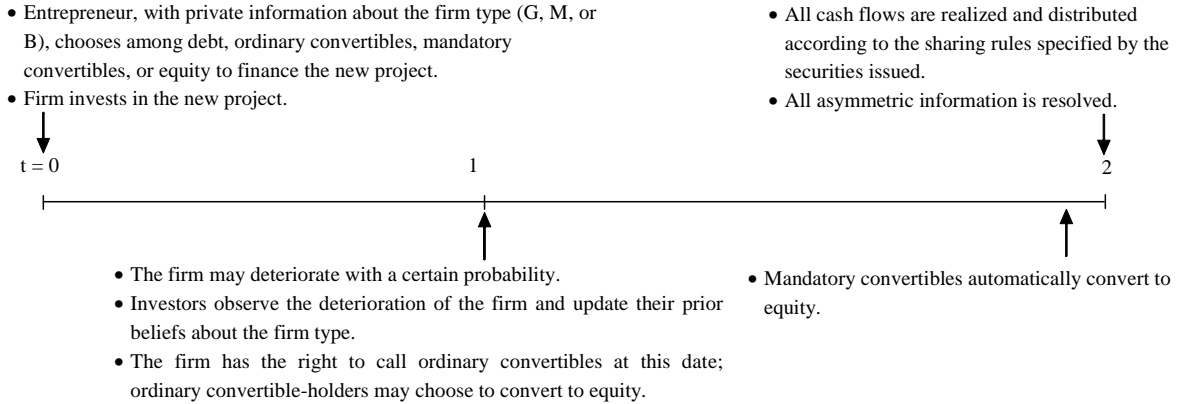


Figure 3: Sequence of Events

observe whether a firm has deteriorated or not. Based on this additional information, they engage in Bayesian updating about the type of the firm. At time 2, all asymmetric information is resolved (since all cash flows are realized at this time). The sequence of events is depicted in figure 3.

3.2 Menu of Securities

The entrepreneur can issue one of four different securities to raise the required external financing I : straight risky debt (“straight debt” denoted by DT), ordinary callable convertible debt (“ordinary convertibles” denoted by OC), mandatory convertibles (denoted by MC), or equity (denoted by EQ).

If the entrepreneur chooses to issue debt, he receives an amount I up-front at time 0, and promises to pay an amount P_d to the debt holder at time 2. If he chooses to issue ordinary callable convertible debt, he determines the face value P_c (payable to the convertible holders at time 2), the conversion ratio n_c , and the call price K at time 0. At time 1, he has the right to redeem (“call”) the convertibles at the call price K . If investors convert, they receive a ratio n_c of the total equity. If the convertibles are not called, they are equivalent to straight debt, with the issuing firm obligated to pay P_c to investors at time 2. In other words, P_c is the sum of the principal and

coupon if the convertible remains as straight debt. Alternatively, if the entrepreneur chooses to issue equity, he exchanges a fraction n_e of the total equity to investors for an amount I .

Finally, if the entrepreneur chooses to finance the amount I by issuing mandatory convertibles, these convertibles mandatorily convert to the firm's equity in two periods (prior to the resolution of information asymmetry at time 2). In this case, investors are promised a fraction n_m ("the exchange ratio") of the firm's equity upon conversion, provided the market value of this equity exchanged is less than a "cap" amount U_m . Here, we assume $n_m \leq \bar{n}$, where \bar{n} is the maximum possible exchange ratio, $\bar{n} \leq 1$. $\bar{n} = 1$ implies that the entrepreneur is willing to allow his entire equity holding in the firm to be exchanged for mandatory convertibles upon conversion; $\bar{n} < 1$ implies that the entrepreneur chooses to retain a certain fraction of equity for himself (perhaps due to incentive reasons or due to considerations of maintaining control in the firm). If the market value of the promised fraction of equity at time 1 is greater than U_m , then investors receive only shares worth the amount U_m . In addition, mandatory convertible holders receive an aggregate amount D of dividends over the life of the convertible.⁸ For analytical simplicity, we will assume that the actual payment of this amount D takes place at time 2.⁹ We also assume that $D \leq \bar{d}x_L$, where \bar{d} is the maximum possible fraction of the firm's sure cash flow that can be paid out as dividends.¹⁰

Clearly, $\bar{d} \leq 1$; $\bar{d} = 1$ implies that the firm is free to pay out its entire cash flow to investors as

⁸ In practice, the dividend paid on mandatory convertibles is greater than that on common equity. For simplicity, we assume here that the dividend paid on common equity is zero. Thus, one can think of this dividend D paid to mandatory convertible holders in our model as the dividend amount paid in excess of that paid to common equity.

⁹ This assumption is made only to minimize the complexity of our analysis. Since the discount rate is zero, assuming that the dividends are paid in quarterly amounts over the life of the mandatory convertible (as is the case in practice) is equivalent to a single payout at time 2, and will not change the nature of our results.

¹⁰ We use the term dividends here only to refer to sure (certain) cash flows proposed to investors, which is why we assume that dividends can at most equal x_L (which happens only when $\bar{d} = 1$). We will see later that it is the sure cash flow promised to investors that is important in the design of mandatory convertibles. Of course, in practice, firms can promise additional (uncertain) cash flows to investors over and above this amount. But investors will see these additional dividends as no different from other cash flows available at time 2. We therefore choose to refer only to the sure cash flow promised to investors as dividends, clubbing all uncertain cash flows available to investors together as the "distribution of residual project cash flows" at time 2.

dividends when the cash flow realized is low, and $\bar{d} < 1$ implies that the firm needs to retain part of its realized cash flows (perhaps to cover other operational expenses or implement other projects), and pay only the rest as dividends. In our model, both the cap U_m and the dividends paid D of the mandatory convertibles are determined endogenously in equilibrium.

If straight debt is issued by the entrepreneur, or if ordinary convertibles are issued and the firm does not force conversion at time 1 (in which case, the ordinary convertibles are equivalent to straight debt), costly financial distress may occur at time 2. If the firm's cash flow at this date is not sufficient to pay the promised payment to debt holders or convertible holders in full, the firm will be forced into financial distress (bankruptcy). In this case, an exogenous deadweight cost of financial distress $C > 0$ is imposed on the entrepreneur. Remember that since $x_L < I < x_H$, financial distress occurs only if the cash flow at time 2 turns out to be low. For example, the ex-ante (time 0) probability of financial distress in the case of raising the investment amount by issuing straight debt is the same as the ex-ante probability of earning a low cash flow, which is $\phi_k + \delta_k(1 - \phi_k)$ for a type k firm; $k \in \{G, M, B\}$.

Of the above menu of contracts, the security actually issued by the firm will be determined in equilibrium: i.e., not all securities will be offered in all situations. We assume that the firm first chooses the security to be issued from the above menu (at time 0). Further, in the case where an ordinary convertible is issued, the firm chooses at time 1 whether or not to force conversion by calling the convertible; also, investors choose whether or not to convert the convertible to equity at this date. Finally, we assume that any firm issuing mandatory convertibles (a new financial innovation) will incur a small marketing cost ε to sell these securities to investors, which they do not need to incur if the security issued is equity, ordinary convertibles, or straight debt.¹¹

¹¹ The marketing cost for new financial innovations like mandatory convertibles that we assume here is meant to capture the notion that a firm is unlikely to issue innovative securities if it is as well off by issuing conventional

3.3 The Objective of Entrepreneur and Outside Investors

The entrepreneur's objective is to maximize the expected long-term (time 2) value of the equity held by him (or equivalently, his time 0 expectation of the cash flows accruing to the equity retained by him at time 2), net of the cost of financial distress associated with the external financing of the amount I and the marketing cost ε in case he issues mandatory convertibles. The entrepreneur has four options to finance his firm's new project: straight debt, ordinary convertibles, mandatory convertibles, or common equity.¹² The entrepreneur will choose to issue that security from the above menu which maximizes the value of his objective.

We now discuss the value of the entrepreneur's objective if he chooses to issue each of the above securities. In each case, the entrepreneur will choose the specifics of the security issued (face value for debt; face value and conversion features for ordinary convertibles; fraction of equity sold when equity is issued; and exchange ratio, cap, and dividends for mandatory convertibles) in order to raise the amount I of external financing required, subject to satisfying the outsider investors' break-even constraint.¹³

If a type k firm chooses to issue debt, then it maximizes its expected payoff at time 0:

$$\Pi_k(DT) = (1 - \delta_k)(1 - \phi_k)(x_H - P_d) - [1 - (1 - \delta_k)(1 - \phi_k)]C, \quad (1)$$

where $k \in \{G, M, B\}$ is the true type of the firm. The outsider investors' break-even constraint is given by:

$$(1 - \delta_j)(1 - \phi_j)P_d + [1 - (1 - \delta_j)(1 - \phi_j)]x_L \geq I, \quad (2)$$

securities such as equity, debt, or ordinary convertibles. There is some evidence that firms (or investment banks acting on their behalf) incur such costs of marketing innovative securities in practice. In the setting of our model, our results go through even if such marketing costs are extremely small.

¹² Since the entrepreneur is the decision maker in the firm and chooses the security to issue, we will often talk about the firm and the entrepreneur interchangeably from now on.

¹³ In the case of separating equilibria, the specifics of the security issued also be affected by incentive compatibility conditions, which we discuss in section 3 (when we analyze the details of each equilibrium).

where δ_j and ϕ_j are the ex-ante probability of deterioration and the ex-ante probability of low cash flow in case of no deterioration, respectively, under the investors' belief about the firm type(s) j that may issue debt. In other words, j refers to the set of firm types that investors perceive to have issued a specific security; $j \in \{G, M, B, GM, GB, MB, GMB\}$. For example, if investors believe that type k firm issues debt, then $j = k$. On the other hand, if investors believe that all three types of entrepreneurs are likely to issue debt, then $j = GMB$, in which case $\delta_{GMB} = \frac{\gamma_G(1-\phi_G)\delta_G + \gamma_M(1-\phi_M)\delta_M + \gamma_B(1-\phi_B)\delta_B}{\gamma_G(1-\phi_G) + \gamma_M(1-\phi_M) + \gamma_B(1-\phi_B)}$, and $\phi_{GMB} = \gamma_G\phi_G + \gamma_M\phi_M + \gamma_B\phi_B$. Or if investors believe that both type M and type B firms are likely to issue debt, then $j = MB$, in which case $\delta_{MB} = \frac{\gamma_M(1-\phi_M)\delta_M + \gamma_B(1-\phi_B)\delta_B}{\gamma_M(1-\phi_M) + \gamma_B(1-\phi_B)} (= \delta')$, and $\phi_{MB} = \frac{\gamma_M\phi_M + \gamma_B\phi_B}{\gamma_M + \gamma_B}$. The objective function (1) equals the residual cash flow after paying P_d to investors at time 2, net of the expected financial distress cost, $[1 - (1 - \delta_k)(1 - \phi_k)]C$.¹⁴ The constraint (2) ensures that the promised payment to investors at time 2, P_d , should at least be large enough that investors break-even from their investment under their beliefs about the set of firms that are likely to issue debt.

If a type k entrepreneur chooses to issue equity, then the value of his objective at time 0 is given by:

$$\Pi_k(EQ) = [(1 - \delta_k)(1 - \phi_k)(x_H - x_L) + x_L](1 - n_e). \quad (3)$$

The outside investors' break-even constraint is then given by:

$$V_j(1 - n_e) \geq I. \quad (4)$$

Here, V_j is the ex-ante (time 0) value of the firm under outside investors' belief that a firm issuing equity is of type(s) j : $V_j = (1 - \phi_j)(1 - \delta_j)(x_H - x_L) + x_L$. For example, if the market perceives a

¹⁴ Recall that we assume $\phi_G = \phi_M \equiv \phi$, $\phi_B \equiv \phi'$, $\delta_G \equiv \delta$, and $\delta_M = \delta_B \equiv \delta'$. Thus, the objective function (1) would be $\Pi_G(DT) = (1 - \delta)(1 - \phi)(x_H - P_d) - [1 - (1 - \delta)(1 - \phi)]C$ for the type G entrepreneur; $\Pi_M(DT) = (1 - \delta')(1 - \phi)(x_H - P_d) - [1 - (1 - \delta')(1 - \phi)]C$ for the type M entrepreneur; and $\Pi_B(DT) = (1 - \delta')(1 - \phi')(x_H - P_d) - [1 - (1 - \delta')(1 - \phi')]C$ for the type B entrepreneur.

firm issuing equity as either a type B or a type M firm, then $V_j = V_{MB} = (1 - \phi_{MB})(1 - \delta_{MB})(x_H - x_L) + x_L$.

If a type k entrepreneur chooses to issue ordinary convertibles, the value of his objective at time 0 is given by:

$$\Pi_k(OC) = (1 - \phi_k)[(1 - \delta_k)x_H + \delta_k x_L](1 - n_c) - \phi_k C, \quad (5)$$

which equals the entrepreneur's expected value of the equity retained by him at time 2 after the redemption of the convertibles, net of the expected financial distress cost, $\phi_k C$, incurred by him. This objective function takes into consideration that at time 1, the firm chooses to force conversion (by calling the convertibles) only when the value of the equity foregone in exchange for the convertibles is less than the sum of the promised payment on the convertibles, and the fact that the firm will be in financial distress if (and only if) the convertibles are not called at time 1. Note that the entrepreneur will choose to force conversion only if his firm does not deteriorate at time 1 (in other words, the entrepreneur will force conversion with an ex-ante probability $1 - \phi_k$ for a type k firm). The outside investors' break-even constraint is given by:

$$n_c(1 - \phi_j)[(1 - \delta_j)x_H + \delta_j x_L] + \phi_j x_L \geq I, \quad (6)$$

where j is the firm type(s) perceived by investors to issue ordinary convertibles.

Finally, if a type k firm chooses to issue mandatory convertibles, the value of entrepreneur's objective is given by:

$$\Pi_k(MC) = \phi_k(1 - n_m)(x_L - D) + (1 - \phi_k)[1 - \min(\frac{U_m}{V_j^1 - D}, n_m)](V_k - D) - \varepsilon. \quad (7)$$

The outside investors' break-even constraint is given by:

$$n_m(x_L - D)\phi_j + \min(\frac{U_m}{V_j^1 - D}, n_m)(V_j^1 - D)(1 - \phi_j) + D \geq I, \quad (8)$$

where j is the set of firm type(s) perceived by outside investors as issuing mandatory convertibles. In the objective function (7), $(1 - n_m)(x_L - D)$ is the value of equity received by the entrepreneur if the firm deteriorates at time 1, and $[1 - \min(\frac{U_m}{V_j^1 - D}, n_m)](V_k - D)$ is the value of the equity received by the entrepreneur if the firm does not deteriorate. Here, V_j^1 is the expected time 1 value of a firm if it does not deteriorate at time 1, under the outside investors' belief that a firm issuing mandatory convertibles is of type j ; $V_j^1 = (1 - \delta_j)x_H + \delta_j x_L$; the corresponding time 0 value $V_j = (1 - \phi_j)V_j^1 + \phi_j x_L$. For example, if investors believe that all three firm types issue mandatory convertibles, then $j = GMB$. In this case, $V_j^1 = V_{GMB}^1 = (1 - \delta_{GMB})(x_H - x_L) + x_L$.

Thus, at time 0, the entrepreneur strategically chooses the type of security to issue by comparing the above four expected payoffs: we will discuss the trade-offs faced by the entrepreneur in making the above choice in the next section. The objective of investors is to maximize the expected value of the cash flows they obtain from the firm. Thus, holders of ordinary convertibles choose to convert to equity only when the value of equity obtained through the exchange exceeds the promised payment (face value) of the convertibles.

4 The Equilibrium

Equilibrium strategies and beliefs in our model are defined as those constituting a Pareto dominant or Efficient Perfect Bayesian Equilibrium (PBE) which survives the Cho-Kreps intuitive criterion. Before going on to characterize the equilibria of our model, we analyze the problem faced by each type of firm.¹⁵

In the rest of the paper, we assume that $\frac{\delta'}{\delta} > \frac{\phi'}{\phi}$. This assumption implies that the difference

¹⁵ Thus, we look for Perfect Bayesian Equilibria which maximize the objective of higher type firms, by minimizing the dissipative costs incurred by them. See Fudenberg and Tirole (1991) for a formal definition of a PBE, and Milgrom and Roberts (1986) for an application of Pareto dominant or Efficient PBE to signaling games. The Cho-Kreps Intuitive Criterion is formally defined in Cho and Kreps (1987).

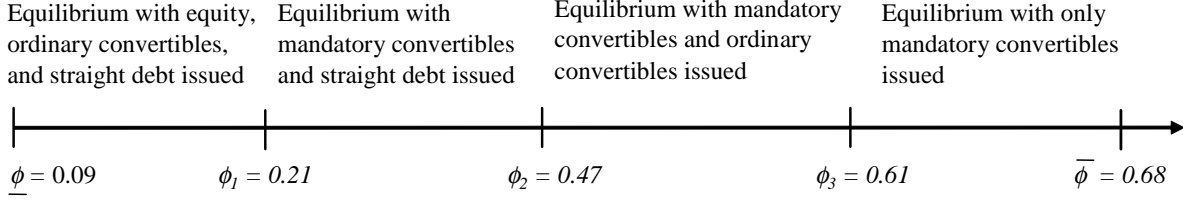


Figure 4: Effect of a Decrease in Asymmetric Information and Increase in Financial Distress Probability on the Securities Issued. In this numerical example, $\delta = 0.1$, $\delta' = 0.8$, $\phi' = 0.7$, $x_H = \$80$, $x_L = \$10$, $I = \$20$, $C = \$30$, $\varepsilon = \$0.1$, $\gamma_G = 0.2$, $\gamma_M = 0.3$, and $\gamma_B = 0.5$. This figure depicts the parameter space for various equilibria when the probability of bankruptcy of the type G firm ϕ increases (or equivalently when the information asymmetry faced by the type G firm decreases). When $\phi \in (0.09, 0.21)$, the securities issued in equilibrium consist of equity, ordinary convertibles and straight debt (proposition 1). When $\phi \in [0.21, 0.47)$, the securities issued in equilibrium consist of mandatory convertibles and straight debt (proposition 2). When $\phi \in [0.47, 0.61)$, the securities issued in equilibrium consist of mandatory convertibles and ordinary convertibles (proposition 3). Finally, when $\phi \in [0.61, 0.68)$, the security issued in equilibrium consists of only mandatory convertibles (proposition 4).

in intrinsic values between the type G and the type M firm is greater than that between the type M and the type B firm. We also assume that the deadweight cost of bankruptcy is large enough that $C > \frac{(\delta' - \delta)(I - x_L)}{\delta'(1 - \phi)(1 - \delta)}$. The above two parametric assumptions are equivalent to assuming that $\underline{\phi} < \phi < \bar{\phi}$, where $\underline{\phi} = \frac{\delta\phi'}{\delta'}$ and $\bar{\phi} = 1 - \frac{(\delta' - \delta)(I - x_L)}{\delta'(1 - \delta)C}$. We further assume that $\delta \leq \bar{\delta}$, so that the type G firm has only a small probability of realizing a low cash flow in the case of no deterioration; and also that $\delta' \geq \underline{\delta}$, implying that the type M and B firms have a significant probability of realizing a low cash flow in the case of no deterioration. $\bar{\delta}$, $\underline{\delta}$ and the threshold values for various parameters in the propositions below are defined in the Appendix. Finally, we assume that $\bar{n} = 1$ and $\bar{d} = 1$; we relax this assumption in section 4.

In the rest of this section, we first analyze the trade-offs faced by each type of firm in choosing from the menu of securities available to it (section 3.1). We then characterize the equilibria under different situations. When we increase ϕ from $\underline{\phi}$ to $\bar{\phi}$ (keeping all other parameters constant), the probability of the type G firm realizing a low cash flow (through deterioration) increases. This has

two consequences. First, the probability of financial distress (and therefore the expected cost of financial distress) of the type G firm if it issues straight debt or ordinary convertibles increases. Second, the difference in intrinsic values between the type G and the lower type firms decreases, so that the extent of asymmetric information facing the type G firm decreases. We will present the equilibria starting from a situation where the extent of asymmetric information faced by the type G firm is severe, while its financial distress probability is low (this will be the case when ϕ is very low and close to $\underline{\phi}$) and ending with the situation where the extent of asymmetric information faced by the type G firm is small, while its financial distress probability is high (this will be the case when ϕ is large and close to $\bar{\phi}$). We study four different kinds of equilibria (in terms of the securities issued by various firm types) depending on the range of values of ϕ in sections 3.2, 3.3, 3.4, and 3.5 respectively: thus, we characterize the equilibria for the entire range of values of $\phi \in (\underline{\phi}, \bar{\phi})$. In the appendix proofs of various propositions, we show that the equilibrium is unique in each range of values of ϕ (see figure 4, which provides a numerical illustration of the range of values of ϕ where various kinds of equilibria prevail).

4.1 Analysis of the Firm's Problem

We now analyze the trade-offs faced by the three types of firms in arriving at their equilibrium choice of security to issue. In our discussion below, we will focus primarily on the type G firm's problem and discuss the type M and the type B firm's problems only briefly.

4.1.1 The Type G Firm's Problem

The type G firm has a higher intrinsic value than the type M and type B firms. Thus, the type M and type B firms have an incentive to mimic the type G because, if they pool with the type G, their securities would be overvalued. On the other hand, the type G firm has an incentive to separate itself from the type M and type B firms since its securities would be undervalued if it

pools with these lower type firms. It can do this by issuing a security which the type M and B firms find sub-optimal to offer. Debt or ordinary convertibles are such securities, since the type M and type B firms have to incur costs of financial distress with some probability if they mimic the type G firm by issuing these two securities. However, issuing debt or ordinary convertibles could also result in the type G firm incurring a cost of financial distress. Thus, the equilibrium choice of the security to issue made by the type G entrepreneur is determined by the trade off between the cost of undervaluation (if the type G chooses to pool with the other types) and the expected cost of financial distress (if the type G chooses to separate itself from the other types). In the following, we discuss the trade-offs faced by the type G entrepreneur in choosing between debt, equity, ordinary convertibles, and mandatory convertibles.

We first discuss the case where the type G firm chooses to separate itself from the type M and type B firms. One way to achieve this separation is by issuing straight debt. The type M and B firms have less incentive to issue straight debt compared to the type G firm, since they are more likely than the type G to incur a financial distress cost if they issue debt. In particular, the type M and B firms have probabilities $[1 - (1 - \delta')(1 - \phi)]$ and $[1 - (1 - \delta')(1 - \phi')]$, respectively, of incurring a financial distress cost at time 2, which occurs when the firm realizes the low cash flow x_L at that time. When the expected financial distress cost is substantially large, the benefit to the type M and B firms from mimicking the type G (i.e., overvaluation of their securities) is lower than the expected financial distress cost arising from doing so. While the type G firm also incurs a financial distress cost with a probability $[1 - (1 - \delta)(1 - \phi)]$ if it issues debt, its probability of financial distress is smaller than that of the type M and type B.

The type G firm could also issue ordinary convertibles to achieve separation from lower firm types. The key difference between straight debt and ordinary convertibles is that ordinary con-

vertibles allow the issuing firm to call its convertibles (thus forcing conversion to equity) at time 1, so that ordinary convertibles would induce a lower probability of financial distress to the issuing firm compared to straight debt. In particular, in the event that the issuing firm does not deteriorate at time 1, the firm's share price will go up at that time, so that the firm will be able to force conversion to equity by calling back the convertible, thereby avoiding incurring financial distress costs in this scenario. Note, however, that issuing ordinary convertibles does not completely eliminate the probability of financial distress for the issuing firm: if the issuing firm deteriorates at time 1, the convertibles remain as straight debt, since the conversion value of the convertibles will be below the call price K in this case, and the firm is unable to call back the convertible.¹⁶

Thus, the probability of financial distress associated with ordinary convertibles may still enable the type G firm to separate itself from the other firm types. However, compared to straight debt, ordinary convertibles have the disadvantage of being less effective in achieving separation, since ordinary convertibles entail a lower probability of financial distress to the type B and M firms as well: if the type M and B firms mimic the type G by issuing ordinary convertibles, they face only probabilities ϕ and ϕ' , respectively, of falling into financial distress. But ordinary convertibles also have an advantage: due to the reasons discussed above, the type G firm can reduce the probability of financial distress to itself by issuing ordinary convertibles rather than straight debt (the type G only faces a probability ϕ of deterioration at time 1).

In summary, the type G firm can potentially achieve separation from the lower type firms by issuing either straight debt or ordinary convertibles. Its choice between these two securities to achieve separation depends on the severity of asymmetric information it faces in the financial

¹⁶ A deteriorated firm will not only be unable to use the call provision to force conversion, but will also be unable to use this provision to exchange the convertible for cash. Since $x_L < K$, the call price exceeds the market value of the firm upon deterioration, so that the firm will not be able to raise the cash at time 1 to pay the call price K .

market (i.e., the benefit to the lower types from mimicking it, which is greater when the difference in intrinsic values between the type G and the lower type firms is greater) and its own probability of financial distress (which, in turn affects its expected cost of financial distress). When the extent of asymmetric information it faces is high while its financial distress probability is low, the type G firm chooses to achieve separation by issuing straight debt. In this case, it can be shown that the firm cannot achieve separation by issuing ordinary convertibles (since the benefit to the lower type firms from mimicking the type G exceeds the expected value of financial distress costs incurred by them). On the other hand, if the extent of asymmetric information faced by the type G firm is low, while its low cash flow probability is large, the type G firm chooses to achieve separation by issuing ordinary convertibles. In this case, the type G firm could have achieved separation by issuing straight debt as well, but chooses to use ordinary convertibles instead since the expected value of its financial distress cost would be higher if it attempts to achieve separation by issuing straight debt.

We now discuss the case where the type G firm is better off pooling with the lower firm types rather than separating from them: this occurs when the probability of financial distress (low cash flow) is large than in the situation discussed above, while the extent of asymmetric information it faces in the financial market is smaller than in those situations. In case the type G firm chooses to pool with the lower firm types, it chooses that security to issue when pooling with these lower firm types to maximize its objective: i.e., the security will minimize the expected value of the sum of its adverse selection (undervaluation), financial distress, and marketing costs.

As long as the deadweight cost of financial distress is significant, the type G firm will not choose to pool with the type M and B firms by issuing straight debt or ordinary convertibles, since issuing those securities to pool with lower type firms involves the type G firm incurring the above dissipative

cost of financial distress with a significant probability without obtaining any commensurate benefit (since any benefits to the type G arising from issuing the above securities in a pooling equilibrium would be overcome by the significant financial distress costs that it would have to incur by doing so). The type G firm will choose to pool with lower type firms by issuing only equity or mandatory convertibles, since both of these allow the firm to avoid incurring any probability of financial distress (i.e., expected financial distress costs will be zero). The type G firm's choice between issuing equity or mandatory convertibles if it chooses to pool with the lower type firms will therefore depend on a comparison of the magnitude of the dissipative costs arising from asymmetric information (i.e., undervaluation) that it will incur if it issues each security (assuming that the marketing costs associated with issuing mandatory convertibles are very small). We discuss the type G's choice between these two securities to pool with lower type firms below.

If it chooses to issue equity, the type G maximizes $\Pi_G(EQ)$ as in (3), subject to the outsider investors' break-even constraint (4). Substituting constraint (4) holding as an equality into (3), the type G's objective can be transformed to:

$$\Pi_G(EQ) = V_G - I - \frac{V_G - V_{GMB}}{V_{GMB}} I. \quad (9)$$

$\frac{V_G - V_{GMB}}{V_{GMB}} I$ captures the cost of undervaluation for the type G firm if it issues equity to pool with the type M and the type B. On the other hand, if the type G firm issues mandatory convertibles, it maximizes $\Pi_G(MC)$ as in (7), subject to the outside investors' break-even constraint (8). Substituting constraint (8) holding as an equality into (7), we get:

$$\Pi_G(MC) = (V_G - I) - \frac{V_G - V_{GMB}}{V_{GMB} - D} (I - D) + \frac{(V_{GMB}^1 - D)(\phi_{GMB} - \phi_G)(x_L - D)}{V_{GMB} - D} [n_m - \min(\frac{U_m}{V_{GMB}^1 - D}, n_m)] - \varepsilon. \quad (10)$$

The second and the third terms in (10) capture the undervaluation cost for the type G firm from issuing mandatory convertibles. Comparing (9) and (10), we can show that, if the type G firm

designs mandatory convertibles so that $n_m \geq \frac{U_m}{V_{GMB-D}^1}$ and $D \leq x_L$, then $\Pi_G(MC) > \Pi_G(EQ)$, i.e., the adverse selection cost (undervaluation) from issuing mandatory convertibles is smaller than that from issuing equity (since $\frac{V_G - V_{GMB}}{V_{GMB-D}}(I - D) < \frac{V_G - V_{GMB}}{V_{GMB}}I$ and $\frac{(V_{GMB}^1 - D)(\phi_{GMB} - \phi_G)}{V_{GMB-D}} > 0$).

The smaller adverse selection cost for mandatory convertibles comes from two special features of mandatory convertibles. First, unlike equity, mandatory convertibles provide a “cap” on the “upside” cash flow paid to security holders. The effect of this cap is to reduce the difference in the cash flow obtained by investors in the high and the low cash flow scenarios. This, in turn, means that the difference between the intrinsic (true) values of mandatory convertibles issued by the type G and the lower type firms (type M and B) is less than the corresponding difference in the intrinsic values of the equity issued by the type G and lower type firms. Thus, the subsidization of the type M and B firms by the type G is lower if the type G issues mandatory convertibles, so that the undervaluation of the type G firm’s securities due to asymmetric information is less if it issues mandatory convertibles rather than equity (in other words, the market value of mandatory convertibles is less sensitive to asymmetric information compared to that of equity). The effect on the adverse selection cost from the cap U_m is captured by the third term in (10). When the cap is binding, i.e., $\frac{U_m}{V_{GMB-D}^1} < n_m$, the third term is positive.

Second, mandatory convertibles also compensate investors for the capped upside cash flow with an incremental dividend payment, D . When $D \leq x_L$, the cash flow D is risk free and thus not affected by the asymmetric information in the financial market. The effect of the dividend feature is captured in the second term in (10). Due to the existence of risk-free dividends, the remaining part of the cash flow given to outsider investors in mandatory convertibles entails a smaller cost of undervaluation compared to the case when equity issued, i.e., $\frac{V_G - V_{GMB}}{V_{GMB-D}}(I - D) < \frac{V_G - V_{GMB}}{V_{GMB}}I$. In sum, because of the capped upside payments and the incremental dividend payment, mandatory

convertibles are a better security for the type G firm to issue if it chooses to pool with the lower firm types.

4.1.2 The Type M Firm's Problem

The type M has two choices to make when deciding the security to issue to fund its project: First, whether to attempt to pool with the type G firm or separate from it; and second, whether to allow the type B firm to mimic it (i.e., to pool with it) or to separate from it as well. Whether the type M chooses to separate or pool with the type G is determined by the security issued by the type G firm: if the type G firm chooses to separate itself from the type M and type B, the security it issues (straight debt or ordinary convertibles) will be such that any benefit to the type M from mimicking the type G (arising from overvaluation) is smaller than the expected cost of doing so (arising from its expected cost of financial distress). Thus, if the type G chooses to separate itself from lower type firms, the type M's choice of security to issue is determined by whether it wishes to pool or separate from the type B firm.

The type M firm's choice between separating and pooling with the type B depends on the relative magnitudes of its adverse selection costs (undervaluation) and its expected costs of financial distress. Similar to the type G firm's problem, if the extent of asymmetric information faced by the type M is large (i.e., the intrinsic value of the type B is significantly lower than that of the type M) while the type M's probability of financial distress is low, then the type M firm chooses to separate itself from the type B. While such a separation from the type B firm can be accomplished by issuing either straight debt or ordinary convertibles (as discussed under the type G firm's problem), the type M would prefer to achieve this separation by issuing ordinary convertibles (if possible), since this would minimize its own expected costs of financial distress. If, on the other hand, the extent of asymmetric information faced by the type M firm is smaller (i.e., the difference in intrinsic values

of the type M and type B firms is smaller) while its probability of financial distress is higher than that in the above situation, then the type M firm chooses to pool with the type B. As discussed under the type G firm's problem, the type M will choose to pool with the type B only by issuing equity or mandatory convertibles as long as the deadweight cost of financial distress is significant (as assumed throughout the paper). Further, it can be shown that, as long as the marketing cost of issuing mandatory convertibles is small, the type M will prefer to pool with the type B by issuing mandatory convertibles rather than equity, since issuing mandatory convertibles can minimize the adverse selection costs of pooling with the type B (as discussed under the type G firm's problem).

Finally, if the type G firm chooses to pool with the lower firm types (type M and type B) by issuing mandatory convertibles, the type M will also choose to issue mandatory convertibles. In addition to the advantage to the type M of using mandatory convertibles to pool with the type B as discussed above, in this case, issuing mandatory convertibles has the added advantage to the type M of allowing it to pool with a higher firm type as well.

4.1.3 The Type B Firm's Problem

The choice faced by the type B firm is whether to pool or separate from higher type (type M and G) firms. Whether the type B chooses to separate or pool with the higher types of firms is determined by the latter firms: if the type G and the type M choose to separate themselves from the type B, the securities they issue (e.g., straight debt issued by the type G, and ordinary convertibles issued by the type M) will be such that any benefit to the type B from mimicking the higher types (arising from overvaluation) is smaller than the expected cost of doing so (arising from its expected cost of financial distress). In this case, the type B firm will be forced to separate itself by issuing a security different from the higher firm types. Since there is no benefit to the type B from incurring any probability of financial distress if it chooses to separate itself, it will choose

to issue either equity or mandatory convertibles in this case (issuing straight debt and ordinary convertibles will require it to incur some financial distress probability). Further, given the small marketing cost associated with issuing mandatory convertibles, it will choose to issue equity (since issuing equity does not involve any such costs).

If, on the other hand, the higher type firms (either type M alone, or both type G and type M) prefer to pool with the type B, they will choose to do so by issuing mandatory convertibles (as we discussed under the type G and type M firm's problems). In this case, the type B firm will also choose to issue mandatory convertibles since any small marketing costs it needs to incur in issuing mandatory convertibles will be dominated by the overvaluation benefits arising from pooling with the higher firm types.

4.2 Equilibrium with Straight Debt, Equity, and Ordinary Convertibles

We first consider the case where straight debt, equity, and ordinary convertibles are issued in a separating equilibrium. Proposition 1 characterizes the equilibrium.¹⁷

Proposition 1 (Equilibrium without Mandatory Convertibles) *When $\phi \leq \phi_1$, so that the probability of financial distress of the type G firm is low while the extent of asymmetric information it faces in the financial market is high, the equilibrium is fully separating, and involves the following:*¹⁸

The type G firm: *It issues straight debt at time 0, with a face value $P_d^* = \frac{I-x_L}{(1-\phi)(1-\delta)} + x_L$, maturing at time 2.*

The type M firm: *It issues an ordinary convertible at time 0, with a call price K , $x_L < K < I$, and a face value $P_c > x_L$, which is convertible to a fraction $n_c^* = \frac{I-\phi x_L}{(1-\phi)[(1-\delta')x_H + \delta'x_L]}$ of the firm's equity. It calls back the convertible at time 1, if its conversion value (the market value of the equity obtained upon conversion) at that time is above the call price K .*

The type B firm: *At time 0, it raises the amount I by issuing new equity equal to a fraction $n_e^* = \frac{I}{V_B}$ of the firm's total equity outstanding.*

¹⁷ The intuition driving this equilibrium is similar to that driving the separating equilibrium in Stein (1992), involving the issuance of ordinary convertibles. While ordinary convertibles are not the focus of this paper, this proposition is useful as a starting point for our analysis.

¹⁸ The equilibrium beliefs of outsiders are as follows: If a firm issues straight debt, they infer that it is of type G with probability 1. On the other hand, if a firm issues an ordinary convertible, they infer that it is of type M with probability 1. In this case, outsiders convert to equity at time 1 if the firm does not deteriorate at time 1, inducing the firm to call back the convertible. Further, if a firm issues equity, outsiders infer that it is of type B with probability

The above proposition characterizes the situation where ϕ is small so that the probability of financial distress faced by the type G (as well as type M) firm(s) is small, while the extent of asymmetric information they face in the financial market is severe (since, when ϕ is small, the difference in intrinsic values between the type G, type M, and type B firms is large). In this case, the type G and type M firms find it optimal to distinguish themselves from the type B firm by issuing straight debt and ordinary convertible debt respectively. The type B firm, on the other hand, does not find it optimal to mimic the type G or type M firms by issuing similar securities, since its expected financial distress cost arising from issuing straight debt and ordinary convertibles respectively exceeds its benefit from mimicking the type G and the type M.

Similar to the type B (and the type M), the type G would also incur a financial distress cost (thought with a smaller probability) if it issues straight debt. However, issuing straight debt allows the type G to separate itself from the type M and type B, thus avoiding the undervaluation of its security which would arise if it pooled with the lower firm types. Further, the extent of such undervaluation faced by the type G is larger as the extent of asymmetric information faced by the firm in the financial market is more severe. Thus, when the expected cost of financial distress of the type G is small enough, or when the extent of asymmetric information it faces is severe enough (so that the benefit of separation from the other types is large enough), the type G firm prefers to issue straight debt in equilibrium.

Thus, in this equilibrium, the type G firm maximizes $\Pi_G(DT)$ given by (1), subject to the break-even constraint (2) where $j = G$ and the following incentive compatibility (IC) constraints

1. Finally, if a firm issues mandatory convertibles (an out of equilibrium move), outsiders infer that it is of type B with probability 1. We will show in the appendix that the out of equilibrium beliefs specified in this proposition, as well as in other propositions, satisfy the Cho-Kreps intuitive criterion, as required by our equilibrium definition.

of the type M and type B firms, respectively:

$$\Pi_M(DT) \leq \Pi_M(OC), \text{ and} \tag{11}$$

$$\Pi_B(DT) \leq \Pi_B(EQ). \tag{12}$$

These two constraints ensure that the type M and type B firms do not find it optimal to mimic the type G by also issuing straight debt in equilibrium.

The type M firm chooses to separate itself from the type B by issuing convertible debt, rather than issuing equity and pooling with the type B. Such a separation would result in the type M's securities being correctly valued, rather than being undervalued as in the case of pooling with the type B. The type M firm prefers to separate itself by issuing ordinary convertible debt rather than straight debt. By doing so, it separates itself not only from the type B, but from the type G firm as well, even though mimicking the type G would have enabled it to obtain a higher value for its securities (since pooling with the type G would allow its securities to be overvalued, while separating from both type G and type B would only allow the firm's security to be correctly valued). The type M chooses to issue ordinary convertibles rather than straight debt due to the fact that issuing convertibles allows it to reduce its probability of financial distress. Thus, as long as the reduction in expected costs of financial distress achieved by issuing ordinary convertibles is greater than the benefit of mimicking the type G by issuing straight debt, the type M firm prefers to issue ordinary convertibles.

In summary, the type M firm chooses to issue ordinary convertibles in equilibrium to maximize $\Pi_M(OC)$ given by (5), subject to the break-even constraint (6) where $j = M$, and the following IC

constraints of the type G and type B firms, respectively:

$$\Pi_G(OC) \leq \Pi_G(DT), \text{ and} \quad (13)$$

$$\Pi_B(OC) \leq \Pi_B(EQ). \quad (14)$$

These two constraints ensure that the type G and type B firms do not find it optimal to issue ordinary convertibles in equilibrium.

Finally, in this equilibrium, the type B firm funds its project by issuing equity in order to avoid incurring any probability of financial distress. The type B does not choose to issue mandatory convertibles even though mandatory convertibles are also not associated with any probability of financial distress, since (unlike in the case of issuing equity) it would incur a small marketing cost ε by doing so.

4.3 Equilibrium with Straight Debt and Mandatory Convertibles

We now study the situation where mandatory convertibles emerge as the security issued in equilibrium. There are three scenarios under which mandatory convertibles are issued. In the first scenario, mandatory convertibles are issued together with straight debt (proposition 2) or ordinary convertibles (proposition 3) in partially pooling equilibria. In the second scenario, they are issued alone in a fully pooling equilibrium (proposition 4).

Proposition 2 (Equilibrium with Straight Debt and Mandatory Convertibles) *When $\phi_1 \leq \phi \leq \phi_2$ so that the probability of financial distress of the type G firm is greater than that in proposition 1, while the extent of asymmetric information is lower, the equilibrium involves the following.¹⁹*

The type G firm: *It issues straight debt at time 0, with a face value $P_d^* = \frac{I-x_L}{(1-\delta)(1-\phi)} + x_L$, maturing at time 2.*

The type M and the type B firms: *Both types of firms issue mandatory convertibles at time 0. Upon maturity, the convertibles will be converted to a fraction $n_m^* \geq \frac{I-x_L}{V_{MB}-x_L}$ of the firm's equity mandatorily, but the value of equity exchanged will be no greater than a cap amount $U_m^* = \frac{I-x_L}{V_{MB}-x_L} V_{MB}^1$. The dividends D^* on these mandatory convertibles will be set equal to x_L .*

The above proposition characterizes the situation where ϕ is larger than in proposition 1 but less than a certain threshold value ϕ_2 . The larger value of ϕ results in the type M firm facing a significantly larger financial distress probability than in the previous proposition, while the extent of asymmetric information it faces is significantly less severe. The larger value of ϕ results in the type G firm also facing a somewhat larger probability of financial distress and a smaller extent of asymmetric information than in proposition 1, but as long as $\phi \leq \phi_2$, the change in these two variables for the type G firm is not substantial. Given this, the type G firm behaves in a way similar to that in proposition 1: it finds it optimal to distinguish itself from the type M and B firms by issuing straight debt.

The type G firm cannot issue ordinary convertibles to achieve separation in this equilibrium, given that the extent of asymmetric information it faces with respect to the type M is still significant. If the type G chooses to issue ordinary convertibles, the type M would have an incentive to mimic the type G by issuing the same security. In other words, ordinary convertibles are not sufficient to achieve separation for the type G firm in this case while straight debt can, since straight debt is associated with a larger probability of financial distress for the type M and type B than ordinary convertibles. In sum, the type G firm maximizes $\Pi_G(DT)$ in equilibrium, subject to the outside investors' break-even constraint (2), where $j = G$, and the following IC constraints ensuring that the type M and B have no incentive to mimic the type G firm:

$$\Pi_M(MC) \geq \Pi_M(DT), \text{ and} \tag{15}$$

$$\Pi_B(MC) \geq \Pi_B(DT). \tag{16}$$

¹⁹ Outside investors infer that a firm is of type G with probability 1, if it issues straight debt. If a firm issues a mandatory convertible, they infer that it is of type M with probability $\frac{\gamma_M}{\gamma_M + \gamma_B}$ and type B with probability $\frac{\gamma_B}{\gamma_M + \gamma_B}$. If a firm issues equity (an out of equilibrium move), outsiders infer that it is of type B with probability 1. If a firm issues ordinary convertibles (another out of equilibrium move), outsiders infer that it is of type M with probability 1.

At the same time, given that the type M firm’s financial distress probability is significantly larger than in proposition 1 while the extent of asymmetric information it faces in the financial market is significantly smaller, it no longer finds it optimal to separate itself from the type B; rather it finds it optimal to pool with the type B by issuing mandatory convertibles. The type M firm chooses not to issue straight debt or ordinary convertibles, since issuing these securities would cause it to incur large financial distress probabilities $\phi + \delta'(1 - \phi)$ and ϕ , respectively. In comparison, the type M faces no such danger of financial distress by issuing mandatory convertibles, since conversion to equity is mandatory in this case. Further, the type M prefers to issue mandatory convertibles rather than issuing equity, since the undervaluation of the type M firm’s securities due to asymmetric information is less if it issues mandatory convertibles rather than equity. This is because, unlike equity, mandatory convertibles provide a “cap” on the “upside” cash flow paid to security holders, compensating them for this cap with an incremental risk-free dividend payment, D , (thus reducing the difference between the intrinsic values of mandatory convertibles issued by the type M and type B firms relative to the corresponding differences in the case of equity issued by these two types of firms). Note that, since the marketing cost associated with issuing mandatory convertibles is small, the advantage of mandatory convertibles over equity in terms of reducing the type M firm’s adverse selection costs would dominate this marketing cost. In sum, the type M firm maximizes $\Pi_M(MC)$ given by (7), subject to the outside investors’ break-even constraint (8), where $j = MB$, and the following IC constraint ensuring that the type G has no incentive to mimic:

$$\Pi_G(DT) \geq \Pi_G(MC). \tag{17}$$

Note that, in this equilibrium, the type G firm chooses to achieve separation from the lower type firms while the type M prefers to pool with the type B. This is because the probability of

realizing a low cash flow is lower for the type G than for the type M (so that the expected financial distress cost of issuing straight debt is lower for the type G than that for the type M), and pooling with the lower firm types would impose larger undervaluation costs on the type G firm compared to that on the type M.

4.4 Equilibrium with Ordinary and Mandatory Convertibles

Proposition 3 (Equilibrium with Ordinary and Mandatory Convertibles) *Let $\phi_2 \leq \phi \leq \phi_3$, so that the probability of financial distress of the type G firm is greater than that in proposition 2, while the extent of asymmetric information is lower. Then the equilibrium involves the following:²⁰*

The type G firm: *It issues an ordinary convertible at time 0, with a call price K , $x_L < K < I$, and a face value $P_c > x_L$, which is convertible to a fraction $n_c^* = \frac{I - \phi x_L}{(1 - \phi)[(1 - \delta)x_H + \delta x_L]}$ of the firm's equity. It calls back the convertible at time 1, if its conversion value at that time is above the call price K .*

The type M and the type B firms: *Both types of firms issue mandatory convertibles at time 0. Upon maturity, the convertibles will be converted to a fraction $n_m^* \geq \frac{I - x_L}{V_{MB} - x_L}$ of the firm's equity mandatorily, but the value of equity exchanged will be no greater than a cap amount $U_m^* = \frac{I - x_L}{V_{MB} - x_L} V_{MB}^1$. The dividends D^* on these mandatory convertibles will be set equal to x_L .*

The above proposition characterizes the situation where ϕ is larger than in proposition 2 (but less than a threshold ϕ_3), so that the financial distress probability facing the type G (as well as type M) is significantly larger, while the extent of asymmetric information is significantly smaller. In this situation, while the type G firm continues to prefer to separate itself from the type M and B firms (this will be the case as long as $\phi < \phi_3$), it has two choices available to it to achieve this separation: it can achieve separation from the lower type firms by issuing ordinary convertibles or by issuing straight debt. Given this, the type G will prefer to achieve separation by issuing ordinary convertibles, since its probability of financial distress will be lower if it issues ordinary convertibles rather than straight debt (so that its expected cost of financial distress will be smaller if it issues ordinary convertibles). In sum, the type G firm maximizes $\Pi_G(OC)$ given by (5), subject to the

²⁰ Outsiders infer that a firm is of type G with probability 1 if it issues ordinary convertibles. If a firm issues a mandatory convertible, they infer that it is of type M with probability $\frac{\gamma_M}{\gamma_M + \gamma_B}$ and type B with probability $\frac{\gamma_B}{\gamma_M + \gamma_B}$. If a firm issues equity (an out of equilibrium move), outsiders infer that it is of type B with probability 1. If a firm issues straight debt (an out of equilibrium move), outsiders infer that it is of type G with probability 1.

outside investors' break-even constraint (6), where $j = G$, and the following IC constraints ensuring that the type M and B have no incentive to mimic the type G:

$$\Pi_M(MC) \geq \Pi_M(OC), \text{ and} \quad (18)$$

$$\Pi_B(MC) \geq \Pi_B(OC). \quad (19)$$

On the other hand, similar to proposition 2, the type M firm prefers to pool with the type B by issuing mandatory convertibles for the reasons discussed there. Thus, the type M firm maximizes $\Pi_M(MC)$, subject to the outside investors' break-even constraint (8), where $j = MB$, and the following IC constraint ensuring that the type G has no incentive to mimic:

$$\Pi_G(OC) \geq \Pi_G(MC). \quad (20)$$

4.5 Equilibrium with Mandatory Convertibles Alone

Proposition 4 (Equilibrium with Mandatory Convertibles Alone) *Let $\phi \geq \phi_3$, so that the probability of financial distress of the type G firm is greater than that in proposition 3. Then the equilibrium involves all three types of firms issuing mandatory convertibles at time 0. Upon maturity, the convertibles will be converted to a fraction $n_m^* \geq \frac{I-x_L}{V_{GMB}-x_L}$ of the firm's equity mandatorily, with the value of the equity exchanged subject to a cap amount $U_m^* = \frac{I-x_L}{V_{GMB}-x_L} V_{GMB}^1$. The dividends D^* on these mandatory convertibles will be set equal to x_L .²¹*

The above proposition characterizes the situation where ϕ is larger than in proposition 3, so that the financial distress probability facing the type G (as well as type M) firm(s) is significantly larger, while the extent of asymmetric information facing the firm is significantly smaller. In this case, the type G firm prefers to pool with the type M and B firms rather than separating from them. This is because the benefit to the type G of separating from the lower firm types (in terms of reducing undervaluation due to asymmetric information) is smaller given the smaller extent of

²¹ If a firm issues a mandatory convertible, outside investors infer that it is of type G with probability γ_G , type M with probability γ_M , and type B with probability γ_B . If a firm issues equity (an out of equilibrium move), outsiders infer that it is of type B with probability 1. If a firm issues straight debt or ordinary convertibles (out of equilibrium moves), outsiders infer that it is of type G with probability 1.

asymmetric information it faces in the financial market, while the cost of separating (arising from the expected financial distress cost that it would incur if it were to issue straight debt or ordinary convertibles) is larger.

Further, the type G firm finds it advantageous to pool with the type M and the type B by issuing mandatory convertibles, rather than equity. The intuition underlying this preference is the same as that discussed in the context of the type M pooling with the type B in propositions 2 and 3: it arises from the fact that the cap set on the mandatory convertibles and the risk-free dividends paid make the market value of mandatory convertibles less sensitive to asymmetric information than that of equity, so that this advantage in terms of minimizing the type G firm's adverse selection costs dominates the marketing cost associated with issuing mandatory convertibles.

5 Equilibrium Design of Mandatory Convertibles

In this section, we will analyze in detail the equilibrium design of mandatory convertibles. Due to space considerations, we discuss here only the case of the fully pooling equilibrium (proposition 4). However, the analysis is similar in the case of partial pooling equilibria (propositions 2 and 3).²² In this section, we relax our earlier assumption that $\bar{n} = 1$ and $\bar{d} = 1$; we now allow for $\bar{n} \leq 1$ and $\bar{d} \leq 1$.

Proposition 5 (Equilibrium Design of Mandatory Convertibles) *In a fully pooling equilibrium where all three types of firms issue mandatory convertibles:*

- (a) *The type G firm sets the exchange ratio n_m such that $n_m^* = \bar{n}$ (i.e., the highest possible); the dividend D such that $D^* = \bar{d}x_L$ (the highest possible); and the cap on the mandatory convertible U_m^* is such that it is the lowest possible.*
- (b) *U_m^* is a function of \bar{n} and \bar{d} (given in the appendix). It is decreasing with \bar{d} and \bar{n} .*
- (c) *In particular, if $\bar{d} = 1$ and $\bar{n} = 1$, then $D^* = x_L$, and $U_m^* = \frac{I-x_L}{V_{GMB}-x_L}V_{GMB}^1$, and $n_m^* \geq \frac{I-x_L}{V_{GMB}-x_L}$.*

²² Details of the analysis of the equilibrium design of mandatory convertibles issued in partial pooling equilibria are available to interested readers upon request.

In a fully pooling equilibrium, the type G firm designs the mandatory convertible optimally to minimize its cost of pooling with the type M and the type B firms. This pooling cost arises because outside investors are not aware of the type of the issuing firm and thus price the mandatory convertible according to their prior beliefs. Therefore, to minimize this pooling cost, the mandatory convertible is designed to have a market value that is minimally affected by the asymmetric information characterizing the financial market. This is accomplished by lowering the cap U_m of the mandatory convertible and raising the dividend D and the exchange ratio n_m . When the cap is lowered, the difference in the expected payment to outside investors in the high and the low cash flow scenarios is reduced. This reduces the difference between the intrinsic values of the mandatory convertibles issued by the three types of firms so that the market value of the mandatory convertible becomes less sensitive to the effects of asymmetric information. Of course, when outside investors' upside is capped, they have to be compensated for this through a higher dividend or a higher exchange ratio (or both), so that they break even on their investment in the firm. Since, as long as $D \leq \bar{d}x_L$, the firm is able to pay investors the promised dividend amount with probability 1 regardless of firm type, these dividends are unaffected by asymmetric information, so that the net effect of setting the cap U_m as low as possible, and the dividend D and the exchange ratio n_m as high as possible is to minimize the sensitivity of the market value of the convertible to the effects of asymmetric information.²³ In other words, lowering the cap U_m and raising D and n_m reduce the subsidization of the type M and the type B firms by the type G, thereby reducing the extent

²³ For analytical simplicity, we focus only on the case where $D \leq \bar{d}x_L$ so that the dividends are sure cash flows and therefore unaffected by asymmetric information. As long as there is no uncertainty associated with dividends, lowering the cap and increasing the dividends unambiguously reduces the sensitivity of the market value of the mandatory convertible to asymmetric information. Of course, firms can lower the cap on the mandatory convertible even more by promising investors additional (uncertain) cash flows as dividends, i.e., they can set $D > \bar{d}x_L$. However, in this case, the additional dividends are affected by asymmetric information, so that the effect of doing this on the sensitivity of the market value of the mandatory convertible to asymmetric information is ambiguous.

of undervaluation of the type G firm's mandatory convertibles due to asymmetric information.²⁴

6 Testable Implications

Our model generates several testable implications, which we describe below.

1. *Choice between ordinary versus mandatory convertibles and the probability of financial distress:*

Proposition 3 implies that, in a sample of firms issuing either ordinary or mandatory convertibles, those firms with a larger ex-ante probability of financial distress (type M and B firms in our model) will issue mandatory convertibles, while those with a smaller probability of financial distress (the type G firm in our model) will issue ordinary convertibles. Therefore, in a sample of firms issuing either mandatory or ordinary convertibles, those with a smaller ex-ante financial distress probability will issue ordinary convertibles and those with a larger financial distress probability will issue mandatory convertibles.

2. *Choice between ordinary versus mandatory convertibles and the extent of asymmetric information:*

Proposition 3 implies that, in a sample of firms issuing either ordinary convertibles or mandatory convertibles, those firms facing a smaller extent of asymmetric information will issue mandatory convertibles, and those facing a greater extent of asymmetric information will issue ordinary convertibles. This implication can be tested by using standard proxies for asymmetric

²⁴ The following numerical example demonstrates parts (a), (b), and (c) of the above proposition. We use the same parameters as those in constructing figure 4 and assume $\phi = 0.65$. First, let $\bar{d} = 0.5$ and $\bar{n} = 1$. As characterized in proposition 4, the equilibrium in this case involves all three types of firms issuing mandatory convertibles. The mandatory convertibles will be designed optimally such that the exchange ratio n_m^* is 1; the cap U_m^* is \$39.76; and the dividends D^* is \$5 (thus maximizing the expected time 2 payoff of the type G firm; this payoff is \$1.12). Consider now a second example, with $\bar{d} = 0.75$, and keeping all other parameters the same as before. Then, all three types of firms would still pool by issuing mandatory convertibles, optimally designed such that the exchange ratio $n_m^* = 1$; the cap $U_m^* = \$35.26$; and the dividends $D^* = \$7.5$. In this case, the type G firm's expected payoff is \$1.74. Finally, consider a third example where $\bar{d} = 1$, with all other parameters remaining the same as before. In this case, the equilibrium design of the mandatory convertibles issued by three types of firms is such that the exchange ratio $n_m^* = 1$; the cap $U_m^* = \$30.77$; and the dividends $D^* = \$20$. In this case, the type G firm's expected payoff is \$2.54. Notice from the above three examples that as the cash constraint \bar{d} on the firm is relaxed (so that the firm is able to pay out more and more of its time 2 lower cash flow as dividends), it optimally sets a lower and lower cap on the mandatory convertible. Further, notice that, as the firm sets a lower cap, the subsidization of the lower firm types by the type G firm is reduced, thus increasing the expected payoff to type G firm insiders.

information such as firm size, number of analysts following a firm, standard deviation of analysts forecasts, forecast error, etc.

3. Relationship between firm characteristics and the equilibrium design of mandatory convertibles:

Proposition 5 makes two predictions regarding the characteristics of a firm at the time of issue and the equilibrium design of its mandatory convertibles. First, the greater the extent of asymmetric information facing the firm, the lower the cap on the mandatory convertibles it issues will be in relation to the stock price (this helps to minimize the subsidization of the lower type firms by the higher types), and the higher the level of dividends paid on these convertibles in relation to the dividend paid on common stock (thus compensating investors for the smaller appreciation potential relative to the firm's equity). Second, if the firm faces a more severe financial constraint (so that the cash flow available to pay dividends net of investment requirements is smaller), the firm will set a higher cap or a greater exchange ratio (or both) on its mandatory convertibles (since the dividend paid on the mandatory convertibles in excess of the dividend paid on common stock will be lower in this case).

7 Conclusion

Mandatory convertibles are equity-linked hybrid securities that automatically convert to equity on a pre-specified date, and which have become an increasingly popular means of raising capital in recent years. In this paper, we have presented the first theoretical analysis of mandatory convertibles in the literature. We considered a firm facing a financial market characterized by asymmetric information, and significant costs in the event of financial distress. The firm could raise capital either by issuing mandatory convertibles, or by issuing more conventional securities like straight debt, ordinary convertibles, or equity. We showed that, in equilibrium, the firm issues straight debt or ordinary convertibles if the extent of asymmetric information facing it is more severe,

but the probability of financial distress is relatively small; it issues mandatory convertibles if it faces a smaller extent of asymmetric information but a greater probability of being in financial distress. Our model provides a rationale for the three commonly observed features of mandatory convertibles: mandatory conversion, capped (or limited) capital appreciation, and a higher dividend yield compared to common stock. We also characterized the equilibrium design of mandatory convertibles.

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Appendix

Proof of Proposition 1. In this proof, we first derive firms' equilibrium choices, assuming that IC constraints are satisfied, and then derive the conditions under which these constraints are satisfied in equilibrium.

In this equilibrium, the type G maximizes $\Pi_G(DT)$ subject to the IR constraint (2) where $j = G$, and its IC constraints (11) and (12); the type M maximizes $\Pi_M(OC)$ subject to the IR constraint (6) where $j = M$, and its IC constraints (13) and (14); and the type B maximizes $\Pi_B(EQ)$ subject to the IR constraint (4) where $j = B$, and the non-mimicry constraint for the type G and the type M firms:

$$\Pi_G(DT) \geq \Pi_G(EQ), \text{ and} \tag{A1}$$

$$\Pi_M(OC) \geq \Pi_M(EQ). \tag{A2}$$

It is easy to show that in equilibrium, the type B firm sets $n_e^* = \frac{I}{V_B}$; the type G sets $P_d^* = \frac{I-x_L}{(1-\delta)(1-\phi)} + x_L$; and the type M sets $n_c^* = \frac{I-\phi x_L}{(1-\phi)[(1-\delta')x_H + \delta'x_L]}$, so that IR constraints (2), (4), and (6) are satisfied as equalities. Under these choices, constraint (11) becomes $C \geq c_1 \equiv \frac{1}{\delta'(1-\phi)} \frac{V_G - V_M}{V_G - x_L} (I - x_L)$; constraint (12) becomes $C \geq c_2 \equiv \frac{1}{\phi' + \delta'(1-\phi)} \frac{V_G - V_B}{V_G - x_L} (I - x_L)$; constraint (13) becomes $C \leq c_3 \equiv \frac{1}{\delta(1-\phi)} \frac{V_G - V_M}{V_M - \phi x_L} (I - \phi x_L)$; constraint (14) becomes $C \geq c_4 \equiv \frac{1}{\phi'} \frac{V_M - V_B}{V_M - x_L} (I - x_L)$; constraint (A1) becomes $C \leq c_5 \equiv \frac{1}{\phi + \delta(1-\phi)} \frac{V_G - V_B}{V_B} I$; and constraint (A2) becomes $C \leq c_6 \equiv \frac{1}{\phi} \frac{V_M - V_B}{V_B} I$.

Under our assumption $\frac{\delta'}{\delta} > \frac{\phi'}{\phi}$, $c_2 < c_1$, $c_4 < c_1$, and $c_5 > c_6$. Thus, when $c_1 \leq C \leq \min(c_3, c_6)$, there exist a separating equilibrium with equilibrium choices of securities described in proposition 1. Further, this equilibrium satisfies the Cho-Kreps intuitive criterion. If a firm issues mandatory convertibles (an off-equilibrium move), investors would infer that the firm is of type B. Both the type G and type M have no incentive to deviate by issuing mandatory convertibles since the type B

has a definite incentive to deviate whenever the type G and M have a (weak) incentive to deviate. As a result, the market places no probability weight on a type M or type G issuing mandatory convertibles. Then, given such an off-equilibrium belief on a firm issuing mandatory convertibles, the type B has no incentive to offer mandatory convertibles as well, since the type B would incur an extra cost ε in marketing mandatory convertibles but not in the case of issuing equity.

In the following, we compare the expected payoff in the above PBE with the payoffs in other equilibria, and derive the conditions for the above PBE to be satisfied as an efficient PBE. In the above separating equilibrium, the expected payoff to the three firm types are $\Pi_G(DT) = V_G - I - [\phi + \delta(1 - \phi)]C$; $\Pi_M(OC) = V_M - I - \phi C$; and $\Pi_B(DT) = V_B - I$. The type M firm can also pool with the type B, or the type G firm can pool with both the type M and type B by issuing mandatory convertibles. Both pooling equilibria are characterized and proved later. In the former pooling equilibrium, $\Pi_M(MC) = \frac{V_M - x_L}{V_{MB} - x_L}(V_{MB} - I) - \varepsilon$, and in the latter pooling equilibrium, $\Pi_G(MC) = \frac{V_G - x_L}{V_{GM} - x_L}(V_{GM} - I) - \varepsilon$. However, there does not exist a pooling equilibrium with the type G and M offering ordinary convertibles and the type B offering equity. It can be shown that, when $C \leq c_7 \equiv \frac{1}{\phi} \frac{V_M - V_{MB}}{V_{MB} - x_L}(I - x_L) + \frac{\varepsilon}{\phi}$, $\Pi_M(OC) \geq \Pi_M(MC)$; $c_7 < c_6$. When $C \leq c_8 \equiv \frac{1}{\phi + \delta(1 - \phi)} \left[\frac{V_G - V_{GM}}{V_{GM} - x_L}(I - x_L) \right] + \frac{\varepsilon}{\phi + \delta(1 - \phi)}$, $\Pi_G(DT) \geq \Pi_G(MC)$. Thus, the separating equilibrium characterized above is an efficient PBE when

$$c_1 \leq C \leq \min(c_3, c_7, c_8). \quad (\text{A3})$$

Note that $c_1 \leq C$ is satisfied according to our global assumption on C . Define $\delta_1 \equiv \{\delta \mid C = c_3\}$, $\delta_2 \equiv \{\delta \mid C = c_8\}$, and:

$$\bar{\delta} \equiv \min[\delta_1, \delta_2]. \quad (\text{A4})$$

Thus, given our assumption that $\delta \leq \bar{\delta}$, $C \leq c_3$ and $C \leq c_8$ are satisfied. Further define:

$$\phi_1 \equiv \{\phi \mid C = c_7\}. \quad (\text{A5})$$

Then, when $\phi \leq \phi_1$, $C \leq c_7$ is satisfied and condition (A3) is satisfied as well. ■

Proof of Proposition 2. In this equilibrium, the type G maximizes $\Pi_G(DT)$ subject to the IR constraint (2) where $j = G$, and its IC constraints (17); the type M maximizes $\Pi_M(MC)$ subject to the IR constraint (8), where $j = MB$, and its IC constraints (15); and the type B maximizes $\Pi_B(MC)$ subject to the IR constraint (8) and the non-mimicry constraint (16).

In equilibrium, (8) holds as an equality. Incorporating (8) into $\Pi_M(MC)$, we have $\Pi_M(MC) = (V_M - I) - \frac{V_M - V_{MB}}{V_{MB} - D}(I - D) + \frac{(V_{MB}^1 - D)(\phi_{MB} - \phi_G)(x_L - D)}{V_{MB} - D}[n_m - \min(\frac{U_m}{V_{MB}^1 - D}, n_m)] - \varepsilon$. It can be shown that $\frac{\partial \Pi_M(MC)}{\partial U_m} < 0$ and $\frac{\partial \Pi_M(MC)}{\partial D} > 0$ when $x_L \geq D$. Thus, $D^* = x_L$ and $U_m^* = \frac{I - x_L}{V_{MB}^1 - x_L}(V_{MB}^1 - D)$. Further, the cap U_m satisfies the condition $\frac{U_m}{V_{MB}^1 - D} \leq n_m$ in equilibrium, which implies that $n_m^* \geq \frac{I - x_L}{V_{MB}^1 - x_L}$. On the other hand, the type G designs straight debt in equilibrium so that constraint (2) is satisfied as an equality, i.e., $P_d^* = \frac{I - x_L}{(1 - \delta)(1 - \phi)} + x_L$. Under these securities choices, IC constraint (16) is satisfied as long as IC constraint (15) is binding, which is true when $C \geq c_9 \equiv \frac{(V_G - V_{MB})(V_M - x_L)(I - x_L)}{[\delta'(1 - \phi) + \phi](V_G - x_L)(V_{MB} - x_L)} + \frac{\varepsilon}{[\delta'(1 - \phi) + \phi]}$. Constraint (17) is satisfied when $C \leq c_{10} \equiv \frac{V_G - V_{MB}}{\delta(1 - \phi) + \phi} \frac{I - x_L}{V_{MB} - x_L} + \frac{\varepsilon}{\delta(1 - \phi) + \phi}$; $c_{10} > c_9$ and $c_{10} > c_8$. Also, given our assumption $\delta \leq \bar{\delta}$ (so that $C \leq c_8$), $C < c_{10}$ is satisfied. Thus, when $C \geq c_9$, there exists a PBE as characterized in proposition 2.

Now we derive the conditions for the above PBE to satisfy the Cho-Kreps intuitive criterion. First, the type G firm has no incentive to deviate by issuing equity because the type B and type M firms have incentive to mimic the type G whenever the type G firm deviates by issuing equity. Also, the type M has no incentive to deviate by issuing equity because it would incur a larger underpricing cost by issuing equity compared to issuing mandatory convertibles. As a result, if a firm issues equity (an off-equilibrium move), investors would infer the firm to be of type B. Given such an off-equilibrium belief, the type B will not issue equity in equilibrium, since its mandatory convertibles

are overpriced and its equity would be fairly priced. Second, investors would infer the firm issuing ordinary convertibles to be of type M when $C \leq c_{11} \equiv \frac{1}{\phi} [\frac{V_M - \phi x_L}{V_G - \phi x_L} (V_G - I) - \frac{V_M - x_L}{V_{MB} - x_L} (V_{MB} - I)] + \frac{\varepsilon}{\phi}$ and $C \geq c_4$. When $C \leq c_{11}$, the type G has no incentive to issue ordinary convertibles, since otherwise the type M and B firms always have an incentive to mimic the type G by issuing the same security. When $C \geq c_4$ (which is satisfied as long as $C > c_1$), the large financial distress cost discourages the type B from issuing ordinary convertibles. Thus, for a firm issuing ordinary convertibles, investors assign probability zero to type G and type B, and probability 1 to type M. However, given such an off-equilibrium belief, it is worse off for the type M to deviate and issue ordinary convertibles if $C \geq c_7$. In sum, the PBE characterized above satisfies the intuitive criterion when $\max(c_7, c_9) \leq C \leq c_{11}$.

In the following, we derive the conditions for the above PBE to be an efficient PBE. In this equilibrium, the expected payoff to the type M equals $\Pi_M(MC)$ and the expected payoff to the type G equals $\Pi_G(DT)$. When $C \geq c_7$, $\Pi_M(MC) \geq \Pi_M(OC)$, where $\Pi_M(OC)$ is the expected payoff to the type M by issuing ordinary convertibles in the separating equilibrium (as in proposition 1). When $C \leq c_8$, $\Pi_G(DT) \geq \Pi_G(MC)$, where $\Pi_G(MC)$ is the expected payoff to the type G by issuing mandatory convertibles in a pooling equilibrium. Under our assumption $\delta \leq \bar{\delta}$, $C \leq c_8$ is satisfied. Further, $\Pi_G(DT) < \Pi_G(OC)$, where $\Pi_G(OC)$ is the expected payoff to the type G by issuing ordinary convertibles in a separating equilibrium. As we showed earlier, when $C \leq c_{11}$, such a separating equilibrium with the type G issuing ordinary convertibles does not exist. Thus, the above PBE is an efficient PBE when

$$\max(c_7, c_9) \leq c \leq c_{11}. \quad (\text{A6})$$

Define:

$$\begin{aligned}\underline{\delta} &\equiv \{\delta' \mid C = c_9\}, \text{ and} \\ \phi_2 &\equiv \{\phi \mid C = c_{11}\}.\end{aligned}\tag{A7}$$

Then, given $\delta' \geq \underline{\delta}$, when $\phi_1 \leq \phi \leq \phi_2$, condition (A6) is satisfied. ■

Proof of Proposition 3. In this equilibrium, the type G maximizes $\Pi_G(OC)$ subject to the IR constraint (6) where $j = G$, and its IC constraints (20); the type M's and the type B's maximization problems are the same as described in the proof of proposition 2. In equilibrium, the type M and the type B firms issue mandatory convertibles with $D^* = x_L$, $U_m^* = \frac{I-x_L}{V_{MB}-x_L}(V_{MB}^1 - D)$, and $n_m^* \geq \frac{I-x_L}{V_{MB}-x_L}$. The type G designs ordinary convertibles so that constraint (6) is satisfied as an equality, i.e., $n_c^* = \frac{I-\phi x_L}{(1-\phi)[(1-\delta)x_H + \delta x_L]}$. Under the above securities choices, IC constraint (19) is satisfied as long as IC constraint (18) is binding, which is true when $C \geq c_{11}$. Further, constraint (20) is satisfied when $C \leq c_{12} \equiv \frac{V_G - V_{MB}}{\phi} \frac{I-x_L}{V_{MB}-x_L} + \frac{\varepsilon}{\phi}$. Thus, the above equilibrium exists when $c_{11} \leq C \leq c_{12}$. This equilibrium also satisfies the Cho-Kreps intuitive criterion. First, similar to the discussion in the proof of proposition 2, all three firm types do not issue equity in equilibrium. Second, when $C \geq c_1$ and $C \geq c_2$, the type M and B firms have no incentive to issue debt even if they will be perceived as a type G by doing so. Thus, investors would infer a firm issuing straight debt (an off-equilibrium move) being of type G with probability 1. However, even under this off-equilibrium belief, the type G would be worse off deviating by issuing debt since debt is associated with a larger probability of financial distress compared to ordinary convertibles. As a result, debt is not issued in equilibrium. Note that $c_1 < c_{11}$, and, under our assumption $\frac{\delta'}{\delta} > \frac{\phi'}{\phi}$, $c_2 < c_1$.

In the following, we derive the conditions for the above PBE to be an efficient PBE. In this equilibrium, the expected payoff to the type M equals $\Pi_M(MC)$ and the expected payoff to the type G equals $\Pi_G(OC)$. When $C \geq c_{11}$, $\Pi_M(MC) \geq \Pi_M(OC)$. When $C \leq c_{13} \equiv \frac{1}{\phi} \left[\frac{V_G - V_{GMB}}{V_{GMB} - x_L} (I - \right.$

$x_L)] + \frac{\varepsilon}{\phi}$, $\Pi_G(OC) \geq \Pi_G(MC)$; $c_{13} < c_{12}$. Define:

$$\phi_3 \equiv \{\phi \mid C = c_{13}\}. \quad (A7)$$

Then, when $\phi_2 \leq \phi \leq \phi_3$, $c_{11} \leq C \leq c_{13}$ is satisfied and the equilibrium characterized in proposition 3 is an efficient PBE. ■

Proof of Proposition 4. In this case, mandatory convertibles are issued subject to the IR constraint (8), where $j = GMB$. In equilibrium, mandatory convertibles are designed so that (8) is satisfied as an equality. Incorporating (8) into $\Pi_G(MC)$ yields (10). It can be shown that $\frac{\partial \Pi_G(MC)}{\partial U_m} < 0$ and $\frac{\partial \Pi_G(MC)}{\partial D} > 0$. Thus, in equilibrium, $D^* = x_L$, $U_m^* = \frac{I-x_L}{V_{GMB-x_L}} V_{GMB}^1$, and $n_m^* \geq \frac{I-x_L}{V_{GMB-x_L}}$.

Now we derive the conditions for the above PBE to satisfy the Cho-Kreps intuitive criterion. First, similar to the discussion in the proof of proposition 2, all three firm types do not issue equity in equilibrium. Second, when $C \geq c_{14} \equiv \frac{1}{\phi'} [\frac{V_B - \phi' x_L}{V_G - \phi x_L} (V_G - I) - \frac{V_B - x_L}{V_{GMB-x_L}} (V_{GMB} - I)] + \frac{\varepsilon}{\phi'}$, the type B firm has no incentive to offer ordinary convertibles even if it will be perceived as a type G by doing so. Similarly, when $C \geq c_{15} \equiv \frac{1}{\phi} [\frac{V_M - \phi x_L}{V_G - \phi x_L} (V_G - I) - \frac{V_M - x_L}{V_{GMB-x_L}} (V_{GMB} - I)] + \frac{\varepsilon}{\phi}$, the type M firm has no incentive to offer ordinary convertibles. As a result, investors believe a firm issuing ordinary convertibles to be of type G. However, even under this off-equilibrium belief, when $C \geq c_{13}$, the type G finds it optimal to issue mandatory convertibles rather than deviating by issuing ordinary convertibles. Third, when $C \geq c_{16} \equiv \frac{1}{\phi' + \delta'(1-\phi')} [\frac{V_B - x_L}{V_G - x_L} (V_G - I) - \frac{V_B - x_L}{V_{GMB-x_L}} (V_{GMB} - I)] + \frac{\varepsilon}{\phi' + \delta'(1-\phi')}$ and $C \geq c_{17} \equiv \frac{1}{\phi + \delta'(1-\phi)} [\frac{V_M - x_L}{V_G - x_L} (V_G - I) - \frac{V_M - x_L}{V_{GMB-x_L}} (V_{GMB} - I)] + \frac{\varepsilon}{\phi + \delta'(1-\phi)}$, the type B and M firms have no incentive to offer straight debt even if they will be perceived as a type G by doing so. In this case, investors believe a firm issuing straight debt to be of type G. However, even under this off-equilibrium belief, when $C \geq c_8$, the type G finds it optimal to issue mandatory convertibles rather than deviating by issuing straight debt. $c_{16} < c_{14} < c_1$, $c_{17} < c_{15} < c_{13}$, and $c_8 < c_{13}$.

According to our global assumption on C , $C \geq c_1$. Thus, when $\phi \geq \phi_3$, $C \geq c_{13}$ is satisfied and the pooling PBE in this proposition satisfies the intuitive criterion.

Finally, as discussed in the proofs of previous propositions, when $C \geq c_{13}$, the expected payoff to the type G in this pooling equilibrium is larger than that by issuing ordinary convertibles in the equilibrium in proposition 2, i.e., $\Pi_G(MC) \geq \Pi_G(OC)$. When $C \geq c_8$, $\Pi_G(MC) \geq \Pi_G(DT)$. When $C \geq c_{15}$, $\Pi_M(MC) \geq \Pi_M(OC)$. In addition, if the type G pools with lower firm types by issuing debt, its expected payoff is $\frac{V_G - x_L}{V_{GMB} - x_L}(V_{GMB} - I) - [\phi + \delta(1 - \phi)]C$, which is smaller than $\Pi_G(MC)$ as long as the marketing cost ε is very small. Similarly, it can be shown that the expected payoff in the case when the type G pools with ordinary convertibles is smaller than $\Pi_G(MC)$. Thus, the pooling PBE in this proposition is also an efficient PBE. ■

Proof of Proposition 5. Let $n_m \leq \bar{n}$, $D \leq \bar{d}x_L$. It can be shown that $\frac{\partial \Pi_G(MC)}{\partial n_m} > 0$ for any given $D \leq x_L$, and $\frac{\partial \Pi_G(MC)}{\partial D} > 0$ for any $n_m \leq 1$. Thus, $n_m^* = \bar{n}$, $D^* = \bar{d}x_L$, and $U_m^* = \frac{I - x_L}{V_{GMB} - x_L}(V_{GMB}^1 - \bar{d}x_L) + (1 - \bar{d})\frac{x_L[(1 - \phi_{GMB})(V_{GMB}^1 - x_L) + V_{GMB}^1(1 - \bar{n})\phi_{GMB}]}{V_{GMB} - x_L}$. It is easy to show that $\frac{\partial U_m^*}{\partial d} < 0$ and $\frac{\partial U_m^*}{\partial \bar{n}} < 0$. ■

Table 1: Illustrative Examples of the Mandatory Convertibles in our Sample

| Security & Underwriter | Illustrative Example |
|---|--|
| <p>Automatically Convertible Equity Securities (ACES)</p> <p>Investment Bank:</p> <p>Goldman, Sachs & Co.</p> | <p>Apache Corp. issued 140,000 shares of ACES on 05/12/99. Each unit of ACES comprised of 50 depository shares which was offered to the public at \$31 per share. The market price of the firm's common stock at that time was \$31.0625 per share. The mandatory conversion date for each depository share was May 15th 2002. Each unit of ACES paid a dividend at the rate of 6.5% per annum, payable quarterly. Upon mandatory conversion each depository share of the ACES will be converted into a variable number of shares of Apache common stock. If the common stock price is below the issue price of \$31 then for each depository share of ACES the holder will receive 1 share of Apache common stock (i.e., each unit of ACES will convert to 50 shares of common stock). If the price is between \$31 and \$37.82, then the number of common shares per depository share will be such that the value equals the issue price of \$31. If the stock price is above \$37.82 per share then the holder shall receive 0.8197 ($\\$31/\\37.82) shares of common stock per depository share of the ACES. Hence the cap on the depository shares of the ACES is at the issue price of \$31. The holders of ACES also have an option to convert prior to May 15th 2002 in which case the holder will receive 0.8197 of Apache common stock per depository share of the ACES.</p> |
| <p>Debt Exchangeable for Common Stock (DECS)</p> <p>Investment Bank:</p> <p>Salomon Smith Barney Inc.</p> | <p>Cendant Corporation issued 15,000,000 units of DECS on 07/20/01 at \$50 per share. The market price of the firm's common stock at that time was \$21.53 per share (defined as the reference price). The DECS will be mandatorily convertible to shares of the firm's common stock on August 17th 2004. Each DECS includes a forward purchase contract which obligates the holder to mandatorily convert to the common stock of the company on the mandatory conversion date, and also includes senior notes of the company bearing a principal amount of \$50 which are due on August 17th 2006. Each unit of DECS pays a dividend of 7.75% per year, which comprises of an interest of 6.75% from the senior notes and a 1% contract adjustment fee. The interest on the senior notes will be reset on the mandatory conversion date. Each holder of DECS will receive a variable number of shares on the mandatory conversion date which will be determined as follows. If the stock price is less than or equal to the reference price of \$21.53, then the number of shares will be 2.3223 ($\\$50/\\21.53). If the stock price is less than \$28.42 (a 32% appreciation from the reference price) but greater than the reference price of \$21.53, then the number of shares will such that the value on conversion equals the issue price of DECS of \$50. If the stock price of the company's share is greater than or equal to \$28.42, then each DECS holder will receive 1.7593 ($\\$50/\\28.42) shares of common stock. Hence the cap of the DECS is at the issue price of \$50. Holders of DECS also have an option to convert prior to the mandatory conversion date in which case they will receive 1.7593 shares of Cendant common stock per unit of DECS.</p> |
| <p>Equity Security Units</p> <p>Investment Banks:</p> <p>Joint issue by Goldman, Sachs & Co., J P Morgan, Salomon Smith Barney</p> | <p>Motorola Inc. issued 21,000,000 units of Equity Security Units on 10/26/2001 at \$50 per unit. The market price of the firm's common stock at that time was \$17.28 per share (defined as the reference price). The mandatory conversion date for the Equity Security units is November 16th 2004. Each unit consists of two parts, a purchase contract which obligates the holder to mandatorily purchase the common stock of the company and a senior note due November 16th, 2007 with a principal amount of \$50. Each unit earned a dividend of 7% per year payable quarterly, while the dividend on the common stock of the company was only about 0.95%. Upon mandatory conversion, each unit would be converted into a variable number of shares of Motorola common stock. If the stock price is less than or equal to \$17.28 a holder will receive 2.8935 ($\\$50/\\17.28) shares of the company's common stock. If the stock price is between \$21.08 (a 22% appreciation from the reference price) and \$17.28, then the holder will receive a number of shares having a value equal to \$50. If the average price equals or exceeds \$21.08, each holder will receive 2.3719 ($\\$50/\\21.08) shares of the company's common stock. Hence the cap of the Equity Security Units is at the issue price of \$50. A holder does not benefit from the first 22% appreciation in the market value of the common stock, however if the stock price rises above \$21.08, the holders receive a fraction of any additional appreciation in the market value of the common stock. The holders have the option to settle the purchase contract early at any time prior to the seventh business day of the mandatory conversion date. In such cases the holder receives 2.3719 shares of the company's common stock regardless of the market price of the shares on that date.</p> |

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| <p>Flexible Equity-Linked Exchangeable Preferred Redemption Increased Dividend Equity Securities (FELINE PRIDES)</p> <p>Investment Bank:</p> <p>Merrill Lynch & Co. Inc.</p> | <p>Lincoln National Corp. issued 8,000,000 FELINE PRIDES units on 08/10/1998. The FELINE PRIDES consisted of two separately traded units; 7,000,000 units of Income PRIDES with an issue price of \$25 per unit, and 1,000,000 units of Growth PRIDES with an issue price of \$25 per unit. The market price of the common stock was \$92.875 (defined as the reference price) at the time of issue of the PRIDES. The mandatory conversion date for the FELINE PRIDES was August 16th, 2001. Each Income PRIDES unit consists of a stock purchase contract which obligates the holders to convert mandatorily to the firm's equity, and ownership of a preferred security having a liquidation value of \$25. Each Growth PRIDES unit also consists of a stock purchase contract, and a 1/40th interest in a zero-coupon U.S. Treasury security with a principal amount at maturity equal to \$1000 and maturing on August 15th, 2001. Each unit of Income PRIDES pays a dividend of 7.75% per year payable quarterly, which consists of a 6.4% interest on the preferred security and a contract payment of 1.35% per unit. Each unit of Growth PRIDES pays a contract payment of 1.85% per year, payable quarterly. Upon mandatory conversion each purchase contract of the FELINE PRIDES will receive a certain number of shares depending on the market price of the underlying common stock. If the stock price is less than or equal to \$92.875, then the holder will receive 0.2692 (\$25/\$92.875) shares for each purchase contract. If the average price is between \$92.875 and \$111.45 (a 20% appreciation from the reference price), then the holder will receive a number of shares that produces a value equal to \$25. If the average closing price equals or exceeds \$111.45, then the holder will receive 0.2243 (\$25/\$111.45) shares per purchase contract. Hence the cap on both the components of the FELINE PRIDES is at \$25, which is the issue price per unit of both Income and Growth PRIDES. It is possible for the holders of Income PRIDES to convert their holdings to Growth PRIDES and vice-versa.</p> |
| <p>Mandatory Adjustable Redeemable Convertible Securities (MARCS)</p> <p>Investment Bank:</p> <p>UBS Securities LLC</p> | <p>Coeur d'Alene Mines Corporation offered 6,588,235 shares of MARCS on 03/08/96 at \$21.25 which was also the selling price of the common stock of the company at that time. The mandatory conversion date for the MARCS was March 15th, 2000. Holders of MARCS were entitled to receive cumulative dividends payable quarterly at 7% per annum. This dividend rate was significantly higher than the rate at which dividends historically have been paid on the common stock of the company. Upon mandatory conversion each unit of MARCS will receive a certain number of shares depending on the market price of the underlying common stock. If the stock price is below \$21.25 (the issue price), each unit of MARCS will be converted into 1.111 shares of common stock plus the right to receive cash in an amount equal to all accrued or unpaid dividends on the mandatory conversion date. If the stock price is between \$21.25 and \$25.713 then the holder will receive a number of shares that produces a value of \$21.25. If the stock price exceeds \$25.713, then for each share of MARCS the holder will receive 0.826 (\$21.25/\$25.713) shares of the common stock. Hence the cap of these MARCS is the same as the issue price of \$21.25. The holder also has the option to convert prior to the mandatory conversion date (subject to certain limitations), in which case he will receive 0.826 shares of common stock per share of MARCS which is equivalent to a conversion price of \$25.713. The holders of MARCS have the same voting right as the holders of common stock. The shares of MARCS rank prior to the common stock as to payments of dividends and distribution of assets upon liquidation.</p> |
| <p>Mandatory Enhanced Dividend Securities (MEDS)</p> <p>Investment Bank:</p> <p>J.P.Morgan Securities Inc</p> | <p>Heller Financial issued 7,000,000 MEDS units on 04/26/01 at \$25 per unit, and the market price of their class A common stock was \$32.15 (defined as the reference price) per share at that time. The mandatory conversion date for the MEDS is May 18th, 2004. Each MEDS consists of two components: (1) A contract to purchase shares of the issuing company's class A common stock on the mandatory conversion date (i.e., the MEDS holders are obligated to convert mandatorily to the firm's equity), and (2) A trust preferred security issued by HFI Trust I, due May 2nd 2006. Each unit of MEDS pays a dividend of 7% per year payable quarterly from the trust preferred unit until the mandatory conversion date, after which the distribution rate will be reset. This is substantially greater than the dividend yield of 1.2% per year on the common stock. Upon mandatory conversion each unit of MEDS will receive a certain number of shares depending on the market price of the underlying common stock. If the stock price is less than or equal to \$32.15, then the holder will receive 0.7776 (\$25/\$32.15) shares for each MEDS unit. If the average price is between \$32.15 and \$38.58 (a 20% appreciation from the reference price), then the holder will receive a number of shares that produces a value of \$25. If the average closing price equals or exceeds \$38.58, then the holder will receive 0.6480 (\$25/\$38.58) shares per MEDS unit. Hence the cap on the MEDS units is at \$25 which is its issue price. A MEDS holder does not benefit from the first 20% appreciation in the market value of</p> |

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| | the common stock, however if the stock price rises above \$38.58, the MEDS holder receives a fraction of any additional appreciation in the market value of the common stock. |
| <p>Premium Equity Participating Security (PEPS)</p> <p>Investment Bank:</p> <p>Morgan Stanley Dean Witter</p> | <p>Valero Energy Corp issued 6,000,000 units of PEPS on 06/22/00 at \$25 per unit. The market price of the firm's common stock at that time was \$29.125 per share (defined as the reference price). The mandatory conversion date for the PEPS units is August 18th 2003. Each PEPS consists of two parts, a purchase contract which obligates the holder to mandatorily convert to the common stock of the company and a trust preferred security issued by VEC trust. Each PEPS unit earns a dividend of 7.75% per year payable quarterly while the dividend on the common stock of the company is only 1.10%. Upon mandatory conversion each unit of PEPS will be converted into a variable number of shares of Valero common stock. If the stock price is less than or equal to \$29.125 a holder of PEPS will receive 0.85837 (\$25/\$29.125) shares of the company's common stock. If the stock price is between \$34.95 (a 20% appreciation from the reference price) and \$29.125 then the holder will receive a number of shares having a value equal to \$25. If the average price equals or exceeds \$34.95 each PEPS holder will receive 0.71531 (\$25/\$34.95) shares of the company's common stock. Hence the cap of the PEPS is at the issue price of \$25. A PEPS holder does not benefit from the first 20% appreciation in the market value of the common stock, however if the stock price rises above \$34.95, the PEPS holders receive a fraction of any additional appreciation in the market value of the common stock. The PEPS holders have the option to settle the purchase contract early at any time prior to the seventh business day of the mandatory conversion date. In such cases the holder receives 0.71531 shares of the company's common stock regardless of the market price of the shares on that date.</p> |
| <p>Premium Equity Redemption Cumulative Security (PERCS)</p> <p>Investment Bank:</p> <p>Morgan Stanley Dean Witter</p> | <p>Kmart Corp. issued 5,750,000 shares of PERCS on 08/16/1991. Each unit of PERCS comprised of 4 depository shares which was offered to the public at \$44 per share which was also the market price of the firm's common stock at that time. The mandatory conversion date for each depository share was September 15th 1994. Each unit of PERCS paid a dividend at the rate of 7.75% per annum payable quarterly, while the dividend rate on the common stock was only 4%. Upon mandatory conversion each depository share of the PERCS will be converted into 1 share of Kmart common stock as long as the stock price is less than or equal to the cap price of \$57.20 (an appreciation of 30% above the issue price). If the common stock price is above the cap price of \$57.20 then the number of common shares per depository share will be such that the value equals the cap price of \$57.20. The PERCS rank senior to the company's common stock upon liquidation and holders of PERCS have the same voting rights as the holders of common stock.</p> |
| <p>Premium Income Equity Securities (PIES)</p> <p>Investment Bank:</p> <p>Lehman Brothers Inc.</p> | <p>Tesoro Petroleum Corporation issued 9,000,000 PIES units on 06/26/1998 at \$15.9375 per unit, which was also the selling price of the common stock of the company at that time. The mandatory conversion date for the PIES was July 1st, 2001. Each unit of PIES represents 1/100th of a share of 7.25% mandatorily convertible preferred stock of the company. Holders of PIES were entitled to receive cumulative dividends payable quarterly at 7.25% per annum. Upon mandatory conversion each unit of PIES will receive a certain number of shares depending on the market price of the underlying common stock. If the stock price is less than or equal to \$15.9375 (the issue price), each unit of PIES will be converted into 1 share of common stock. If the stock price is between \$15.9375 and \$18.85 then the holder will receive a number of shares that produces a value of \$15.9375. If the stock price exceeds \$18.85, then for each share of PIES the holder will receive 0.8455 (\$15.9375/\$18.85) shares of the common stock. Hence the cap of the PIES is set at the issue price of \$15.9375. At any time after July 26th, 1998 and prior to the mandatory conversion date the holder also had the option to convert to equity, in which case he received 0.8455 shares of common stock per share of PIES which is equivalent to a conversion price of \$18.85. The holders of PIES however were not entitled to any voting rights.</p> |
| <p>Preferred Redemption Increased Dividend Equity Securities (PRIDES)</p> | <p>MCN Corp issued 5,100,000 shares of PRIDES on 04/22/1996 at \$23 per share, which was also the selling price of the common stock of the company at that time. The mandatory conversion date of the PRIDES was April 30th, 1999. Each security consists of (1) a stock purchase contract under which the holders are obligated to convert mandatorily to the firm's equity on the mandatory conversion date and a commitment from the company to pay yield enhanced payments of 2.25% to the holders of the PRIDES and (2) 6.50 % U.S. Treasury notes having a principal amount equal to the issue price and maturing on the mandatory conversion date. Thus each unit of PRIDES pays a dividend of 8.75% per annum payable semi-annually, whereas historically the average dividend on the common stock</p> |

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| <p>Investment Bank: Merrill Lynch & Co. Inc.</p> | <p>has been around 5%. Upon mandatory conversion each security was to be converted to a variable number of shares of common stock of the company. If the common stock price is less than or equal to the issue price of \$23, then for each unit of PRIDES the holder will get 1 share of the company's common stock. If the stock price is between \$23 and \$27.60 (a 20% appreciation from the issue price) then the holder will receive a number of shares that produces a value of \$23. If the stock price is greater than \$27.60, then the holder will receive 0.833 ($\\$23/\\27.60) shares of common stock per PRIDES unit. Hence the cap of the PRIDES is set at the issue price of \$23. Holders of PRIDES also have an early settlement option, in which case they will receive 0.833 shares of common stock per unit of PRIDES regardless of the market price of the common stock. Holders of PRIDES have no voting rights, unlike the common shareholders.</p> |
| <p>Threshold Appreciation Price Securities (TAPS) Investment Bank: Smith Barney Inc.</p> | <p>MedPartners Inc. issued 18,929,577 shares of TAPS on 09/15/97 at \$22.1875 per share, which was also the selling price of the common stock of the company at that time. The mandatory conversion date of the TAPS was August 31st 2000. Each security consists of (1) a stock purchase contract under which the holders are obligated to convert mandatorily to the firm's equity on the mandatory conversion date and a commitment from the company to pay yield enhanced payments of 0.25% to the holders of the TAPS and (2) 6.25 % U.S. Treasury notes having a principal amount equal to the issue price and maturing on the mandatory conversion date. Thus each unit of TAPS pays a dividend of 6.5% per annum payable semi-annually, whereas historically the common stock of the company has not paid any dividends at all. Upon mandatory conversion each security was to be converted to a variable number of shares of common stock of the company. If the common stock price is less than or equal to the issue price of \$22.1875, then for each unit of TAPS the holder will get 1 share of the company's common stock. If the stock price is between \$22.1875 and \$27.0678, then the holder will receive a number of shares that produces a value of \$22.1875. If the stock price is greater than \$27.0678, then the holder will receive 0.8197 ($\\$22.1875/\\27.0678) shares of common stock per TAPS unit. Hence the cap of the TAPS is set at the issue price of \$22.1875. Holders of TAPS also have an early settlement option, in which case they will receive 0.8197 shares of common stock per unit of TAPS regardless of the market price of common stock. Holders of TAPS have no voting rights, unlike the common shareholders.</p> |
| <p>Trust Automatic Common Exchange Securities (TRACES) Investment Bank: Goldman Sachs & Co.</p> | <p>Estee Lauder Inc. issued 1,734,104 units of TRACES on 2/17/1999 at \$86.50 per unit, which was also the selling price of the common stock of the company at that time. The mandatory conversion date for the TRACES is February 23rd 2002. The TRACES were issued by a trust formed by the Company solely for this purpose, and the trust terminates automatically 10 business days after the mandatory conversion date. Each TRACES unit earns a dividend of 6.25% per year payable quarterly while the dividend on the common stock of the company is only about 0.45%. Upon mandatory conversion, each unit of TRACES will be converted into a variable number of class A common stock of Estee Lauder Inc. If the stock price is less than \$86.50 a holder will receive 1 share of class A common stock. If the stock price is between \$86.50 and \$102.07 (a 18% appreciation from the reference price) and \$86.50, then the holder will receive a number of shares having a value equal to \$86.50. If the average price equals or exceeds \$102.07, each holder will receive 0.8475 ($\\$86.50/\\102.07) shares of the company's class A common stock. Hence the cap of the TRACES is at the issue price of \$86.50. A holder does not benefit from the first 18% appreciation in the market value of the common stock. However, if the stock price rises above \$102.07, the holders of TRACES receive a fraction of any additional appreciation in the market value of the common stock. The holders of TRACES have voting rights with regard to matters of the Trust fund issuing the securities only till the mandatory conversion date, after which they have the same rights as the holders of class A common stock of the company.</p> |