

Concentration of Competing Retail Stores*

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Abstract

The geographical concentration of stores that sell similar commodities is analyzed using a two-dimensional spatial competition model. A higher concentration of stores attracts more consumers with taste uncertainty and low price expectations (a market-size effect), while it leads to fiercer price competition (a price-cutting effect). Our model is general enough to allow for the coexistence of multiple (possibly) asymmetric clusters of stores. We provide sufficient conditions for the nonemptiness of equilibrium store location choices in pure strategies. Through numerical examples, we illustrate the trade-off between the market-size and price-cutting effects, and provide agglomeration patterns of stores in special cases.

Keywords: consumer search, market size effect, price cutting effect, taste uncertainty.

JEL classification number: D4, L1, R1, R3.

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1. Introduction

A high concentration of car dealers is commonly observed in American suburbs. Similarly, one finds several fashionable apparel stores in a single shopping mall. In both cases, the competitors' commodities are substitutable, and the consumer typically buys only one unit. Thus, by concentrating at one location, competitive forces drive down the prices of commodities. The questions we ask in this paper are: Why do several stores concentrate at the same location? Why don't they instead keep some distance from other stores and monopolize the nearby customers?

One plausible answer is that there is beneficial trade-off between an increased market size and the cut in prices. Consider the following example: A consumer gets up on Sunday morning wondering if she should get a new fancy car to replace her old Honda. She has some vague idea about fancy cars, but she does not know how much she likes them (relative to their high prices) before she actually visits the dealers and test-drives them. Suppose that she expects that if she visits any one car dealer (BMW, Mercedes, Volvo, and so on), the probability that she will like the cars sold by there well enough to buy is $\frac{1}{4}$ (25%), and these probabilities are independently distributed. Then, if she visits a shopping center with a BMW dealer only, the probability of finding a buyable car is $\frac{1}{4}$ (25%), which is a bit of a costly way to spend her precious Sunday. On the other hand, if a shopping center has both a Mercedes and a Volvo dealer, then the probability of finding a buyable car is $\frac{7}{16}$ (43.7%) (since the probability of *not* finding a buyable car at each dealer is $\frac{3}{4}$ (75%), and if she visits both the probability that she cannot find a buyable car at either dealer decreases to $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$). Given the increased chance of finding a car she likes, she may visit the shopping center with two car dealers even though the location is somewhat far away. Likewise, if there are five car dealers at one shopping center, the probability of finding a buyable car there increases to

$1 - \left(\frac{3}{4}\right)^5$ (76.3%), so that it becomes very likely that she would not waste her Sunday by visiting the shopping center. In such a case, she may not mind going to the shopping center although it may be quite far away from her house. Thus, a high concentration of car dealers can increase the size of the pie (*the market-size effect due to taste uncertainty*), although close proximity implies that they then compete with each other more vigorously (*the price-cutting effect*). Therefore, if the former effect sufficiently exceeds the latter effect, then car dealers can actually make higher profits under higher concentration than by operating alone.

In this paper, we formalize this trade-off between the market-size effect and the price-cutting effect using a spatial oligopoly model with price competition. To determine the number of consumers who visit a given shopping center (the market size), it is necessary to determine the geographical area from which residents visit this shopping center (the market area). To pin down the market size via the market area, we need to introduce an explicit *spatial* structure into our model. The key assumption we use in this paper is that consumers do not know their exact preferences over commodities (consumer taste uncertainty). The structure of the model is as follows: Consumers are distributed over the plane and each consumer can buy at most one unit of a commodity at a shopping center by paying the commuting costs in addition to the price of the commodity. There is a finite number of stores that decide their locations from a set of potential shopping centers (stage I). Consumers can observe the locations of stores, but they know neither their willingnesses-to-pay for commodities nor the prices before they actually visit the stores.¹ Thus, when a consumer decides which shopping center to visit, she calculates the expected utility of searching for commodities at each shopping center by taking commuting costs into account. For simplicity, we assume that

¹The market structure is similar to the ones in Perloff and Salop [32], Wolinsky [43, 44], and Fischer and Harrington [18]. Anderson and Renault [2] synthesize the literature of product diversity and consumer search nicely.

each consumer chooses to visit at most one shopping center (stage II). Once she arrives at the shopping center, the commuting costs are sunk, and at no cost she can try out every commodity sold at that shopping center. Thus, she chooses to buy the commodity that gives her the highest (positive) surplus (her realized willingness-to-pay minus the price of a commodity) among those sold at the shopping center. If no commodity gives her a positive surplus, she does not buy any commodity. Taking consumers' commodity choice behavior into account, stores compete over prices (stage III). If a consumer's willingness-to-pay distributions over various commodities are not perfectly correlated (statistical independence is assumed in this paper), a higher concentration of stores at a shopping center increases the consumer's expected utility of visiting there. This implies that consumers living far away may visit the shopping center, and thus its market area expands. However, since each consumer can choose the commodity which gives her the highest surplus among commodities available at the shopping center, stores will be forced to compete for customers by cutting prices. Thus, our model captures the trade-off between the two effects by featuring both an explicit geographical structure of the economy and price competition among stores.

Looking more closely at the mechanics of geographical concentration of stores, we find that there are two distinct but interconnected incentives for stores to concentrate. First, as we noted in the above example, there is *the market-size effect due to taste uncertainty*: a higher concentration of stores increases the probability of a consumer finding a buyable commodity at the shopping center. Thus, a consumer's expected utility from shopping there increases, resulting in a larger market size at that shopping center. The second effect also operates through an increase in the consumer's expected utility: a higher concentration of stores sends to consumers a signal of lower prices. This increases a consumer's expected utility of choosing the shopping center, and the market size expands again. This may be called *the market-size effect due to lower price expectations*. Thus, consumer

taste uncertainty and imperfect information regarding prices give stores incentives to concentrate their locations.²

In this paper, we first establish the existence of the third-stage equilibrium and an inverse relationship between the number of stores at a shopping center and the equilibrium prices (the price-cutting effect: Proposition 1). Then, we show that the radius of the market of a shopping center increases with the number of stores (the market-size effect: Proposition 2). These two propositions show that our model captures the trade-off between these two effects. Moreover, we can show that the market-size effect can be decomposed into one due to taste uncertainty and one due to lower price expectations (Proposition 2). However, to establish the existence of a subgame perfect equilibrium is more problematic. The main difficulty comes from store's location choice problem (stage I). By the market-size effect, if a shopping center has several stores then it is not profitable to open a store near the shopping center: All potential customers will visit the shopping center, and the new store cannot make any profit (the "urban shadow"). On the other hand, a store can make a positive profit if it is opened at the shopping center, or if it is opened far from any shopping centers. Thus, each store's profit function is not quasi-concave with respect to its location, and we cannot apply the standard fixed point argument to the stores' location choice problem. We provide three existence theorems for a subgame perfect equilibrium in pure strategies without using explicit functional forms, although the conditions are somewhat strong (Propositions

²There is an additional incentive for stores to concentrate that does not arise through the expansion of market size: Suppose that there are two stores each of which has a mutually exclusive customer group. If each of them sells its commodity to its own customer group, then many consumers cannot find a buyable commodity, since each consumer has access to only one type of commodity. However, if these two stores pool their customers, then the consumers' probability of finding a buyable commodity increases as long as consumers' willingnesses-to-pay are not perfectly correlated between two commodities. This implies that these two stores' per store sales and profits will be raised by pooling their consumers, if their prices are kept constant. This effect may be called *the consumer pooling effect*. Although we will not discuss this effect in the paper, our model takes it into account. I thank Parikshit Ghosh for helpful conversations on this effect.

3, 4, and 5).

We then illustrate the relationship between the number of stores at a shopping center, equilibrium prices, market sizes, consumers' probability of finding a buyable commodity, and each store's profit by two numerical examples with the following simple structure: (i) consumers are uniformly distributed over the plane, and (ii) consumers' willingnesses-to-pay are uniformly distributed. The first example assumes that potential shopping centers are far from each other so that their markets do not overlap. The most striking observation is that a marginal increase in the number of stores dramatically expands market size and each store's profit when there are a small number of stores at a shopping center (Table 1). Thus, in this case the market-size effect dominates the price-cutting effect, and there is a strong incentive for stores to concentrate at the same shopping center. In this example, we can also fully characterize the set of subgame perfect equilibria. The result suggests that (a) there will be multiple (quasi-) homogenous shopping centers, and (b) there could be multiple (Pareto-ranked) subgame perfect equilibria due to the coordination problem (Proposition 6). The second example assumes that there are only two potential shopping centers, but their markets can overlap. We observe that a symmetric equilibrium (the same number of stores at each shopping center) and/or clustering equilibria (all stores at the same shopping center) exist depending on the number of stores and the distance between the two shopping centers. If the number of stores is relatively small and the two shopping centers are very close to each other, then a symmetric equilibrium may vanish. On the other hand, if the number of stores is relatively large and if the two shopping centers are not close to each other, then clustering equilibria may vanish. When two types of equilibria coexist, a symmetric equilibrium tends to attain a higher profit for each store.

The rest of the paper is organized as follows: Section 2 provides a brief summary of the literature. Section 3 presents the formal model. Section 4 analyzes each subgame and provides three existence

theorems for a pure strategy subgame perfect Nash equilibrium. Section 5 provides simple examples that illustrate the relationship between the number of stores at a shopping center, equilibrium prices, market sizes, and each store's profits. Section 6 concludes with possible extensions. The Appendix collects the proofs of propositions and lemmas.

2. Summary of the Literature

Since the past literature related to this paper is quite extensive,³ we concentrate on several of the most closely related papers. Although this paper is the first to discuss the trade-off between the two distinct market-size effects (due to taste uncertainty and lower price expectations) and the price-cutting effect in a spatial model, each of these effects has been discussed singly (or in a pair) elsewhere in the literature. Stahl [39] and Wolinsky [43] were the first to discuss the market-size effect due to taste uncertainty. They assume that each type of consumer has different preferences over commodities, but that they do not know which store sells their most preferred commodity. Consumers pick which shopping center to visit only by observing the number of stores at each shopping center. Both papers analyze clustering equilibria. These models lack price related effects (the price-cutting effect and the market-size effect due to lower price expectations), since they either assume that there is no price (Stahl [39]) or that each store charges the same price (Wolinsky [43]).⁴

Dudey [14] considers a (homogeneous commodity) Cournot oligopoly model with finite numbers of consumers and stores. Each consumer has the same demand curve, so two consumers choosing the same shopping center means that the demand curve doubles in scale. Consumers are uninformed

³See Fujita and Thisse [19] for a nice survey of the literature. There are interesting models that explain concentration of retail stores using quite different mechanisms. Rob [34] uses capacity constraint and demand uncertainty to explain concentration of restaurants. Caplin and Leahy [5] stresses the importance of information externality in explaining a rapid (re)vitalization of a specific part of a city.

⁴Although it is not explicitly analyzed in his paper, Wolinsky [43] points out that the price-cutting effect may exist and that it would intensify the concentration of stores (Wolinsky [43], p.277, footnote 3).

about prices, but they choose their preferred shopping center by inferring which shopping center has the lowest prices (the market-size effect due to lower price expectations). Thus, if a store chooses to locate alone, then the store loses all its customers, since transportation costs to any store are zero by assumption.⁵ As a result, all stores concentrate at one location (for the case of the standard linear demand function). Dudey’s model has both the price-cutting effect and the market-size effect due to lower price expectations, but it lacks the market-size effect due to taste uncertainty since he assumes homogeneous commodities.

The model closest to ours can be found in Fischer and Harrington [18], although their main interest is different from the other papers as well as ours. They are interested in interindustry variation in the concentration of stores. They assume that there are two abstract locations: a “cluster” and a “periphery.” Consumers can visit one or both of the two locations. If a consumer visits the cluster, then she can get information about all stores located there at a fixed cost. If she visits the periphery, she can search stores there at the same marginal cost per store. There is no explicit geography in their model, and there is only one possible cluster (shopping center) in the model. In this model, Fischer and Harrington [18] establishes nonemptiness of free entry equilibrium.⁶ Using numerical examples, they illustrate that greater store concentration is associated with industry characterized by greater product variety in equilibrium.⁷

Although Gehrig’s [20] primary interest is tax (transaction fee) competition among marketplaces

⁵In his original paper (Dudey [13]), the model contained transportation costs, but the results are essentially the same as the no-transportation-cost case (Dudey [14], p 1095).

⁶It turns out that we can relate our model to theirs closely by introducing outside opportunities for consumers into our model. (Their periphery market can be interpreted as an internet market.) Such a model will be briefly discussed in Section 6. It is possible to generalize their equilibrium existence result to cover more general (non-uniform) willingness-to-pay distributions by utilizing our Propositions 1 and 2.

⁷In the model in Fischer and Harrington [18], we can actually find all three elements discussed in this paper (the two market-size effects and the price cutting effect). However, they are interested in comparative static exercises on concentration of stores at the cluster with respect to product heterogeneity, and they do not discuss the trade-off between these effects. There is also no explicit geography in their model.

such as financial markets in London and Frankfurt, he also employs a price competition model with consumer taste uncertainty in order to generate an agglomeration of transactions at marketplaces. Using a Hotelling-type spatial model, he demonstrates the existence of a free entry equilibrium for certain parameter values, in which (i) firms cluster at the two opposite ends of space with an equal number of firms and (ii) all firms charge the same price. Gehrig's multiple cluster equilibrium result is novel in the literature.⁸ Although we also show the existence of equilibrium with multiple shopping centers, we allow these shopping centers differ in the number of stores and prevailing market prices,⁹ and we establish three existence theorems without specifying explicit functional forms. Unlike other papers cited here, Gehrig [20] assumes that consumers know the prices of commodities prior to their search.¹⁰ Thus, the market-size effect due to lower price expectations is not present in his model.¹¹

The common feature that the above papers and ours share is the assumption regarding information available to consumers before they arrive at the stores: consumers have imperfect information regarding the types (and the prices) of commodities sold there, which is the very source of the concentration of stores in those models. Consumers thus need to engage in search activities to choose where to shop.¹² There exists a huge literature following Hotelling [22] on spatial and price

⁸Dudey [15] provides an example in which stores may locate separately, but with a very special population distribution.

⁹Even if population distribution is uniform, we can still have an equilibrium with asymmetric two shopping centers (see an example with $n = 14$ in section 5.2 below).

¹⁰For his motivation (financial markets), perfect information on prices is essential and is quite convincing. However, for competing retail stores, this assumption may not be appropriate.

¹¹Schulz and Stahl [37] also assume that consumers know prices prior to their search in a related model with consumer taste uncertainty (although their objective is very different from ours again). In the oligopolistic model in Schulz and Stahl [37], the market price increases with the number of stores (see Rosenthal [36]), but the market size eventually shrinks as the number of stores increases despite the market-size effect due to taste uncertainty. Using this oligopoly model, Schulz and Stahl [37] show that multiproduct monopoly can be better than free entry oligopoly equilibrium.

¹²For search theory, see Stigler [40], Kohn and Shavell [25], Stuart [41], and Wolinsky [44]. The original idea without search behavior can be found in Eaton and Lipsey [16].

competition with perfect information. However, in those models, there may not be an equilibrium in pure strategies, or even if it exists, there is no clustering equilibrium in most cases (see d’Aspremont et al. [9]); stores tend to choose different locations.¹³ A notable exception is de Palma et al. [10]: By employing a discrete choice model (see Anderson, de Palma, and Thisse [1]), they introduce heterogeneous-taste consumers into the Hotelling model. De Palma et al. show that there is a clustering equilibrium at the center even under perfect information, if consumers’ tastes are sufficiently dispersed.¹⁴

In summary, this paper makes three new contributions to the literature. First, this is the first paper that discuss the trade-off between the two distinct market-size effects and the price-cutting effect in a spatial model. Second, our model allows an equilibrium with multiple shopping centers that differ in the number of stores and prevailing market prices. Third, our paper provides three existence theorems that do not rely on functional forms of the model.

3. The Model

Let $N = \{1, 2, \dots, n\}$ be the set of stores. We assume a continuum of consumers distributed over the (two-dimensional) plane $Z = \mathfrak{R}^2$ according to a density function $g : Z \rightarrow \mathfrak{R}_+$.¹⁵ Each store can produce one type of indivisible good at the same marginal cost. We normalize the marginal cost to zero without loss of generality. Different stores sell different types of goods. Each consumer buys at

¹³See d’Aspremont, et. al. [9], Economides [17] and Kats [24] for the existence of such equilibria. Anderson, de Palma, and Thisse [1] has a complete literature survey.

¹⁴Note that the mechanism of generating the concentration of stores in the de Palma et al. model is very different from the one described by the market-size effect due to taste uncertainty. In de Palma et al. [10], even though consumers’ tastes are differentiated, each consumer knows each store’s commodity type and price before her search. As a result, there is no benefit for stores to be close to other stores. The reason that a concentration of stores at the center of the landscape occurs is that the central location is so attractive for stores that they want to stay there as long as competition does not bring down prices too much. Price competition force is weakened by sufficiently dispersed consumers’ tastes.

¹⁵We use a two-dimensional spatial model in order to avoid discontinuities of profit functions due to linear transportation costs (see also footnotes 17 and 19).

most one unit of at most one indivisible commodity. Each consumer has identical ex ante preferences over the types of indivisible goods, but ex post willingnesses-to-pay can vary. Her willingness-to-pay for the indivisible commodity sold by each store $i \in N$ (type i commodity) is assumed to be a random variable \mathbf{v}_i with probability density $f(v)$ on the interval $[0, 1]$, and $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ are identically and independently distributed (i.i.d.).¹⁶ A vector (v_1, v_2, \dots, v_n) describes the consumer's ex post willingness-to-pay for each commodity. We assume for simplicity that each consumer chooses at most one location to shop. Although this assumption is not the most realistic, it simplifies our analysis dramatically. A consumer needs to pay the commuting cost, which is the Euclidean distance from her location to the chosen shopping location.¹⁷ Although the consumer has no information on commodities (her willingnesses-to-pay and the prices) ex ante, once she arrives at a shopping center she obtains this information on all the commodities sold there without any additional search cost. Thus, if a consumer at location $a \in Z$ goes to location $d \in Z$ and buys a type i commodity with price p_i , then her payoff is $v_i - p_i - \|d - a\|$.

Each store can choose its location from a *finite* set of locations (shopping centers) D embedded in the same location space for consumers ($D \subset Z$).¹⁸ This assumption can be justified by an assumption of local zoning policies. Stores' location choice is described by a vector $s = (s_1, s_2, \dots, s_n) \in D^n$. Let $N_d(s)$ be the set of stores that choose location d under s ; $N_d(s) \equiv \{i \in N : s_i = d\}$. Consumers can observe stores' locations s before they make their decisions about where to shop. The game goes as follows:¹⁹

¹⁶We assume that the law of large numbers applies to the case of a continuum of i.i.d. random variables (see Judd [23]).

¹⁷This assumption is for simplicity. Multiplying a transportation cost coefficient $t > 0$ does not change anything. Even assuming any strictly increasing transportation cost function does not alter the results in Section 4.

¹⁸We assume finiteness of D for simplicity. As we discussed in Section 1, each store's profit function is not quasi-concave in its location even if D is convex.

¹⁹The order of Stages II and III is not essential in the following analysis under our assumptions on consumers' available information before search. Here, we adopt this order for the sake of simpler explanations.

Stage I (store's location choice decision): Each store $i \in N$ chooses its location from the set D simultaneously.

Stage II (Consumers' shopping location decision): Knowing s , each consumer chooses a shopping center from $D^* = D \cup \{\emptyset\}$ (she goes to a shopping center in D , or decides not to shop (chooses \emptyset)). In making her decision, she infers the prices of commodities at $d \in D$ and calculates the expected utility from shopping at $d \in D$. Denoting the expected price of the type i commodity by p_i^e , the expected payoff of a consumer at $a \in Z$ who shops at d with $N_d(s) \neq \emptyset$ can be written as:

$$EU_a(d, s) = E \left(\max_{i \in N_d(s)} (\mathbf{v}_i - p_i^e) \right) - \|d - a\|.$$

If she shops at d with $N_d(s) = \emptyset$, then her expected payoff is $EU_a(d, s) = -\|d - a\|$, since she finds no store there. Of course, nobody chooses this option since it is dominated by the option of not visiting any shopping center ($\emptyset \in D^*$). A consumer chooses the location $d^*(a, s) \in D^*$ that yields the highest expected payoff:

$$(i) \quad d^*(a, s) = \arg \max_{d \in D} EU_a(d, s),^{20}$$

if $\arg \max_{d \in D} EU_a(d, s) \geq 0$, and

$$(ii) \quad d^*(a, s) = \emptyset,$$

if $\arg \max_{d \in D} EU_a(d, s) < 0$ (she does not visit any shopping center).

²⁰We do not need to specify the tie-breaking rule. It is not important since the measure of consumers who are indifferent between two options is zero under a two-dimensional spatial structure. (If the spatial structure is one dimensional then the d'Aspremont et al. problem occurs, if the transportation cost function is linear. See d'Aspremont et al. [9].)

We assume that each consumer perfectly foresees the prices when she makes the decision: $p_i^e = p_i$ (subgame perfection). In Stage II, the population of consumers who shop at $d \in D$ is determined by $G(d, s) \equiv \int_{\{a' \in Z: d^*(a', s) = d\}} g(a) da$.

Stage III (Price competition among stores at the same location): Stores in $N_d(s)$ simultaneously decide the prices of their commodities $(p_i)_{i \in N_d(s)}$, and then, each consumer decides whether to buy a commodity as well as which type of commodity to buy after the realization of her willingness-to-pay $(v_i)_{i \in N_d(s)}$.²¹ She buys the type i commodity only if $i \in \arg \max_{i \in N_d(s)} (v_i - p_i)$ and $v_i - p_i \geq 0$. Note that the commuting costs are *sunk* when consumers make their shopping decisions.

Our equilibrium notion utilized in this paper is a subgame perfect Nash equilibrium (SPNE).

In the next section, we analyze the equilibria of our game.

4. Equilibria of the Game

We proceed by backward induction.

4.1. Stage III

Consider a location d in D . Since consumers have already decided which locations to visit, the measure of consumers who visit d (the size of market at d) is already determined, and the size of the market at d is $G(d, s)$. Since firms are symmetric, we focus our attention on symmetric

²¹We can divide Stage III into three. In substage (i), stores in each d decide their prices. In substage (ii), Nature plays and consumers' willingnesses-to-pay realize, and in substage (iii), consumers make their purchase decisions. Since we assume that the law of large numbers applies to our situation, stores face no uncertainty when they decide their prices.

Nash equilibria in the price competition (Bertrand) game. *Note that the Nash equilibrium price at location d is affected by the number of stores at d , $N_d(s)$, but not by the market size of d , $G(d, s)$.*

This is because we assume that commodities are produced at constant (zero) marginal costs. This assumption greatly simplifies our analysis. To guarantee the existence and the uniqueness of Nash equilibria with a desirable property, we impose the following two conditions on the probability density:

Assumption 1. *The probability density f is log concave.*

This condition is used to guarantee the existence of an equilibrium by utilizing the Prékopa Theorem (see Bagnoli and Bergstrom [3], Caplin and Nalebuff [6], Dierker [12], and Anderson, de Palma, and Thisse [1]). It is satisfied by a variety of commonly used probabilistic distributions.

Assumption 2. *The probability density f is continuously differentiable, and satisfies $f(v) > 0$ and $-2 \leq \frac{vf'(v)}{f(v)} \leq 1$ for every $v \in [0, 1]$.*

This condition is more restrictive but is the simplest sufficient condition that guarantees the uniqueness of symmetric equilibrium (the lower bound) and an inverse relationship between the number of stores and the equilibrium prices (the upper bound). These require that $f(v)$ does not increase or decrease too fast. Of course, the uniform distribution satisfies Assumptions 1 and 2.

Assumptions 1 and 2 will be maintained throughout the paper. The main result of this subsection is the following:²²

²²For the existence of a symmetric price equilibrium in stage III, we need only that v_i are identically but interdependently distributed according to a symmetric log concave density (see Theorem A.1 in the appendix). Propositions 3, 4, and 5 (below) can be proved under the same assumption as well. However, in order to show the uniqueness of the symmetric price equilibrium and the inverse relationship between the number of stores and the equilibrium prices, we need independence of \mathbf{v}_i s and Assumption 2.

Proposition 1. *Let $k > 0$ be the number of stores at location d . There is a unique symmetric price equilibrium at which all stores charge p_k^* . Moreover, p_k^* is decreasing in k ($p_k^* > p_{k+1}^*$).*

This proposition guarantees uniqueness of the symmetric price equilibrium in the third stage of the game, and shows the monotonicity of p_k^* in k (the price-cutting effect). The reason price competition does not lead stores to a zero price equilibrium can be seen in a two-firm case. See Figure 1. Even if store j is charging a very low price, store i still can make profits by charging a higher price due to consumers' taste uncertainty. Store i can get customers in the shaded area of Figure 1 ($v_i - p_i \geq v_j - p_j$ and $v_i \geq p_i$). This is why zero price equilibrium does not occur.²³ In the following, we select this unique symmetric price equilibrium as the outcome of each stage III subgame.²⁴

4.2. Stage II

In this subsection, we analyze the consumer's decision of where to shop (stage II), which determines the market size of each location $G(d, s)$ for any $d \in D$. First, we find which consumers can expect a *positive* payoff from visiting location d , and later we consider consumers' choice between two locations $d, d' \in D$. The expected utility of a consumer who commutes to d where there are k stores is:

$$EU_a(d, s) = k \int_{p_k^*}^1 (v - p_k^*) f(v) F(v)^{k-1} dv - \|d - a\|.$$

²³De Palma et. al. [10] and Bester [4] discuss the relationship between dispersed tastes and positive markups in different models.

²⁴The proposition does not say that there is no asymmetric price equilibrium. If $k = 2$ and f is uniform, then we can easily show that the game is a supermodular game (see Milgrom and Roberts [29], Vives [42], and Milgrom and Shannon [30], and the symmetric price equilibrium is the unique equilibrium. However, in general, we cannot make such a statement. For example, with a general density function f , we can easily construct a game that violates supermodularity even if $k = 2$.

Let $\mu(k, p) \equiv k \int_p^1 (v - p) f(v) F(v)^{k-1} dv$ be the expected utility from shopping at a center with k stores and prevailing price p (no transportation costs). Let $r_k^* \equiv \mu(k, p_k^*)$. It is apparent that $EU_a(d, s) = \mu(k, p_k^*) - \|d - a\| \geq 0$ holds if and only if $\|d - a\| \leq r_k^*$. Thus, r_k^* denotes the radius of area within which consumers can get *positive* expected payoffs by shopping at d when there are k stores at d (the potential market size). The significance of r_k^* is that consumers in the circle with its center at d and with radius r_k^* would visit the shopping center d unless there is another shopping center d' that gives higher payoffs. Thus, if all possible locations for stores are far apart from each other, the population in the circle determines the size of the market. We are interested in how r_k^* changes as k increases. To see this, we need to see how r_k^* changes as k increases. We can conveniently decompose the total change of r_k^* into two effects: the market-size effect due to taste uncertainty and the one due to lower price expectations:

$$\begin{aligned} r_{k+1}^* - r_k^* &= \mu(k+1, p_{k+1}^*) - \mu(k, p_k^*) \\ &= [\mu(k+1, p_k^*) - \mu(k, p_k^*)] + [\mu(k+1, p_{k+1}^*) - \mu(k+1, p_k^*)]. \end{aligned}$$

The contents of the first bracket in the above equation denote a **market-size effect due to taste uncertainty**, which is purely based on the consumers' benefits from having a greater variety of commodities. This effect is positive since increasing the number of options for a fixed price raises the expected utility. This effect is also described by Stahl [39] and Wolinsky [43]. On the other hand, the contents of the second bracket is a **market-size effect due to lower price expectations**, which is described by Dudey [14]. This effect is also positive, since we know $p_k^* > p_{k+1}^*$ from Proposition 1. Consumers are attracted by the lower equilibrium prices at shopping centers with higher concentration of stores. Thus, the (total) market size effect (the *LHS*) contains these two

effects discussed separately in the previous literature. Obviously, the (total) market size effect is positive as well, and we have the following proposition.

Proposition 2. *The radius of potential market r_k^* is increasing in the number of stores k ($r_k^* < r_{k+1}^*$ for $k > 0$).*

Potential customers of shopping center d are in the area of a circle centered at d with radius $r_{n_d}^*$, where $n_d = |N_d(s)|$. For each $d \in D$, we can draw circles. If $a \in Z$ does not belong to any circle, consumers at a do not visit any shopping center. If a belongs to only one circle, consumers there definitely visit that shopping center. If a belongs to multiple circles, then among the circles to which a belongs consumers visit the shopping center d that maximizes

$$EU_a(d, s) = \mu(n_d, p_{n_d}^*) - \|d - a\| = r_{n_d}^* - \|d - a\|.$$

Thus, we can describe the consumer's choice by

$$d^*(a, s) \in \arg \max_{d \in D^*} r_{n_d}^* - \|d - a\|,$$

if $r_{n_{d^*}}^* \geq \|d^*(a, s) - a\|$.

4.3. Stage I

We denote the market area covered by stores at location $d \in D$ by $A(d, s) = \{a \in Z : d = d^*(a, s)\}$. We also let π_k^* be the profit of a store when it shares the same location with other $k - 1$ stores (at the location there are k stores) and the market size is one (profit per unit demand).

There certainly exists a mixed-strategy equilibrium in this game (in the first stage mixed strategies

are used). However, in general, it is difficult to show the nonemptiness of equilibrium in pure strategies. Nonetheless, we can show the nonemptiness of equilibrium in pure strategies in the following three special cases: a non-overlapped market case (Proposition 3), a two-shopping-center case (Proposition 4), and the case where potential shopping centers are in a small area (Proposition 5). Note that we make no assumption on the distribution of consumers $g(a)$.

First, we state Proposition 3. To guarantee non-overlapped market structure, we define the minimum distance between any pair of shopping centers: $\bar{r}_n \equiv \max_{k \in \{1, \dots, n\}} (r_k^* + r_{n-k}^*)$ (see Figure 3).²⁵

Proposition 3. *Suppose that D is a finite set that satisfies $\|d - d'\| \geq \bar{r}_n$ for any $d, d' \in D$ ($d \neq d'$). Then, there is an SPNE in pure strategies.*

[Insert Figure 3 here]

The proof is based on the potential function approach developed by Rosenthal [35].²⁶ Unfortunately, this method applies only in the case of non-overlapped market (no interaction between firms at d and d' ($d \neq d'$)). The following proposition offers an alternative assumption, which gives us an existence theorem:

Proposition 4. *If $\#D = 2$ (i.e., $D = \{d, d'\}$), then there is an SPNE in pure strategies.*

²⁵ Although \bar{r}_n is not a primitive, it can be uniquely calculated by utilizing unique p_k^* (and unique r_k^*). We can replace \bar{r}_n by 2 (a condition on primitives). It is because the expected payoff is bounded above by 1, and the radius of a shopping center is bounded above by 1.

²⁶ See also Monderer and Shapley [31] and Konishi, Le Breton, and Weber [26]. For an application in the field of industrial organization, see Slade [38]. Note that this proof could be extended to more general cases such as a multiple branch case by applying Rosenthal's [35] technique.

The proof of this proposition is based on d'Aspremont et al. [8]. The following proposition says that there is a clustering equilibrium if the area of D is small enough.

Proposition 5. *Suppose that there is $d \in D$ such that $\|d - d'\| \leq r_{n-1}^* - r_1^*$ for any $d' \in D$. Then, there is an SPNE with a cluster of stores at d ($N_d(s) = N$).*

[Insert Figures 4a and 4b here]

Proposition 5 applies if the area is very congested. Although such clustering of stores forces them to cut their prices and earn smaller profits, they cannot move to a nearby shopping center unilaterally. As is seen in Figure 4a, if n stores concentrate at shopping center d , then the market size is r_n^* . If a store tries to move to location d' , then the market size of d shrinks to r_{n-1}^* . However, the deviator's potential market (a circle centered at d') is still completely covered by the market d . No consumer would visit a deviated store, since the cluster of stores gives consumers higher expected utilities despite location differences (Figure 4b). This proposition simply says that if the populated area is small enough then there will be at least one clustering equilibrium. If there are multiple potential shopping centers that satisfy the stated condition, then there are at least as many clustering equilibria as the number of those potential shopping centers.²⁷ Other than these clustering equilibria, there can be equilibria with multiple shopping centers as well.²⁸

²⁷Since we assume a general population density over the plane, one of the shopping centers could be better than others in the sense that the shopping center can cover a more populated area. This is simply a coordination problem.

²⁸This tendency is observed in our numerical examples in Subsection 5.2.

5. Examples: Double Uniform Distribution Assumption

In this section, we provide explicit calculations for a class of examples by assuming that $g(a) = 1$ for any $a \in Z$: i.e., consumers are geographically uniformly distributed. The probability density function f is also specified as a uniform distribution over the interval $[0, 1]$. We consider two special cases in which the SPNE is guaranteed to be nonempty: The first is the case with non-overlapped markets (Proposition 3), and the other is the case with only two potential shopping centers (Proposition 4).

5.1. Non-Overlapped Markets

In this subsection, we assume that markets of different shopping centers $d, d' \in D$ do not overlap. This condition is satisfied, for instance, if $\|d - d'\| \geq 2$ for any $d, d' \in D$. Given the uniform distribution f , we can solve for p_k^* numerically (Lemma 1 below), which in turn determines r_k^* (Proposition 2). Equilibrium prices in the third stage are calculated as follows:²⁹

Lemma 1. *Under the hypotheses of this section, the equilibrium price p_k^* , the market size r_k^* , and each store's profit $\Pi_k^* = \left(\int_{a' \in \{a \in A: \|d-a\| \leq r_k^*\}} 1 da' \right) \times \pi_k^*$ for each $k = 1, 2, \dots$ satisfy: (i) p_k^* is a solution of $-(p_k^*)^k - kp_k^* + 1 = 0$, (ii) $r_k^* = \frac{k}{(k+1)} (1 - p_k^* - (p_k^*)^2)$, and (iii) $\Pi_k^* = \pi \times (r_k^*)^2 (p_k^*)^2$. Moreover, the probability of no purchase (a consumer does not find a buyable commodity) is $\text{prob}(\text{No}) = (p_k^*)^k$.*

The following table describes the equilibrium prices, the radius of the market, the equilibrium profits, and the probability of not finding a buyable commodity for each k :

[Insert Table 1 here]

²⁹The π in (iii) of Lemma 1 is the circumference ratio, $\pi = 3.1415\dots$ (not profit per demand).

This table shows the trade-off between the price-cutting effect and market-size effect, and describes how the concentration of stores affects the stores' profits. We also attach a consumer's probability of not finding a buyable commodity ("-" in Table 1 denotes negligibly small numbers). Uniformity of g is needed to calculate Π_k^* 's. As we know from Propositions 1 and 2, price and market size move in different directions, but the market size expands sharply when k is small. As a result, the equilibrium profit goes up very quickly for small k and attains maximum at $k = 3$. After that the profit starts to decline slowly. Until $k = 13$, the equilibrium profit is still greater than in the monopoly case. This shows that the concentration effect is strong since stores make location decisions noncooperatively. The probability that a consumer cannot find a buyable commodity decreases very quickly with k . This suggests that the market-size effect is quite significant relative to the price-cutting effect for small number of ks .

In the following, we characterize the equilibrium store distribution structure in our special case in order to draw out the implications for the concentration of stores. Since markets are not overlapped and consumers are uniformly distributed, each store's profit is determined solely by the number of competitors at the same shopping center. Thus, without loss of generality, a store distribution structure can be essentially described by a list of integers $\{n_1, n_2, \dots, n_\ell\}$, where (i) $n_j > 0$, (ii) $\sum_{j=1}^{\ell} n_j = n$, and (iii) ℓ is a positive integer. Actually, it is easy to characterize every Nash equilibrium in this particular case.

Proposition 6. *Under the hypotheses of this section, a store distribution structure $\{n_1, n_2, \dots, n_\ell\}$ is an equilibrium store distribution structure if and only if (i) $\min\{n, 3\} \leq n_j \leq 13$ for any $j = 1, \dots, \ell$, and (ii) $|n_j - n_k| \leq 1$ for any $j, k = 1, 2, \dots, \ell$.*

The first condition says that every shopping center has the profit-maximizing number of stores ($k = 3$) or more if n is not less than three. The second condition asserts that the equilibrium number of stores can differ at most by one among nonempty shopping centers (so at most two sizes can be observed).³⁰ Thus, in this particular case, we can claim that shopping centers will be more or less homogeneous and the number of stores at each shopping center will be no less than three (if $n \geq 3$). For example, if $n = 13$, there could be a few equilibria. One is a grand coalition $\{13\}$; others are $\{6, 7\}$, $\{4, 4, 5\}$ and $\{3, 3, 3, 4\}$. Obviously, the average profit is highest under $\{3, 3, 3, 4\}$ and lowest under $\{13\}$. These equilibria are therefore Pareto ranked from the stores' perspectives. On the other hand, if $n = 14$, the grand coalition vanishes, and there are only three equilibria $\{7, 7\}$, $\{4, 5, 5\}$ and $\{3, 3, 4, 4\}$. The reason $\{14\}$ is not an equilibrium is that every store has an incentive to move out of the grand coalition. Then, can $\{1, 13\}$ be an equilibrium? The answer is no. Since $k = 2$ is much more profitable than $k = 13$, a store will move from the bigger shopping center to a smaller one.

5.2. Two Potential Shopping Centers

In this subsection, we assume that there are only two potential shopping centers d and d' but that there could be market overlap between them. When market areas overlap, a store's move from one shopping center to the other can have a large impact on stores' profits by altering market areas of the shopping centers, and hence equilibrium allocation can be influenced as well. Let δ be the distance between d and d' . We first set $n = 10$, and find equilibria for various δ s. If $\delta \leq \bar{r}_{10} = 2r_5^* = 1.2668$, then markets may overlap with each other. Thus, we find equilibria in the

³⁰Obviously, this result depends crucially on the assumption that population density is the same everywhere. Otherwise, there can be equilibria that have multiple shopping centers with widely varying numbers of stores.

cases of $\delta = 0.2, 0.4, \dots, 1.2$. We find only two types of equilibria: a symmetric equilibrium (each shopping center has the same number of stores) or a clustering equilibrium (one of the shopping centers has all the stores). There is no asymmetric non-clustering equilibrium in this example. The numbers in the column describe each store's equilibrium profit times 100.

[Insert Table 2-1 here]

The bottom row corresponds to the non-overlapped market case. We can make a few observations about the results in Table 2-1: First, each store's profit is higher in a symmetric equilibrium (if one exists). This is not surprising given the profit levels of the non-overlapped market case. Second, there is always a clustering equilibrium. This is because the number of stores is small. From Table 1, we know that even if there is no overlap, a ten-store cluster gives each store a higher profit than standing alone (for any $n \leq 13$, $\Pi_n^* > \Pi_1^*$ holds). Thus, there is no incentive for stores to deviate from the cluster unilaterally (with overlap, a deviator may get even less than Π_1^*). Third, there is no symmetric equilibrium for $\delta = 0.2$ (see Figure 5).

[Insert Figure 5 here]

Under symmetric allocation, five stores each locate at shopping centers d and d' , and consumers in the shaded area visit d' . If a store moves from d to d' , then the new allocation becomes six stores at d' and four stores at d . As is seen in Figure 5, the market area for d' with six stores is much larger than the one for d' with five stores. Despite the price-cutting effect, stores have an incentive to move to a symmetric allocation to break the tie. Thus, there is no symmetric equilibrium in

this case. This observation implies that if the two shopping centers are very close and the number of stores is small, then incentive for stores to concentrate is stronger due to competition between shopping centers.

In Table 2-2, we assume that $n = 20$. We focus on the case where $\delta \leq \bar{r}_{20} = 2r_{10}^* = 1.61818$. Again we find only symmetric and clustering equilibria in this example.

[Insert Table 2-2 here]

In this case, the number of stores is relatively large in the sense that clustering is less attractive for each store than standing alone, and there is no clustering equilibrium in the non-overlapped market case. There are two main differences from Table 2-1. First, there is a symmetric equilibrium in the case of $\delta = 0.2$. This is because a shopping mall with 11 stores cannot sufficiently dominate the other one with 9 stores, and a store's profit does not increase by breaking a tie in a symmetric allocation. Thus, two ten-store shopping centers can coexist even when the shopping centers are very close to each other. Second, there is no clustering equilibrium for δ more than 0.8. This is because clustering is less attractive than standing alone. So, if the two shopping centers are far enough from each other, then a clustering allocation cannot be supported as an equilibrium.

We list two other cases for interested readers. First, there can be an asymmetric equilibrium in an intermediate case. When $n = 14$ and $\delta = 0.2$, there are three types of equilibria: a symmetric equilibrium $\{7, 7\}$ (profit: 2.0162), clustering equilibria $\{14, 0\}$ (profit: 1.1907), and asymmetric equilibria $\{5, 9\}$ (profit: 2.3186 for 5 stores, 2.0265 for 9 stores). Interestingly, the asymmetric equilibrium Pareto-dominates the symmetric equilibrium and the clustering equilibrium in this particular case. Second, the clustering equilibrium may survive even without the sufficient condition

in Proposition 5. By the argument in the case of $n = 10$, we know that a clustering equilibrium always exists when $n \leq 13$. However, if $n > 13$ and if the sufficient condition is violated, we do not know whether a clustering equilibrium exists. The following example says that it may. When $n = 14$, the sufficient condition is $\delta \leq r_{13}^* - r_1^* = 0.7266$ (Proposition 5). We find that even if $\delta = 0.8$, there is a clustering equilibrium in this case. This is because a deviating store from the cluster may not be able to attract many customers due to competition between the two shopping centers (see Figure 6).³¹

[Insert Figure 6 here]

6. Concluding Remarks

In this paper, we analyze the mechanism of geographical concentration of retail stores using a spatial model that incorporates taste uncertainty. The trade-off between the two market-size effects and the price-cutting effect is the focus of the paper. We establish existence of subgame perfect equilibria in pure strategies without imposing explicit functional forms, and show that multiple shopping centers can emerge in stores' location choice equilibrium through numerical examples.

Although we have assumed that consumers can visit at most one shopping center, it seems possible to obtain similar results even if we abandon this simplifying assumption. However, it turns out to be a difficult task to incorporate sequential search into a model with an explicit geographical structure when consumers are uninformed about prices and their willingnesses-to-pay.³² Consumers'

³¹This observation may not be robust for a larger number of stores, since the cluster would have a weaker competitive power against a deviating store. Actually, if $n = 20$, the sufficient condition in Proposition 5 is $\delta \leq r_{19}^* - r_1^* = 0.772368$. However, we found that for $\delta = 0.772369$, there is already no clustering equilibrium. It is probably because a store's deviation would pay even if the resulting market area for the store after the deviation is very small (the profit level at the cluster is too low).

³²For consumers' sequential search, see Diamond [11], Wolinsky [44], Fischer and Harrington [18] and Anderson and Renault [2]. These models do not have an explicit geography in their models.

optimal stopping rules in sequential search depend on their residential locations, and a store's price decision at a shopping center affects stores at other shopping centers as well as at the same shopping center. Konishi and Yu [27] consider a two-stage game of a one-dimensional spatial model with two stores and consumers who search sequentially at stores. Interestingly, due to consumers' sequential search, the equilibrium price in each subgame specified by the distance between the two stores is not a monotonic function with respect to the distance even in a very simple model. They show that stores collocate in symmetric perfect Bayesian equilibrium. This result suggests that retail stores may have incentive to concentrate geographically even if sequential search is allowed.³³

We can apply our model to interesting real-world problems. We can predict how Internet shopping opportunities for consumers can affect the geographical concentration of stores. We can include Internet shopping as an outside option into our model: instead of visiting a shopping mall, a consumer may search commodities sequentially within the Internet market by purchasing and returning them with a positive transaction cost for each purchase and return (such as a waiting cost and/or a mailing cost). We can show that the concentration of retail stores would be increased as transaction costs of Internet shopping decrease.³⁴ We can also explain through our model why owners of shopping malls usually restrict the number of stores in the same category.³⁵ Assuming that mall owners can effectively exploit their tenants' profits, it is in the owners' interests that their tenants earn more. Note that a restriction on the number of stores effectively increase the profits of stores, and mall owners can thus increase their rents.

³³If there are more than two stores, or if geographical space is two-dimensional, then stores' concentration incentives would be stronger.

³⁴It is because the market size and the equilibrium price at a shopping center with a fixed number of retail stores also decrease. As a result, stores have more incentive to concentrate at the same shopping center to provide greater variety of commodities to attract consumers who could have chosen to shop on the Internet.

³⁵See Henderson [21] for the land-developers' entrepreneurship in the context of local public goods economy.

Appendix

Proof of Proposition 1

First, we prove the existence of a symmetric price equilibrium price p_k^* for each $k = 1, 2, \dots$. We show that no store has an incentive to deviate from p_k^* given that every other store chooses p_k^* . Since we can prove the existence of an equilibrium without assuming independence of distribution, we define the joint probability density on the space $B^k = [0, 1]^k$ ($k = 1, \dots, n$). Let $h^k : B^k \rightarrow \mathfrak{R}_+$ be probability density. We say $h^k(v^k)$ is *symmetric* iff $h^k(v^k) = h^k(\tilde{v}^k)$ for any $v^k \in B^k$ and any permutation \tilde{v}^k of v^k . The second condition represents the symmetry of probability density. Let $\pi_i^k(p_i, p)$ be such that the profit that store i makes from market size 1 (Thus the profit of store i at shopping center d is $G(d, s) \times \pi_i^k(p_i, p)$) when other firms at d choose p and store i chooses p_i , given that there are k stores at d . Given $p_i \in [0, 1]$ and $p \in [0, 1]$, the set of consumers who buy commodity i is described by $B_i^k(p_i, p) \equiv \{v^k \in B^k : v_i \geq p_i \text{ and } v_i - p_i \geq v_j - p \text{ for any } j \neq i\}$. Thus, $R_i^k(p_i, p) = p_i \times \int_{B_i^k(p_i, p)} h^k(v^k) dv^k$. We prove the existence of a symmetric equilibrium by utilizing Prékopa's theorem:

The Prékopa Theorem (Prékopa [33]). *Let ψ be a probability density function on \mathfrak{R}^m with convex support C . Take any measurable sets A_0 and A_1 in \mathfrak{R}^m with $A_0 \cap C \neq \emptyset$ and $A_1 \cap C \neq \emptyset$. For any $0 \leq \lambda \leq 1$, define $A_\lambda = (1 - \lambda)A_0 + \lambda A_1$, the Minkowski average of the two sets.³⁶ If $\psi(\alpha)$ is log concave, then*

$$\log \int_{A_\lambda} \psi(\alpha) d\alpha \geq (1 - \lambda) \log \int_{A_0} \psi(\alpha) d\alpha + \lambda \log \int_{A_1} \psi(\alpha) d\alpha.$$

³⁶The Minkowski average A_λ is defined as all points of the form $x_\lambda = (1 - \lambda)x_0 + \lambda x_1$, with $x_0 \in A_0$, $x_1 \in A_1$, and $0 \leq \lambda \leq 1$.

We prove the following theorem by utilizing the Prékopa theorem.³⁷

Theorem A.1. *There is a symmetric price equilibrium in the third stage if $h^k(v^k)$ is log concave and has convex support on B^k and symmetric for any $k = 1, 2, \dots$.*

Proof. It is easy to see that $B_i^k((1-\lambda)p_i + \lambda p'_i, p) \supseteq (1-\lambda)B_i^k(p_i, p) + \lambda B_i^k(p'_i, p)$ for any $p, p_i, p'_i \in [0, b]$.³⁸ By using the Prékopa Theorem, we obtain:

$$\begin{aligned} \log \int_{B_i^k((1-\lambda)p_i + \lambda p'_i, p)} h^k(v^k) dv &\geq \log \int_{(1-\lambda)B_i^k(p_i, p) + \lambda B_i^k(p'_i, p)} h^k(v^k) dv \\ &\geq (1-\lambda) \log \int_{B_i^k(p_i, p)} h^k(v^k) dv + \lambda \log \int_{B_i^k(p'_i, p)} h^k(v^k) dv. \end{aligned}$$

Thus, we conclude that $\int_{B_i^k(p_i, p)} f(v^k) dv^k$ is log concave in p_i . Since $\log \pi_i^k(p_i, p) = \log p_i + \log \int_{B_i^k(p_i, p)} h^k(v^k) dv$, $\pi_i^k(p_i, p)$ is log concave in p_i as well. This implies that $\pi_i^k(p_i, p)$ is quasi-concave in p_i . Since $h^k(v^k)$ is a density function, $\pi_i^k(p_i, p)$ is continuous. By the Maximum Theorem, the best response correspondence $\beta_i : [0, 1] \rightarrow [0, 1]$ is upper hemi-continuous. Quasi-concavity of π_i^k implies convex-valuedness of β_i . By the intermediate value theorem, there is a fixed point $p_k^* \in [0, b]$ such that $p_k^* \in \beta_i(p_k^*)$. ■

Now, we turn to the uniqueness and the inverse relationship between the number of stores and the equilibrium prices. From here on, we need statistical independence of $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ and Assumption 2. We derive explicit formulas for $\pi_i^k(p_i, p)$. Since there is a difference between raising

³⁷This result can be regarded as a special case of a result in Dierker [12], although we need to show the existence of *symmetric* equilibrium. Caplin and Nalebuff's [6] technique is also very closely related. The easiest reference for this result is Theorem 6.3 (and the following discussion on page 168) in Anderson, de Palma, and Thisse [1].

³⁸The first inclusion could be strict if $p_i < p < p'_i$.

the price more than others and lowering the price less than others, we analyze these two cases separately. Let $\pi_i^{k+}(p_i, p)$ be store i 's profit given that the market size is equal to 1 when $p_i \geq p$.

This can be written as follows:

$$\pi_i^{k+}(p_i, p) = p_i \int_{p_i}^1 f(v_i) F(v_i - (p_i - p))^{k-1} dv_i,$$

where $F(u) = \int_0^u f(v_i) dv_i$.

[Insert Figure 1 here]

Note that $F(v_i - (p_i - p))$ denotes the probability that a consumer prefers type i commodity over type $j (\neq i)$ commodity given that her realization of \mathbf{v}_i is v_i . Thus, $F(v_i - (p_i - p))^{k-1}$ denotes the probability that a consumer prefers type i commodity to any other type at location d given that her realization of \mathbf{v}_i is v_i . Next, we consider the other case. Let $\pi_i^{k-}(p_i, p)$ be store i 's profit given that the market size is equal to 1 when other firms at d choose p and store i chooses $p_i \leq p$. This can be written as follows:

$$\pi_i^{k-}(p_i, p) = p_i \int_{p_i}^{1-(p-p_i)} f(v_i) F(v_i - (p_i - p))^{k-1} dv_i + p_i \int_{1-(p-p_i)}^1 f(v_i) dv_i.$$

[Insert Figure 2 here]

The last term shows that if $v_i \in [1 - (p - p_i), 1]$, then with a probability of one consumers will buy type i goods ($\int_{1-(p-p_i)}^1 f(v_i) 1^{k-1} dv_i$). It is easy to see that $\pi_i^{k+}(p, p) = \pi_i^{k-}(p, p)$. Moreover, by

letting $\varepsilon = p_i - p$ go to zero we have the following:

$$\begin{aligned}
\varphi_k(p) &= \frac{\partial \pi_i^{k+}(p_i, p)}{\partial p_i} \Big|_{p_i=p} \\
&= \frac{\partial \pi_i^{k-}(p_i, p)}{\partial p_i} \Big|_{p_i=p} \\
&= \int_p^1 f(v)F(v)^{k-1} dv - pf(p)F(p)^{k-1} - (k-1)p \int_p^1 f(v)^2 F(v)^{k-2} dv.
\end{aligned}$$

This is a continuous function in the interval $[0, 1]$. Note that every symmetric price equilibrium must satisfy the first-order condition $\varphi_k(p_k^*) = 0$. Thus, if $\varphi_k(p)$ is a monotonic function, we can conclude that there is at most one p_k^* with $\varphi_k(p_k^*) = 0$, which is the unique candidate for a symmetric equilibrium price. From Theorem A.1, there exists a symmetric equilibrium price, and p_k^* must be the unique symmetric equilibrium price. It is easy to see $\varphi_k(0) = \int_0^1 f(v)F(v)^{k-1} dv > 0$ and $\varphi_k(1) = -f(1) < 0$. Thus, we need only show that φ_k is non-increasing. Differentiating φ_k we obtain:

$$\begin{aligned}
\varphi_k'(p) &= -f(p)F(p)^{k-1} - f(p)F(p)^{k-1} - pf'(p)F(p)^{k-1} - (k-1)pf(p)^2F(p)^{k-2} \\
&\quad - (k-1) \int_p^1 f(v)^2 F(v)^{k-2} dv + (k-1)pf(p)^2F(p)^{k-2} \\
&= -2f(p)F(p)^{k-1} - pf'(p)F(p)^{k-1} - (k-1) \int_p^1 f(v)^2 F(v)^{k-2} dv \\
&= -f(p)F(p)^{k-1} \left(2 + \frac{pf'(p)}{f(p)} \right) - (k-1) \int_p^1 f(v)^2 F(v)^{k-2} dv.
\end{aligned}$$

Thus, $\varphi_k'(p) < 0$ follows for any $p \in [0, b)$, since $-2 \leq \frac{pf'(p)}{f(p)}$ is guaranteed by Assumption 2 ($\varphi_k'(p) = 0$ would not happen because the last term of $\varphi_k'(p)$ is negative for any $p \in [0, 1)$).

Therefore, there is unique symmetric equilibrium price $p_k^* \in (0, 1)$ that satisfies $\varphi_k(p_k^*) = 0$.

Finally, we show $p_k^* > p_{k+1}^*$ for each $k = 1, 2, \dots$. By integrating the last term of $\varphi_k(p)$ by parts,

we obtain:

$$\begin{aligned}
\varphi_k(p) &= \int_p^1 f(v)F(v)^{k-1}dv - pf(p)F(p)^{k-1} \\
&\quad - p \left\{ \left[f(v)F(v)^{k-1} \right]_p^1 - \int_p^1 f'(v)F(v)^{k-1}dv \right\} \\
&= \int_p^1 f(v)F(v)^{k-1}dv - pf(p)F(p)^{k-1} \\
&\quad - pf(1) + pf(p)F(p)^{k-1} + p \int_p^1 f'(v)F(v)^{k-1}dv \\
&= \int_p^1 f(v)F(v)^{k-1}dv + p \int_p^1 f'(v)F(v)^{k-1}dv - pf(1) \\
&= \int_p^1 f(v)F(v)^{k-1} \left(1 - \frac{pf'(v)}{f(v)} \right) dv - pf(1).
\end{aligned}$$

If we can show $\varphi_{k+1}(p_k^*) < 0$, then we can show $p_{k+1}^* < p_k^*$ since $\varphi_{k+1}(p)$ is non-increasing in p . However, since $\varphi_k(p_k^*) = 0$, it suffices to show $\varphi_{k+1}(p_k^*) < \varphi_k(p_k^*)$. Actually, we can show that $\varphi_{k+1}(p) < \varphi_k(p)$ for any $p \in [0, 1]$:

$$\varphi_k(p) - \varphi_{k+1}(p) = \int_p^1 f(v)F(v)^{k-1} (1 - F(v)) \left(1 - \frac{pf'(v)}{f(v)} \right) dv.$$

It is easy to see that $\varphi_k(p) - \varphi_{k+1}(p) \geq 0$ follows if $1 \geq \frac{pf'(v)}{f(v)}$ for any $v \geq p$. Assumption 2 supposes that $1 \geq \frac{vf'(v)}{f(v)}$ for any $v \in [0, 1]$. Thus, for any $v > p$, $1 \geq \frac{vf'(v)}{f(v)} > \frac{pf'(v)}{f(v)}$ must follow if $f'(v) > 0$. If $f'(v) \leq 0$, then $1 - \frac{pf'(v)}{f(v)} > 0$ anyway. Therefore, we conclude $p_k^* > p_{k+1}^*$. This completes the proof of Proposition 1. ■

Proof of Proposition 3

Since $\|d - d'\| \geq \bar{r}_n$ for any pair $d, d' \in D$ ($d \neq d'$), the assumption in the statement guarantees no overlap among market areas for any strategy configuration. This implies $A(d, s) = \{a \in A :$

$\|d - a\| \leq r_{n_d}^*$. Given that, it is easy to see that for each $d \in D$, for each k , we can determine the profits of each store at d : $\Pi_d(k) = \pi_k^* \int_{a \in A(d,s)} g(a) da$. Let $\Psi((n_d)_{d \in D}) = \sum_{d \in D} \sum_{k=1}^{n_d} \Pi_d(k)$. We claim that the strategy configuration that maximizes Ψ is an SPNE: $(n_d^*)_{d \in D} \in \arg \max_{(N_d)_{d \in D}} \Psi((n_d)_{d \in D})$. Suppose not. Then, there are $d', d'' \in D$ with $d' \neq d''$ and $\Pi_{d'}(n_{d'}) < \Pi_{d''}(n_{d''} + 1)$. Now, consider a strategy configuration $(n'_d)_{d \in D}$ that satisfies: (i) for any $d \neq d', d''$, $n'_d = n_d^*$, (ii) $n'_{d'} = n_{d'}^* - 1$, and (iii) $n'_{d''} = n_{d''}^* + 1$. Then,

$$\begin{aligned}
\Psi((n'_d)_{d \in D}) &= \sum_{d \in D} \sum_{k=1}^{n'_d} \Pi_d(k) \\
&= \sum_{d \in D} \sum_{k=1}^{n_d^*} \Pi_d(k) - \Pi_{d'}(n_{d'}^*) + \Pi_{d''}(n_{d''}^* + 1) \\
&= \Psi((n_d^*)_{d \in D}) - \Pi_{d'}(n_{d'}^*) + \Pi_{d''}(n_{d''}^* + 1) \\
&> \Psi((n_d^*)_{d \in D}).
\end{aligned}$$

This contradicts the definition of $(n_d^*)_{d \in D}$. Therefore, $(n_d^*)_{d \in D}$ is an SPNE store distribution. \blacksquare

Proof of Proposition 4

In this special case, the number of firms at d , say, $k = n_d$, describes the pattern of strategy configurations, and payoffs are determined solely by k . The payoffs of stores at d and d' are described by

$$\begin{aligned}
\bar{\Pi}_d(k) &= \pi_k^* F(d; (n_d, n_{d'})) = \pi_k^* \int_{a \in A(d; (n_d, n_{d'}))} g(a) da, \\
\bar{\Pi}_{d'}(k) &= \pi_{n-k}^* F(d'; (n_d, n_{d'})) = \pi_{n-k}^* \int_{a \in A(d'; (n_d, n_{d'}))} g(a) da,
\end{aligned}$$

where $A(d; (n_d, n_{d'})) = \{a' \in Z : \|d - a'\| \leq r_k^*, r_k^* - \|d - a'\| \geq r_{n-k}^* - \|d' - a'\|\}$, and $A(d'; (n_d, n_{d'})) = \{a' \in Z : \|d' - a'\| \leq r_{n-k}^*, r_{n-k}^* - \|d' - a'\| \geq r_k^* - \|d - a'\|\}$. Let us consider the case where $k = 0$

(all stores are at d' , and no store is at d). If $\bar{\Pi}_{d'}(0) \geq \bar{\Pi}_d(1)$, then $k = 0$ is an equilibrium (of the first stage of the game). Thus, we assume $\bar{\Pi}_{d'}(0) < \bar{\Pi}_d(1)$. Now, one store moves to d ($k = 1$). If $\bar{\Pi}_{d'}(1) \geq \bar{\Pi}_d(2)$, then it is an equilibrium since a store at d does not want to move back to d' by assumption. Thus, we assume $\bar{\Pi}_{d'}(1) < \bar{\Pi}_d(2)$. We move one additional store to d ($k = 2$). Again, if $\bar{\Pi}_{d'}(2) \geq \bar{\Pi}_d(3)$, then it is an equilibrium by the same reason, and we assume $\bar{\Pi}_{d'}(2) < \bar{\Pi}_d(3)$, and so on. If all of $k = 0, 1, 2, \dots, n-1$ violate $\bar{\Pi}_{d'}(k) \geq \bar{\Pi}_d(k+1)$, then we conclude $\bar{\Pi}_{d'}(n-1) \leq \bar{\Pi}_d(n)$. This implies that if all stores choose d , then no store wants to deviate from the allocation. Hence, there is an SPNE in this game. ■

Proof of Proposition 5

We need only show that if all n stores are at d , no store wants to move to any $d' \neq d$ unilaterally. Suppose that store i moves to d' alone. Then, store i 's potential market is a circle with its center at d' and radius r_1^* . However, the shopping center d 's potential market area is a circle with its center at d and radius r_{n-1}^* , and the latter circle contains the former circle. This implies that store i cannot attract any customer by this move, and so it has no incentive to leave the cluster. Thus, there is an SPNE with a cluster at d . ■

Proof of Lemma 1

We utilize the first-order condition for profit-maximization defined in the proof of Proposition

1. Given that $f(v) = 1$ for any $v \in [0, 1]$, we can write:

$$\begin{aligned}
\varphi_k(p) &= \int_p^1 v^{k-1} dv - pp^{k-1} - (k-1)p \int_p^1 v^{k-2} dv \\
&= \left[\frac{v^k}{k} \right]_p^1 - p^k - p \left[v^{k-1} \right]_p^1 \\
&= \frac{1}{k} \left(-p^k - kp + 1 \right).
\end{aligned}$$

Thus, p_k^* is implicitly defined by $\varphi_k(p_k^*) = 0$ for each k . We calculate $\mu(k, p_k^*)$ by utilizing this relationship.

$$\begin{aligned}
\mu(k, p_k^*) &= k \int_{p_k^*}^1 v^{k-1} (v - p_k^*) dv \\
&= k \left[\frac{v^{k+1}}{k+1} - \frac{p_k^* v^k}{k} \right]_{p_k^*}^1 \\
&= k \left[\frac{1}{k+1} - \frac{p_k^*}{k} - \frac{(p_k^*)^{k+1}}{k+1} + \frac{(p_k^*)^{k+1}}{k} \right] \\
&= \frac{k}{k+1} [1 - p_k^* - (p_k^*)^2].
\end{aligned}$$

In the last transformation, we used $\frac{1}{k} (-(p_k^*)^k - k p_k^* + 1) = 0$. Finally, we turn to the equilibrium profit Π_k^* . Note that $\Pi_k^* = \left(\int_{a' \in \{a \in A: \|d-a\| \leq r_k^*\}} 1 da' \right) \times R_k^*$. It is easy to see the value in the parenthesis is $\pi(r_k^*)^2$. Thus, what is left is to calculate π_k^* . By using the expression in the proof of Proposition 1, we obtain:

$$\pi_k^* = \pi_i^{k+}(p_k^*, p_k^*) = p_k^* \int_{p_k^*}^1 (v)^{k-1} dv = p_k^* \left(\frac{1 - (p_k^*)^k}{k} \right) = (p_k^*)^2.$$

In the last transformation, we used $\frac{1}{k} (-(p_k^*)^k - k p_k^* + 1) = 0$. This completes the proof. \blacksquare

Proof of Proposition 6

The proof is a variation of one in Conley and Konishi [7]. It is easy to see that a store distribution structure that satisfies (i) and (ii) is an equilibrium. Thus, we concentrate on the other direction. First, note that the relevant range of k is $1 \leq k \leq 13$, since if $k \geq 14$ a store would try to be independent given that Z is assumed to be large enough. Second, notice that Π_k^* is single-peaked

at $k = 3$ in the relevant range (actually, it is globally single-peaked). Third, Π_k^* increases in k very quickly until $k \leq 3$ and then decreases slowly. By using these observations, we will characterize the set of equilibria. Since it is trivial to see the statement is true for $n < 3$, we assume $n \geq 3$ in the following.

We claim that there are at most two sizes of shopping center that can be an equilibrium. Suppose that there are three sizes $k < k' < k''$ in equilibrium. If $k \geq 3$, then a store in a size k'' shopping center joins a size k shopping center. Thus, it cannot happen in the equilibrium. Thus, $k < 3$ holds. First, let us assume that $k = 2$. Then, $k'' \geq 4$ must hold. As a result, a store in a size k'' shopping center joins a size k shopping center. This implies $k = 1$. However, since a firm in a size k shopping center does not move to a size k' shopping center, we have $\Pi_1^* \geq \Pi_{k'+1}^* \geq \Pi_{k''}^*$. Since we know $\Pi_2^* > \Pi_1^*$, we conclude $\Pi_2^* > \Pi_{k''}^*$, and a store in a size k'' shopping center joins a size k shopping center. This is a contradiction. Hence, there are at most two sizes of shopping centers that can be in equilibrium.

Now, let these two sizes be k, k' with $k < k'$. We need only show (1) $k \geq 3$ and (2) $k' = k + 1$ to show (i) and (ii). For (1), let us assume $k < 3$. Since stores have no incentive to move, $\Pi_k^* \geq \Pi_{k'+1}^*$ and $\Pi_{k+1}^* \leq \Pi_{k'}^*$ hold. However, since $k < 3$, we have $\Pi_k^* < \Pi_{k+1}^*$. Thus, $\Pi_{k'+1}^* \leq \Pi_k^* < \Pi_{k+1}^* \leq \Pi_{k'}^*$ must hold. However, since profits increase very quickly until 3 and decrease slowly after that (see Table 1), we cannot find any $k < 3$ and $k' \geq 3$ that satisfies these inequalities. Hence, (1) is proved. It is easy to see that (2) holds given (1): Suppose that $k < k + 1 < k'$. By (1), we know $\Pi_k^* > \Pi_{k+1}^* > \Pi_{k'}^*$. Thus, a store in a size k' shopping center will move to a size k shopping center. ■

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k	p_k^*	r_k^*	$\Pi_k^* (\times 100)$	$prob(\text{No})$
1	.5	.125	1.2272	50%
2	.41421	.27615	4.1104	17.2%
3	.32219	.4305	6.0439	3.34%
4	.24904	.55115	5.9187	0.385%
5	.19994	.6334	5.0385	0.032%
6	.16666	.69048	4.1602	-
7	.14286	.73214	3.4368	-
8	.125	.76389	2.8644	-
9	.11111	.78889	2.4137	-
10	.1	.80909	2.0566	-
13	.076923	.85165	1.3483	-
14	.071429	.8619	1.1907	-

Table 1

δ	symmetric eq. profit	clustering eq. profit
0.2	None	2.0566
0.4	3.5149	2.0566
0.6	3.9795	2.0566
0.8	4.4007	2.0566
1.0	4.7556	2.0566
1.2	5.0021	2.0566
1.2668~	5.0385	2.0566

Table 2-1 ($n = 10$)

δ	symmetric eq. profit	clustering eq. profit
0.2	1.1897	0.6395
0.4	1.3486	0.6395
0.6	1.5024	0.6395
0.8	1.6481	None
1.0	1.7824	None
1.2	1.9008	None
1.4	1.9967	None
1.6	2.0551	None
1.61818~	2.0566	None

Table 2-2 ($n = 20$)

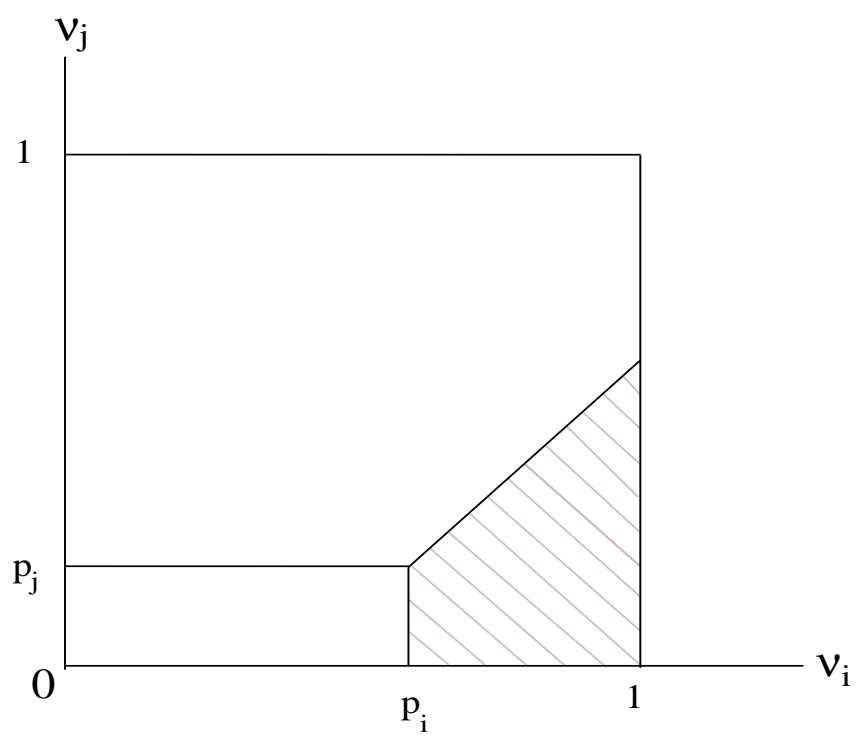


Figure 1
($p_i \geq p_j$)

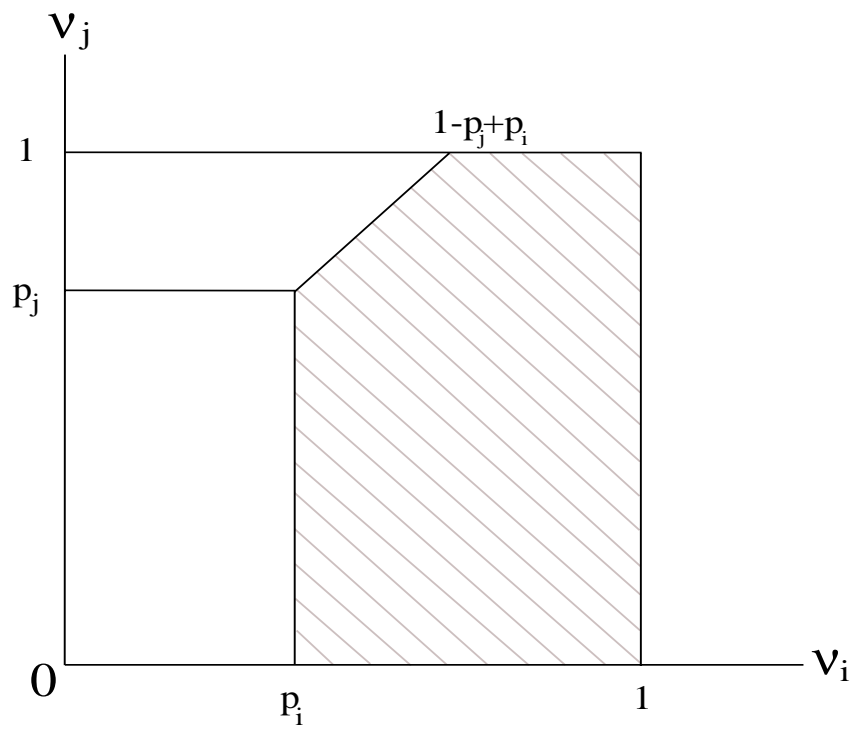


Figure 2
($p_j \geq p_i$)

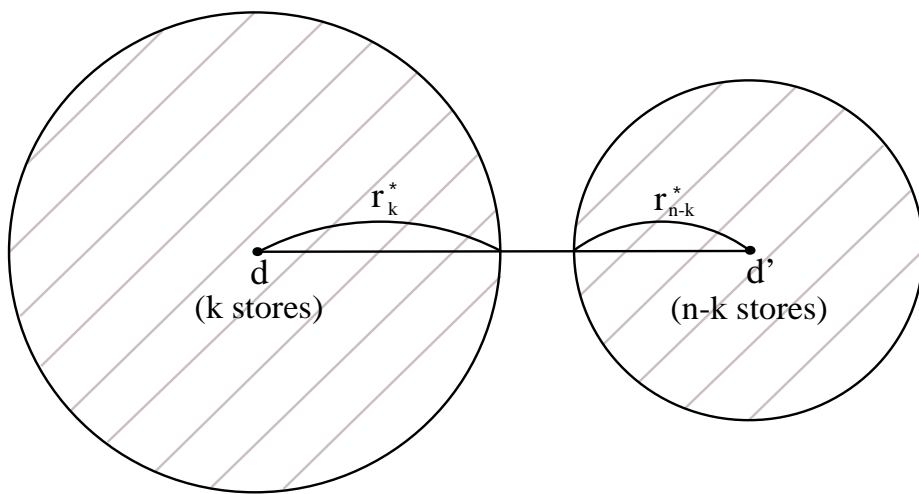


Figure 3
Non-overlapped markets
(Proposition 3)

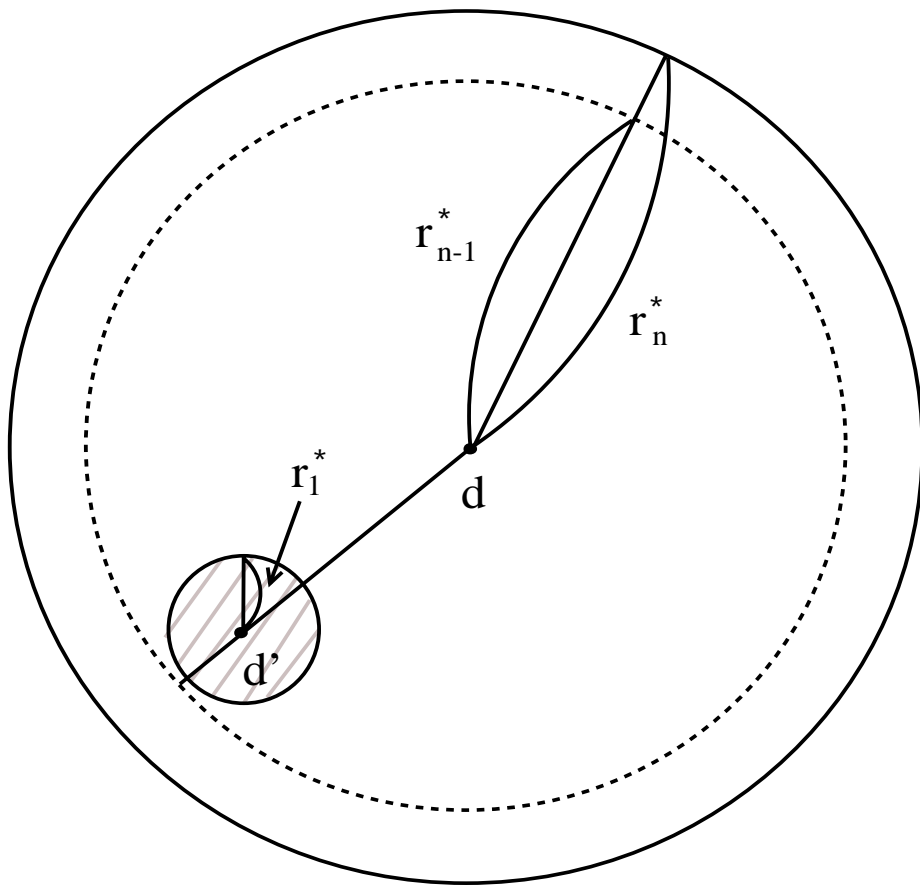


Figure 4a
Absorbed market
(Proposition 5)

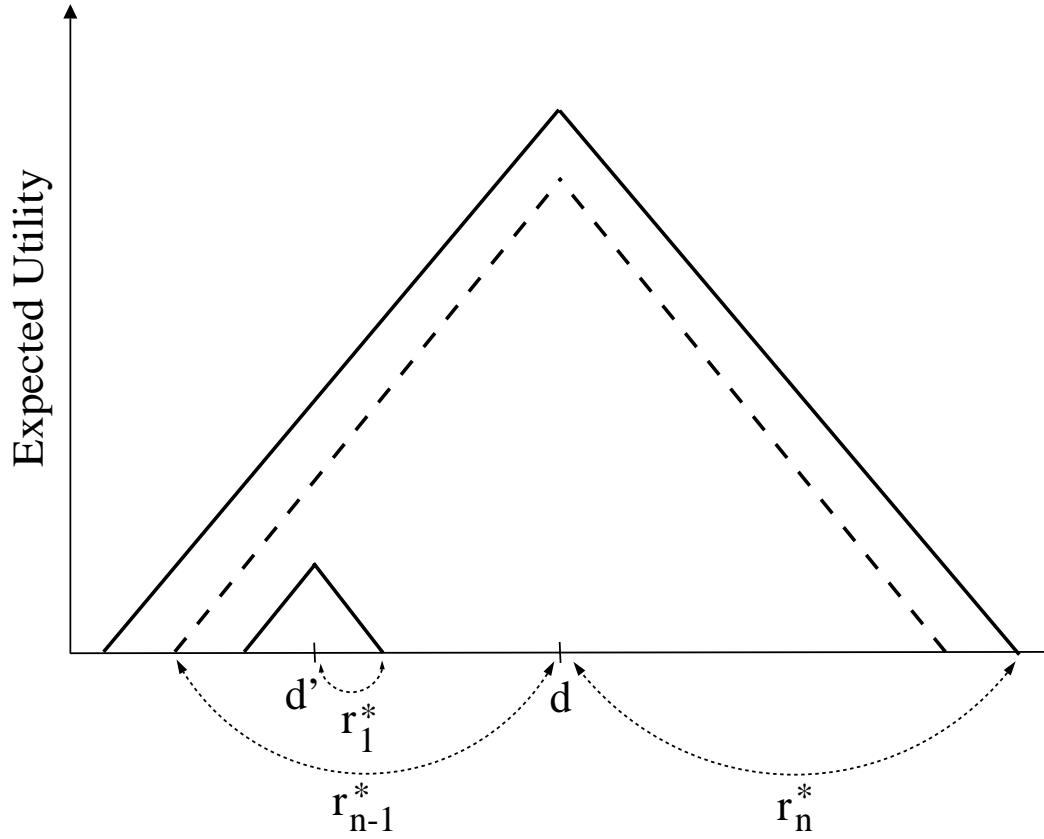
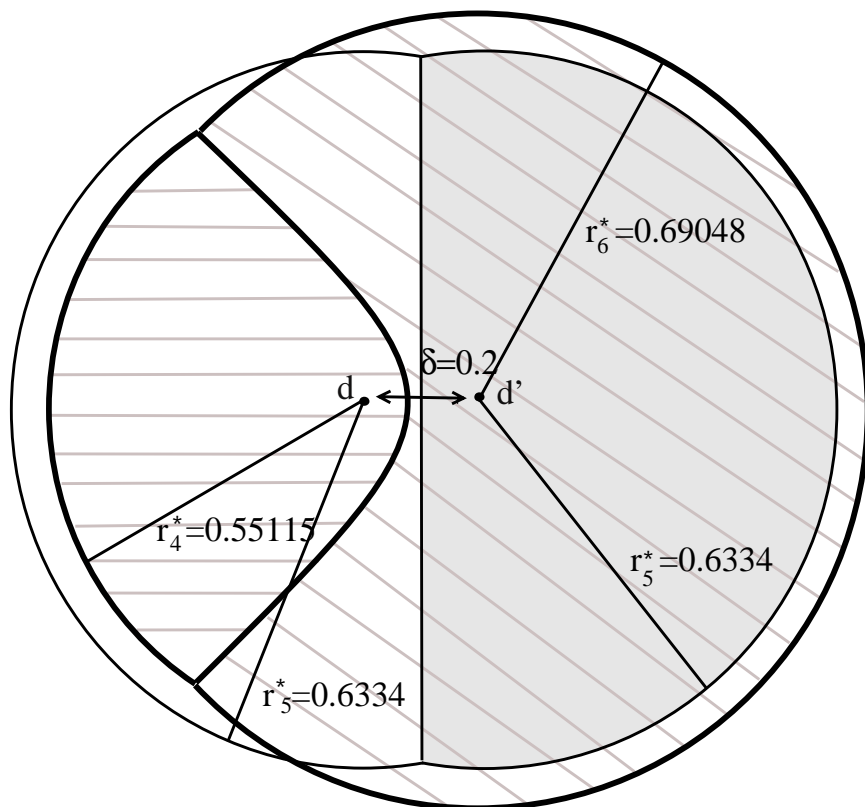


Figure 4b
 Absorbed market
 (Proposition 5)






- d: 4 stores 
- d': 5 stores 
- d': 6 stores 

Figure 5

(A symmetric allocation cannot be an equilibrium)

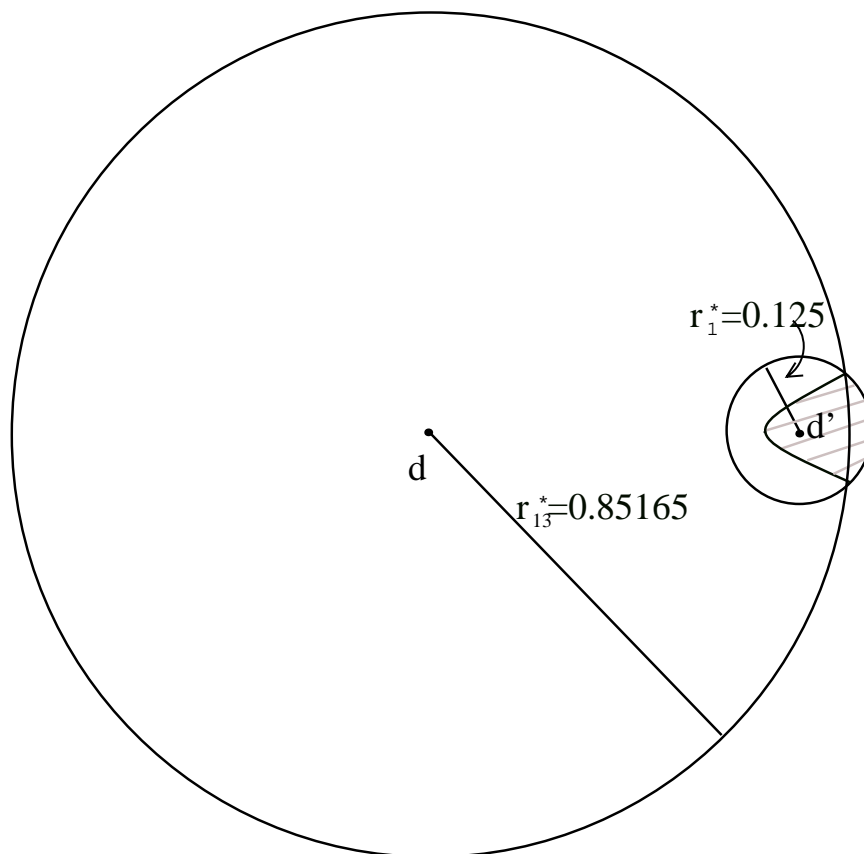


Figure 6
(A clustering equilibrium exists)