**DCF analysis**
**Chpt. 4: problems 12, 18, 26, 30, 42, 48**
**Chpt. 5: problems 4, 7, 9, 13, 16, 22, 23, 27, 30**

## I. Discounted cash flow basics.

### *1. Discount rates & effect of compounding:*

***Effective annual interest rate*** **(EAR) takes into account the compounding effects of more frequent interest payments.**

***APR*** **= periodic rate * # periods per year**

$$EAR = \left[1 + \frac{APR}{m}\right]^m - 1$$

### *2. Present and future value*

**The basis relationship between present & future value is expressed:**

$$PV = \frac{FV_n}{(1+r)^n}$$

**The present (future) value of a stream of cash flows is simply the sum of the present (future) values of the individual cash flows.**

### *3. Annuities*

**Annuity: constant cash flow (A) occurring at regular intervals of time.**

**The <u>present value of a simple annuity</u> is calculated:**

$$PV = CF * \frac{1 - \frac{1}{(1+r)^n}}{r} = CF * \frac{1 - PV_r^t}{r} = CF * A_r^t$$

**where $A_r^t$ is known as the present value of annuity factor.**

**Important! This formula assumes the first payment in the annuity is received one period after the present value date.**

**Example: Suppose your monthly mortgage payments are $1,028.61 for 360 months, and the monthly interest rate is 1%. What is the value of the mortgage today?**

**<u>Future value of an annuity</u>:**

$$FV = CF * \frac{(1+r)^n - 1}{r} = CF * FVA_r^t$$

**where $FVA_r^t$ is the future value of annuity factor.**

**Example: You are very concerned about retirement. You plan to set aside $2000 at the end of each year in your IRA account for the next 40 years. If the interest rate is 15% how much will you have at the end of the 40th year?**

**FV =**

**Other useful formulas:**

**Growing annuity: cash flows growing at a constant rate and paid at regular intervals of time.**

$$PV = A * (1+g) * \left[ \frac{1 - \frac{(1+g)^n}{(1+r)^n}}{r-g} \right]$$

**Perpetuity: constant cash flows at regular intervals forever.**

$$PV = \frac{CF}{r}$$

**Growing perpetuity: constant cash flow, growing at a constant rate, and paid at regular time intervals forever.**

$$PV = \frac{CF}{r-g}$$

## II. Examples of DCF valuation: Bond Valuation

**Payments to the bondholder consist of:**

**1. Regular *coupon* payments every period until the bond matures.**

**2. The *face value* of the bond when it matures.**

**If a bond has five years to maturity, an $80 annual coupon, and a $1000 face value, its cash flows would look like this:**

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Coupons | | $80 | $80 | $80 | $80 | $80 |
| Face Value | | | | | | $1000 |
| Total | | | | | | $1080 |

**The cash flows on a bond do not change as interest rates change!**

**How much is the bond worth if the going rate on bonds like this one is 10%?**

**Bond jargon:**

**Coupon rate**

**current yield**

**yield to maturity**

**Whenever the YTM and coupon rate are the same, price = face value.**

**When YTM is greater than the coupon rate, the bond sells for less than its face value (par value) and is called a *discount* bond.**

**When coupon rate is greater than the YTM, the bond price is greater than the face amount (greater than par) and is called a *premium* bond.**

## III. Examples of discounted cash flow valuation: Stock Valuation

**If dividends to grow over time at a constant rate g, then**

$$P_0 = [D_0(1+g)]/(r-g) = D_1/(r-g)$$

**This is known as the *dividend growth model.***
**We can rewrite this equation to find the required rate of return:**

$$r = \frac{D_1}{P_0} + g$$

**$D_1/P_0$ = *Dividend yield***
**and**
**g = rate of growth of dividends, which can also be interpreted as the *capital gains yield.***

**Example with constant growth:**

**Suppose a stock has just paid a $4 per share dividend. The dividend is projected to grow at 6% per year indefinitely. If the required return is 10%, then the price today is:**

$P_0 = D_1/(r-g)$

$= \$4 \times (1.06) / (.1-.06)$
$= \$4.24/.04$
$= \$106.00$ **per share**

**What will the price be in a year? It will rise by 6%:**

$P_t = D_{t+1}/(r-g)$
$P_1 = D_2/(r-g) = (\$4.24 \times 1.06)/(.10 - .06) = \$112.36$

**Example with nonconstant growth:**

**Suppose a stock has just paid a $4 per share dividend. The dividend is projected to grow at 8% for the next two years, then 6% for one year, and then 4% indefinitely. The required return is 12%. What is the stock value?**

| Time | Dividend |
|---|---|
| 0 | $4.00 |
| 1 | $4.32 |
| 2 | $4.66 |
| 3 | $4.95 |
| 4 | $5.14 |

**At time 3, the value of the stock will be:**

$P_3 = D_4/(r-g) = \$5.14\ /(.12 - .04) = \$64.25$

**The value of the stock is thus:**

$P_0 = D_1/(1+r) + D_2/(1+r)^2 + D_3/(1+r)^3 + P_3/(1+r)^3$

$= \$4.32/1.12 + \$4.66/1.12^2 + \$4.95/1.12^3 + \$64.25/1.12^3$

$= \$56.83$

**Other Definitions:**

**Term Structure of Interest Rates**

**Risk Structure of Interest Rates**