Chapter 5 Discrete Probability Distribution

Random variables
Discrete Probability Distributions
Expected Value and Variance
Binomial Distribution
Poisson Distribution

A random variable is a **numerical** description of the outcome of an experiment
Ex. randomly picks a person and measures his height
Ex. tossing a fair coin, define the outcome x=1if head and x=0 if tail

A discrete random variable is a variable which can take on a **countable number (maybe infinite number)** of values. Alternatively, a discrete random variable is a variable which can take on a finite number of values or an infinite sequence of values.
Countable: if $x \in [1, 3]$, it is uncountable.

- Discrete random variable with a finite number of values
  Ex: Let x = number of days working in one week.
  where x can take on 7 values (0, 1, 2, 3, 4, 5, 6, 7)
- Discrete random variable with an infinite sequence of values
  Let x = number of customers arriving in one day,
  where x can take on the values 0, 1, 2, . . .
  We can count the customers arriving, but there is no finite upper limit on the number that might arrive.

More examples: discrete random variables:

Toss a fair coin 3 times, x=the possible number of heads obtained
x=Number of cars in a family
x=Own dog or cat
= 1 if own no pet;
= 2 if own dog(s) only;
= 3 if own cat(s) only;
= 4 if own dog(s) and cat(s)
x=education level
=1 if <=HS
=2 if <=Ba/Bs
=3 if > Ba/Bs

A continuous random variable is a variable which can assume any numerical value in one or more given intervals.
Ex. individual's height, weight,
Traveling distance from home to school

Discrete probability distribution
The **probability distribution** for a discrete random variable describes how probabilities are dis`tributed over the values of the random variable.
The **probability distribution** is defined by a probability function, denoted by f(x). it provides the probability for each value of the random variable.

f(x) needs to satisfy 2 conditions:

1. 1>=f(x)>=0 (because f(x) is a probability)
2. sum of f(x)over all the possible values of x is 1: $\sum_{i=1}^{n} f(x_i) = 1$

Ex 1. Tossing a coin once, let x=1 if head, x=0 if tail, then x has the following probability distribution:

x	f(x)
0	0.5
1	0.5

Ex 2. Tossing a coin twice, let x be the number of heads obtained, then x has the following probability distribution:

x	f(x)
0	0.25
1	0.5
2	0.25

Since 0, 1, 2 are all the possible outcomes for x, they make up of the sample space; therefore the sum of their probability should be 1.

We can describe a discrete probability distribution with **a table, graph, or equation**.

Tabular representation of Probability Distribution

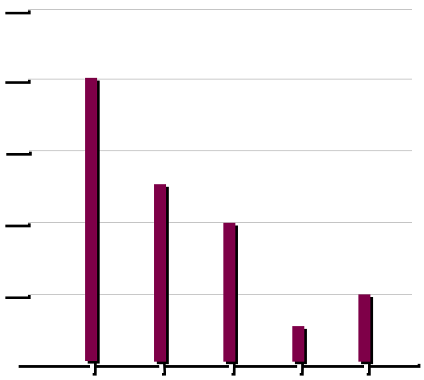
Ex3: Using past data on TV sales, …

A tabular representation of the probability distribution for TV sales was developed.

Units Sold	Number of Days
0	80
1	50
2	40
3	10
4	20
	200 ➔

x	f(x)
0	.40
1	.25
2	.20
3	.05
4	.10
	1.00

Graphical Representation of Probability Distribution



Function /equation representation
The discrete uniform probability distribution is the simplest example of a discrete probability distribution given by a formula.
The discrete uniform probability function is

Prob f(x)=1/n.

where:

n = the number of values the random variable may assume

It implies the random variable x takes on n equally likely values; therefore, f(x) is a constant.

Expected value and variance
The expected value, or mean, of a discrete random variable is a measure of its central location. $E(x) = \mu = \sum_{i=1}^{n} x_i f(x_i)$

It's a weighted average, and the weight is the probability.
The variance summarizes the variability in the values of a random variable.

$$Var(x) = \sigma^2 = \sum_{i=1}^{n} (x_i - \overline{x})^2 f(x_i)$$

Standard deviation is the positive square root of variance

$$\sigma = \sqrt{\sigma^2}$$

For a uniformly distributed discrete random variable $E(x) = \mu = \frac{1}{n}\sum_{i=1}^{n} x_i$

$$Var(x) = \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \overline{x})^2$$

Ex 2 cont' E(x)=?

x	f(x)	xf(x)
0	0.25	0
1	0.5	0.5
2	0.25	0.5
	1	μ =1

Ex 2 cont' Var(x)=?

x	$\bar{x}$	x-$\bar{x}$	(x-$\bar{x}$)^2	f(x)	(x-$\bar{x}$)^{2}f(x)
0	1	-1	1	0.25	0.25
1	1	0	0	0.5	0
2	1	1	1	0.25	0.25
Total		**0**		**1**	Var(x)=0.5

Ex3 Cont' E(x)=?

x	f(x)	xf(x)
0	.40	.00
1	.25	.25
2	.20	.40
3	.05	.15
4	.10	.40

E(x) = 1.20➔ expected number of TVs sold in a day

Ex 3 cont' Var(x)=?

x	x - μ	(x - μ)^2	f(x)	(x - μ)^{2}f(x)
0	-1.2	1.44	.40	.576
1	-0.2	0.04	.25	.010
2	0.8	0.64	.20	.128
3	1.8	3.24	.05	.162
4	2.8	7.84	.10	.784

Variance of daily sales = σ^2 = 1.660