

# Teaching Mathematics Graduate Students to Teach: An International Perspective

Solomon Friedberg\*  
Department of Mathematics  
Boston College  
Chestnut Hill, MA 02467-3806  
USA  
friedber@bc.edu

High-quality teaching of mathematics at the university level is important to every country. Almost all of the best jobs in the global economy require fluency in mathematics, and a great many require advanced technical knowledge. High-quality university math instruction contributes to high student achievement in the quantitative disciplines and prepares students for good jobs. Moreover, university faculty in most countries teach the students who will themselves become the next generation of pre-collegiate math teachers. If these teachers are to succeed with their own students they must have, along with teaching skills, detailed mathematical knowledge, understanding, and skills. So it is important that these teachers-to-be attend well-taught mathematics classes at the university. Thus every nation should be vitally interested in high-quality mathematics teaching at the university level.

Such considerations suggest that faculty development as regards teaching might be useful. Though this is certainly true, it is also true that math faculty find themselves with a wide range of responsibilities and time constraints and might not be receptive to such development. Instead, it may be more practical to focus on future math faculty.

Mathematics graduate students go on to become the next generation of university faculty. I wish to argue here that it is possible to develop programs for such graduate students that equip them to do an excellent job as future university teachers. Moreover, it is possible to do so without unduly impacting their mathematics studies. The implementation of such a program—one whose goal is to prepare its graduates to teach a class independently and excellently upon graduation—can thus be a high-reward, low-cost opportunity.

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The question of what to do to achieve this goal, then, is the concern here. Certainly, students in such a program should practice giving mathematical explanations. One can bring graduate students to the blackboard to explain some problem or concept, and then give them critical feedback. In addition, one should talk about technical aspects of teaching, such as how to plan a course, how to write a syllabus, how to write and fairly grade a test, and how to assign course grades. To go beyond this, one might read advice about how to teach well, such as the book by Krantz [4], and students might then practice the teaching of a specific mathematical topic, balancing examples, heuristics, theorems, explanations or proofs, and problems that extend the topic. One might also wish to discuss things specific to one's institution, such as university rules as regards registration and grading, accommodations for students with disabilities, privacy of student records, and cheating. All university mathematics faculty must be aware of such matters.

Though such steps are valuable, one can rightly ask if they are sufficient to promote excellence in teaching. They do not prepare novice teachers, for example, for crises in the classroom, for student responses that were not expected, for students who are not motivated to learn mathematics or who have difficulty in learning mathematics, for classrooms with a range of student abilities, interests, and knowledge levels, for what to do when the students do not understand what has been taught. In the rest of this article, I will discuss an approach to developing excellent teaching that seeks to go beyond the steps outlined in the preceding paragraph but that functions well in concert with them, that takes full account of the need for individuals to develop their own styles of excellent teaching, and that addresses teaching crises and issues such as the ones mentioned above.

The approach is based on *case studies*. Case studies are written (possibly fictionalized) depictions of university student-TA (Teaching Assistant) interactions in undergraduate mathematics instruction. Each case presents a teaching difficulty or a key moment of decision in the classroom—in short, a teaching crisis—together with a context for understanding or examining the crisis. Typically a number of issues are intertwined, sometimes subtly. The cases are to be read and analyzed by a group consisting of the teaching assistants and a faculty facilitator. The goal of such a discussion is to consider different approaches and to invite different judgments concerning the teaching crisis. In doing so, the participants are led to think deeply about teaching and to learn from the collective experiences and judgments of the full group of participants.

It is important to emphasize that the case studies method is open to different approaches to teaching excellence. Indeed, just as not all first-rate mathematics researchers have the same style in their approach to research, there is more than one way to be an excellent teacher. Choices, governed by personal style, mathematical taste, and individuals' understandings of the goals of their teaching (among other factors), play a role. Such choices often appear in the case discussion, and participants will understand that there are a range of well-thought-out decisions that could be appropriate to a given situation.

As an example, here is a brief outline of one case study. A graduate student runs a problem session for Calculus students. He is concerned that students are interested only in obtaining the answers to the homework at the problem session. He wants to promote their understanding without undercutting the principle that they should do the homework themselves. What should he do? (The full case is approximately 10 pages long and describes the TA's interactions with his students during a specific class meeting in detail. It raises not just one but a number of pedagogical issues.)

The key point of the case studies method is that the group analyzes the case events. They need not agree but should be able to support their ideas. The case discussion is a variation on the Socratic method<sup>1</sup>. What is behind such a method? It is this. Beyond the fundamental requirement of subject matter knowledge, and beyond technical skills such as the ability to write clearly, *good teaching requires good judgment*. A good teacher must decide how to blend examples and theorems to maximize understanding of the result and its applications, how to balance the needs of students of different levels and abilities, how to assign homework that will get students to work to the maximum of their abilities, what to say when a student is in difficulty. Good judgment enters fundamentally into the classroom choices that make the difference between excellent and mediocre teaching.

Can good judgment in the context of teaching be taught? There is a great deal of evidence that, surprising, it can be, using the case study method described above. Case studies originated in US business and law schools, professions where good judgment is a key to success. In those disciplines, students read written depictions of key decisions in commerce and legal proceedings and analyze them, typically in a group discussion format. The persistence of this method attests to its usefulness. The case studies method has been applied to university-level teaching since the 1960s, much of this work originating with Prof. C. Roland Christensen of the Harvard Business School [1]. Once again, the goal is to develop good judgment by the group analysis of cases, this time concerning teaching. However, the issues and situations that appear in Christensen's cases are far from the ones of concern to most mathematics graduate students.

This author led a large-scale project to develop case studies that were directed at mathematics graduate students in US universities<sup>2</sup>. This work has been published in the book [3] (available in both faculty and graduate student editions) and it is now being used in a wide range of US universities. The cases in [3] give mathematics graduate students the chance to analyze complicated realistic teaching situations, to think in advance about how to handle teaching crises so that they can deal with them when they arise in real life, to formulate their own approach to teaching, and to view teaching as non-trivial and a worthy subject of serious discussion.

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<sup>1</sup>Socratic method: "A pedagogical technique in which a teacher does not give information directly but instead asks a series of questions, with the result that the student comes either to the desired knowledge by answering the questions or to a deeper awareness of the limits of knowledge."—American Heritage Dictionary of the English Language, Fourth Edition, Houghton Mifflin Company.

<sup>2</sup>Development team members: Avner Ash, Elizabeth Brown, Solomon Friedberg, Deborah Hughes Hallett, Reva Kasman, Margaret Kenney, Lisa A. Mantini, William McCallum, Jeremy Teitelbaum, and Lee Zia.

The cases we produced were evaluated for effectiveness. Among the conclusions was that the cases were effective in promoting thought and dialogue about teaching, and that they were also effective at communicating to foreign graduate students many aspects of university culture in the United States that impact teaching. We did find that beginning graduate students, many of whom have not thought about how to teach mathematics, did not all possess the experience base to use the cases effectively, but also that it was possible to address this by including a more experienced TA in the discussion group. Lastly, we found that leading a case discussion requires the faculty leader to use very different skills—the skills of a discussion leader—than come in to play in most mathematics teaching.

To address this last point, the author, Dr. Diane Herrmann of the University of Chicago, and the American Mathematical Society have teamed up to offer a series of workshops for faculty who work with TAs on teaching. The latest of these was at the 2005 joint math meetings, the annual conference sponsored by all the major American mathematical societies. Attendees included 4 faculty from non-English speaking countries.

This brings us to the topic of applying these methods internationally. There are two separate reasons to do so. The first is to prepare individuals to go to the United States for graduate school or to hold postdoctoral or other faculty positions. These individuals are more likely to be successful if they are able to teach US students well (for graduate students, for example, good teaching is critical to receiving and retaining financial support). Doing so requires language ability in English, knowledge of the culture and expectations of United States universities, and skill in the teaching of mathematics. The case studies in [3] contribute to all of these. Besides contributing to the development of mathematics teaching skills, they serve as a catalyst to reflect on how the culture of US universities will affect the way that someone from another culture teaches in the US. They also contribute to language ability if the discussion takes place in English, particularly if participants are asked to directly model or role-play their proposed response to a classroom situation. So it would be valuable to implement a TA-development program using the case studies in [3] for this reason.

The second reason to apply the case study method internationally is to improve the teaching of mathematics graduate students worldwide. As we have argued above, this method has the potential to be highly effective when used with mathematics graduate students, and implementing such a program can be of value to every country. However, it seems likely that some additional, country-specific, work with the cases will be necessary to achieve the full potential of the method. The first question is to how to produce high quality relevant case studies for other, non-English speaking, countries. A starting point would be to translate the materials we have produced into other languages. Some of the cases could be translated with minimal changes. Others might reflect aspects of university culture in the United States that are not relevant elsewhere; for example, issues related to teacher-student interactions that are different in different cultures. An alternative approach would be to generate a set of case studies specific to another country's university teaching. The cases in [3] were developed through an extensive process of writing, feedback from graduate students and faculty, and rewriting. This process is described

in detail in [3]. It might be possible to shortcut the process described there somewhat, but the following guiding principles are important: (i) the cases (though possibly fictional) must be completely true-to-life; (ii) the cases must raise non-trivial questions in teaching; (iii) the cases must not be one-sided, but rather raise questions about which it is possible for intelligent people to disagree. Feedback from graduate students is crucial in evaluating draft cases.

Another issue for applying the case study method internationally concerns how to lead a discussion. Faculty must create an environment in which graduate students are willing to express their opinions candidly, and they must learn to facilitate discussions that truly promote good judgment in making teaching decisions. Doing so requires a range of skills related to discussion-leading. There are several ways to develop such skills. One is to learn the method from someone who is skilled in using it with mathematics graduate students. The number of such individuals is increasing, but most are based in the United States. A second is to seek out those who have discussion-leading skills in one's own country. For example, discussions are a common feature in the humanities, and many US universities provide workshops for faculty on how to lead discussions. If such opportunities are available elsewhere it might be valuable to take advantage of them. A third method is to learn about discussion-leading by reading about it, and then to experiment. To learn the method by reading, one might consult the faculty edition of our book [3], which contains extensive information on how to use case studies (as well as detailed teaching notes on each of our cases), or Christensen's classic text [1]. The volume [2] contains additional information about discussion leadership and the potential impact of discussions.

In conclusion, it is time for the mathematics profession to articulate the goal that each new Ph.D. in mathematics be an excellent mathematics teacher upon completion of their doctoral studies. Case studies for mathematics graduate students can contribute to achieving this goal. It would be of great interest to implement this method in a variety of cultures and countries.

## References

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