Exponential Probability Distribution

Poisson distribution applies to a number of events occurring in a specified interval of time or space if these events occur with a **known constant average rate.**

Exponential distribution applies to the same situation. But instead of describing the probability of a number of events occurring in a specified interval of time or space, it is used to model the time/ distance between two successive events.

Time between two consecutive events is sometimes called duration.

Relationship between the Poisson and Exponential Distributions

- The Poisson distribution provides an appropriate description of the number of occurrences per interval.
- The exponential distribution provides an appropriate description of the length of the interval between occurrences

Examples of events that can be modeled as Poisson distributions include:

- The number of cars that pass through a toll booth during a given period of time.
- The number of incoming phone calls at a call center per minute.
- The number of road-kill found per unit length of road.
- the number of blemishes or flaws occurring in each 100 feet of material

Accordingly, exponential distribution can be used to describe:

- The time between two successive cars arriving at a toll booth.
- The time between two successive incoming calls
- The distance between two adjacent road-kills on a given street
- The length of material between two adjacent flaws.

In real-world scenarios, the assumption of a constant rate (or probability per unit time) is rarely satisfied. For example, the rate of incoming phone calls differs according to the time of day. But if we focus on a time interval during which the rate is roughly constant, such as from 2 to 4 p.m. during work days, the exponential distribution can be used as a good approximate model for the time until the next phone call arrives.

Probability Density Function

$$f(x) = \frac{1}{\mu} e^{-x/\mu}, \text{ for } x \geq 0, \mu > 0$$

where, μ = mean, the expected duration/time between events,$1/\mu$ is a rate parameter

e = 2.71828

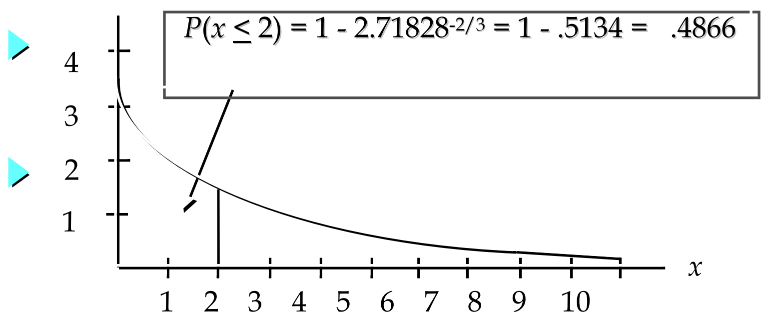
Cumulative Probability Function

$$\Pr ob(x \le c) = 1 - e^{-c/\mu}$$

where, c = some specific value of x, measured in the same units as μ.

The exponential distribution is skewed to the right.
The skewness measure for the exponential distribution is 2.

Example: The time between arrivals of cars at a gas station follows an exponential probability distribution with a mean time between arrivals of 3 minutes. What is the probability that the time between two successive arrivals will be 2 minutes or less? What is the probability that the time between two successive arrivals will be 3 minutes or less? What is the probability that the time between two successive arrivals will be exactly 3 minutes? What is the probability that exactly one car arriving in 3 minutes? What is the probability that one or more cars arriving in 3 minutes?



Time Between Successive Arrivals (mins.)

A property of the exponential distribution is that the mean, m, and standard deviation, s, are equal.
Thus, the standard deviation, s, and variance, s 2, for the time between arrivals at Al's gas station are:

$$\mu = \sigma = 3 \text{ min.}$$
$$\sigma^2 = 9$$