

Referrals in Search Markets*

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Abstract

This paper compares equilibrium outcomes in search markets with and without referrals. Although consumers would benefit from honest referrals, it is not at all clear whether firms would unilaterally provide information about competing offers since such information could encourage a consumer to purchase the product elsewhere. In a model of a horizontally differentiated product and sequential consumer search, we show that valuable referrals can arise as a part of equilibrium: firm will give referrals to consumers whose ideal product is sufficiently far from the firm's offering. The effect of referrals on the equilibrium prices is examined, and it is found that prices are higher in markets with referrals. Although consumers can be made worse off by the existence of referrals, referrals lead to a Pareto improvement as long as search cost is not too low relative to product heterogeneity. The effects of referral fees and third-party referrals are examined, and policy implications are drawn.

Keywords: horizontal referrals, consumer search, information, matching, broker commission.

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1 Introduction

In a number of industries, consumers have to incur substantial costs to learn about available products and their characteristics. Examples include high-tech products such as digital cameras or specialized services such as picture framing, as well as professional services in areas of law, accounting, real estate, and health care. These products and services are typically purchased infrequently, or else their characteristics change often and are thus difficult to assess. As a result, uninformed consumers need to conduct a costly search in order to find the products and services they want.¹

In contrast, agents in the industry know the industry well and can gather information on products and services provided by their competitors without much effort: they obtain useful information simply by engaging in their everyday business, or through the economies of scale in information gathering. Referral systems reduce consumers' search costs tremendously, since sales agents can ascertain their clients' preferences in conversation, and they know which agents in the industry the clients should visit. However, referrals made by competing firms or intermediaries are usually subject to the problem of moral hazard. A referring firm faces an incentives problem when it faces a trade-off between serving a consumer or referring the consumer to another seller (sales agent). Referrals can also be influenced by kickbacks – side payments in the form of referral fees or commissions.

An example of referral activities is found in real estate brokerage.² Unless she decides to

¹Consumers may even need to do research on products or services of the industry before conducting searches in order to acquire some necessary knowledge in judging products or services.

²Although this type of referral activity is usually not called “referral” in the real estate industry, it makes a simple illustration of our approach. Note also that the story here is greatly simplified: for example, buyer agents help their clients in negotiating with sellers and choosing a house in addition to showing them available properties. They provide “referrals” of mortgage companies, house inspectors, and other service professionals, too. We ignore these important factors in the example.

sell the house directly herself, a homeowner is interested in selling her property can contact an agent in a realtor office. The agent then becomes a *seller broker* of the property, representing the seller's interest, and the office posts the property in a list of available properties. As seller broker, the agent can then show the property to potential buyers. If a buyer purchases the property, the agent as seller broker obtains a commission from the sale of the property (usually 5-7% of the sales price in the U.S. real estate industry).³ However, if the agent (or the office) has no properties suitable for the buyer, the agent then shows the buyer properties that are handled by other agents, and acts as a *buyer broker*, now representing the buyer. If the buyer purchases a property from the agent she is now referred to, the commission from the sale is split into two (usually equal) shares. Thus, the real state agent (and the office) obtains a full commission if she can sell a property that she handles as a seller broker, while she obtains a half commission as a buyer broker if she refers the buyer to a property handled by another agent and succeeds in the sales transaction. An agent obviously prefers to sell a property as a seller broker, but if she knows that a buyer will not purchase any of the properties she handles, then she works hard to find a property suitable for her buyer-client for a half-commission. Without this system of referrals, a buyer would need to visit many realtors' offices until finding a suitable property. There are also exclusive buyer agents who do not handle any properties as seller broker (or who commit not to show clients any properties that they handle as seller agents). They work solely for a half-commission (referral

³If the agent has signed a contract to represent the buyer, the agent represents both the seller and the buyer, and is called a dual agent (or a transaction agent). Although we do not consider these issues, in the real world, dual agency generates serious conflict of interests: the agent cannot help clients in negotiating the deals and in choosing which house to purchase due to this conflict of interest (if he/she helps the buyer then it may hurt the seller, and the agent knows too much information about the clients who are negotiating). Thus, neither client can receive the services they expected from the agent when dual agency occurs.

fees). Thus, exclusive buyer agents act as third-party referral brokers.⁴

Although this referral practice would seem to save buyers' search costs substantially, it also affects consumer demand and market prices. Moreover, it is not clear under what circumstances competing firms would inform consumers about products offered by other firms, since directing consumers to products that match their tastes can result in a loss of business. In this paper, we ask the following questions. What are the equilibrium referral policies? Are referral practices good for buyers? Are they good for sellers? Do referrals increase or decrease market equilibrium prices? Do the sizes of referral fees matter for equilibrium price? Are third-party referrals good for consumers and for the industry? Although the pros and cons of regulating referral practices are often discussed by professional associations and regulatory authorities (as we will see below), there is not much economics theory on the topic.⁵ In particular, search markets with referrals have not yet been studied in the literature. To explore the above questions, we examine referrals among horizontally differentiated sellers (horizontal referrals)⁶ in a model of sequential consumer search (Wolinsky, 1984). The focus is on the effects referrals have on equilibrium prices, profits, and consumer benefits. To better understand the economics of referrals in markets with imperfectly informed consumers, we construct a model of a search market in which firms can refer consumers to other sellers.

⁴In the real world, the best benefit to the buyer of working with an exclusive buyer agent is that he/she will always help the buyer in negotiation and in choosing a house since he/she would never face a dual-agency problem.

⁵Few exceptions include Pauly (1979), Spurr (1990), and Garicano and Santos (2004). An empirical study by Spurr (1990) on referral practices among lawyers examines the proportion of cases referred between lawyers, as dependent on the value and nature of a claim, advertising activity, and other factors.

⁶Garicano and Santos (2004) examine referrals between vertically differentiated firms (vertical referrals). Due to complementarity between the value of an opportunity and firms' skills, efficient matching involves assigning more valuable opportunities to high-skill firms. The authors show that flat referral fees can support efficient referrals from high-quality to low-quality firms but not in the opposite direction. The low-quality firm has incentives to keep the best opportunities to itself rather than refer them to a high-quality firm. Income-sharing contracts can solve the incentives problem but the first-best is usually impossible to achieve in their model due to the free-riding problem in team production.

It is assumed that firms inside the industry know competitors' (horizontally differentiated) product offerings as part of doing regular business, while consumers do not know which firms offer which products. If they want to, firms can provide such information to consumers who visit them. In the model, firms set prices and choose referral policies, deciding which consumers are served and which are referred to other sellers. A consumer's search strategy specifies what the consumer does at each step of the search, depending on the information available. Given a value for the product, a consumer decides whether to initiate a random (potentially sequential) search. Upon a visit to a firm, the consumer learns the location of the firm. Based on this information and on a referral given by the firm (if any), the consumer decides whether to buy the product, leave the market without a purchase, continue random search, or follow the referral. In the basic model, we assume that referral fees are not allowed and that referrals are honest.⁷ Two parameters characterize the industry: search cost and degree of heterogeneity of horizontally differentiated products. We show which parameter values yield equilibria with and without referrals, and compare them. Then, we introduce referral fees into the model and observe how equilibrium is affected by them. We also trace the effects of third-party referrals.

The results are somewhat surprising. In the basic model, the norm of referring consumers to competitors who best match consumers' tastes tends to increase prices and is preferred by sellers. Thus, although referrals provide consumers with valuable information that saves

⁷As is mentioned in the real estate example, a seller agent has an incentive to sell a property she handles as a seller agent if a buyer is not interested in it. By "honest referrals" we mean that when the realtor thinks that there is no way to sell her properties to a buyer, she makes a referral to a property that is most suitable for the buyer. In the absence of reputation effects, referral fees, and bilateral agreements between firms, referral services do not increase a firm's profit. A firm refers a consumer only if the consumer would otherwise leave the firm to engage in random search. The firm is indifferent where to refer a consumer, so it might as well provide the honest referral.

consumer search costs and improves the product match, consumers may be worse off under referrals. This happens when the benefits from referrals (i.e. lower search costs and better product match) are outweighed by the loss to consumers from the equilibrium price increase. We show that for sufficiently low search costs relative to the degree of product heterogeneity, consumers prefer markets without referrals. At the same time, referrals lead to a Pareto improvement in markets with relatively high search costs (Propositions 3 and 4). If referral fees are introduced, the equilibrium price increases with the referral fees. This is not surprising since sellers can obtain referral fees even if they cannot sell their own products, and therefore raising prices is encouraged (Propositions 5 and 6). Third-party brokers do not face the moral-hazard problem of seller agents, but the presence of third-party referrals increases the equilibrium price. The reason is that price competition is further softened because a firm's demand curve becomes more inelastic: consumers who visit the third-party brokers are assigned to their ideal products by referrals and never consider looking for other products. As a result, even if the moral-hazard problem is improved, consumers can be hurt by third-party referrals (Proposition 7).⁸

There are debates about referrals in various industries. Referral institutions, rules, and regulations address the problems of suboptimal referral intensity and referral biases. Several regulations regarding fee-splitting in referrals have been established to reduce the potentially opportunistic behavior of sellers. For example, the federal anti-kickback law's main purpose is to protect patients and federal health care programs from "fraud and abuse."⁹ Professional

⁸Again, we ignore the services that exclusive buyer agents provide to buyers (see footnotes 2, 3, and 4).

⁹According to the law, anyone who knowingly and willfully receives or pays anything of value to influence the referral of business in federal health care programs such as Medicare and Medicaid conducts a felony and can be punished by up to five years in prison, criminal fines, penalties, and exclusion from participation in federal health care programs.

associations in law, accounting, and real estate have established codes of honor that regulate the referral activity to guard their reputation, avoid conflicts of interest, and protect consumers. For instance, the American Bar Association (ABA) adopted the Code of Professional Responsibility and, more recently, the Model Rules of Professional Conduct, which forbid fee-sharing with non lawyers and prescribe the division of fees by lawyers in proportion to the actual services performed or responsibility assumed. In particular, the division of fees for “pure” referrals is not allowed. Referral fees between lawyers may be prohibited under state codes of professional responsibility unless certain criteria are met, e.g., full disclosure, the client’s consent, and the provision that referred consumers are not charged more than others. Antitrust agencies may recommend changes in the codes of professional conduct when the rules can have anti-competitive effects. For instance, FTC studied the effect of the American Institute of Certified Public Accountants (AICPA)’s conduct rules on competition among public accountants and found a prohibition on referral fees in violation of Section 5 of the FTC Act. Most state governments changed their rules to allow CPAs to accept commissions, provided that they are disclosed.

The rest of the paper is organized as follows. Section 2 presents a model of price competition between firms selling a horizontally differentiated product under imperfect consumer information. We consider search markets with and without referrals and compare the referral and random search equilibria that arise in such markets. In Section 3, we extend the basic model by examining markets with caps on referral fees and with third-party brokers. Section 4 offers concluding comments.

2 The Model

We model competition between firms that produce a horizontally differentiated product. There are n firms located symmetrically on a circle of unit circumference, which produce the product at a zero marginal cost. A unit mass of consumers with unit demand is characterized by consumers' valuation of the product (willingness-to-pay), and their preferences over the horizontally differentiated product brands. Consumers' ideal positions are distributed uniformly over the unit circle. Independently of their spatial preferences, each consumer has a value $v \sim U[0, 1]$ for her ideal product (the product that is a perfect match with her taste).

Prior to visiting firms, consumers are aware of their valuation of the product and their ideal positions. Firms know the product characteristics of all products available in the market (positions of all firms). When a consumer visits a firm, she learns of the firm's position and its price, while the firm learns of the consumer's ideal product (taste). Suppose a consumer whose product valuation is v learns that the firm's brand is located at distance x from her ideal position. Let the consumer's utility for the firm's brand (gross of price and search costs) be $u(x, v) = v - tx$, where $t > 0$ denotes the degree of product heterogeneity.

Consumer search is sequential, with a marginal cost of search $s > 0$ that is common across consumers. Search is with perfect recall and with replacement. We assume that when indifferent between searching or not, consumers search. Firms can refer to other firms consumers who visit them. Since search costs come mainly in the form of learning about product characteristics, referrals lower consumers' search costs. Indeed, we assume that following a referral is costless.

The strategy of a firm is a (nondiscriminatory) price and a referral policy that can be

conditioned on the observed consumer's ideal position. A consumer chooses whether to start a random search given her product value v ; after the first search the consumer learns the firm's brand location and decides whether to continue the random search, follow a referral if it is given, purchase the best examined item, or leave the market without purchasing any product. We start in the next section by analyzing a model of price competition in a random search market.

2.1 Random Search

Let us derive the optimal stopping rule for a consumer who is engaged in a random sequential search. In a symmetric price equilibrium, the optimal stopping rule for an actively searching consumer does not depend on price, so we can first assume zero prices for commodities. If a consumer with reservation utility w engages in a search once, then her expected utility is

$$\begin{aligned} & 2 \int_0^{\frac{v-w}{t}} (v - tx) dx + 2 \int_{\frac{v-w}{t}}^{\frac{1}{2}} w dx - s \\ &= \frac{(v-w)^2}{t} + w - s. \end{aligned} \tag{1}$$

If she does not search, she gets w . Thus, one round of random search pays off in expectation iff $(v-w)^2 \geq ts$. Thus, critical w is a function of v , t , and s : $w^*(v) = v - \sqrt{ts}$. This in turn means $v - tx^* = v - \sqrt{ts}$, and the critical (reservation) distance is $x^* = \sqrt{\frac{s}{t}}$. The optimal stopping rule for the consumer is to stop searching if and only if she draws a product that is closer than distance $x^* = \sqrt{\frac{s}{t}}$ to her ideal position. The probability of finding a product in this range is $2\sqrt{\frac{s}{t}}$. When $\sqrt{\frac{s}{t}} > \frac{1}{2}$, she always stops searching, regardless of the product found.

Now, we need to see what the equilibrium price is under random search. It is useful to follow Wolinsky's (1986) techniques. We will support a symmetric equilibrium price p . The

probability of stopping a search is $2\sqrt{\frac{s}{t}}$ in any other firm (they charge the same price p). If firm i sets a different price, $p_i \neq p$, then it can affect consumers' search behavior somewhat. If x satisfies the following, a consumer stops searching at firm i :

$$v - p_i - tx \geq v - p - \sqrt{ts}, \quad (2)$$

or

$$\frac{p - p_i}{t} + \sqrt{\frac{s}{t}} \geq x. \quad (2')$$

As a result, firm i 's demand function per consumer engaged in search activities when there are n firms is written as

$$D_i^n(p_i, p) = \frac{1}{n} \sum_{k=1}^n \left(1 - 2\sqrt{\frac{s}{t}}\right)^{k-1} \left(2 \left(\frac{p - p_i}{t}\right) + 2\sqrt{\frac{s}{t}}\right). \quad (3)$$

Since we assume that consumers engage in random search, firm i could be visited as the k th firm, where k can be any positive integer. By rewriting, we obtain

$$D_i^n(p_i, p) = \frac{1}{n} \left(\frac{1 - (1 - 2\sqrt{\frac{s}{t}})^n}{1 - 1 + 2\sqrt{\frac{s}{t}}} \right) \left(2 \left(\frac{p - p_i}{t} \right) + 2\sqrt{\frac{s}{t}} \right). \quad (3')$$

As the number of firms n grows, the demand falls to zero due to $\frac{1}{n}$, if consumer population is kept constant. To avoid this problem, we replicate the consumer population n times as the number of firms n grows, thus keeping consumer population per firm constant. For zero marginal costs, the profit function of firm i per consumer engaged in search activities is then $\tilde{\pi}_i^n(p_i, p) = np_i D_i^n(p_i, p)$ in the n -firm market. In the limit as $n \rightarrow \infty$, firm i 's profit function becomes

$$\tilde{\pi}_i(p_i, p) = \lim_{n \rightarrow \infty} \tilde{\pi}_i^n(p_i, p) = \frac{p_i}{\sqrt{\frac{s}{t}}} \left(\frac{p - p_i}{t} + \sqrt{\frac{s}{t}} \right). \quad (4)$$

Firm i 's profit is the measure of searchers times the profit function per searcher. A firm's price cannot change the number of consumers who visit the firm. Therefore, firm i chooses

p_i to maximize its profit per searcher, $\tilde{\pi}_i(p_i, p)$. The first-order condition is

$$\left(\sqrt{\frac{s}{t}}\right) \frac{\partial \tilde{\pi}_i(p_i, p)}{\partial p_i} = \frac{p - p_i}{t} + \sqrt{\frac{s}{t}} - \frac{p_i}{t} = 0. \quad (5)$$

Thus, the symmetric equilibrium price ($p_i = p^*$) is

$$p^* = \sqrt{ts}. \quad (6)$$

For which parameter values does a random search equilibrium exist? Since $w^*(v) = v - \sqrt{ts}$, in order for some consumers to have a nonnegative equilibrium expected utility and engage in search, price p^* must be no higher than $w^*(v)$ for some $v \in [0, 1]$, namely $v = 1$. Thus, $p^* = \sqrt{ts} \leq 1 - \sqrt{ts}$, which can be written as $\sqrt{ts} \leq \frac{1}{2}$ or $s \leq \frac{1}{4t}$. For some consumers to search beyond the first firm in the equilibrium, we also need $x^* = \sqrt{\frac{s}{t}} \leq \frac{1}{2}$, or $s \leq \frac{t}{4}$. (The appendix shows that there is no symmetric pure-strategy equilibrium in which consumers search once and for all.) Therefore, condition $s \leq \min\{\frac{1}{4t}, \frac{t}{4}\}$ on search costs ensures that there is a random search equilibrium.

Consumers whose willingness-to-pay v is greater than or equal to a critical value $\bar{v} = 2\sqrt{ts}$ engage in search. Indeed, if $v \geq \bar{v}$ then $v - p^* - \sqrt{ts} \geq 0$, and such a consumer would follow the optimal stopping rule, stopping whenever the distance from the ideal position is less than x^* . This implies that a fraction $1 - \bar{v} = 1 - 2\sqrt{ts}$ of consumers engage in search, and each firm's profit can be written as

$$\begin{aligned} \pi^* &= (1 - 2\sqrt{ts}) \tilde{\pi}_i(p^*, p^*) \\ &= (1 - 2\sqrt{ts}) p^* \\ &= (1 - 2\sqrt{ts}) \sqrt{ts}. \end{aligned} \quad (7)$$

The properties of the random search equilibrium and comparative statics results are stated in Proposition 1.

Proposition 1. *When $s \leq \min\{\frac{1}{4t}, \frac{t}{4}\}$, there exists a unique symmetric random search equilibrium with price $p^* = \sqrt{ts}$ and profits $\pi^* = (1 - 2\sqrt{ts})\sqrt{ts}$; the critical value of willingness-to-pay for market participation is $\bar{v} = 2\sqrt{ts}$. The equilibrium price increases and consumers' market participation decreases in search cost, s , and product heterogeneity, t . Profits can increase or decrease in s and t .*

Proof. The comparative statics results are as follows: $\partial p^*/\partial s > 0$, $\partial p^*/\partial t > 0$, $\partial(1 - \bar{v})/\partial s < 0$, $\partial(1 - \bar{v})/\partial t < 0$. For profits, $\partial\pi^*/\partial t = \frac{1}{2}\sqrt{s/t} - 2s = \frac{1}{2}\sqrt{s/t}(1 - 4\sqrt{ts})$, and $\partial\pi^*/\partial s = \frac{1}{2}\sqrt{t/s}(1 - 4\sqrt{ts})$, and therefore $\partial\pi^*/\partial t > 0$ and $\partial\pi^*/\partial s > 0$ if and only if $ts < \frac{1}{16}$. ■

The comparative statics analysis of Proposition 1 shows that the equilibrium price in the random search equilibrium increases in search cost and product heterogeneity. Interestingly, the equilibrium profits may increase or decrease in s and t depending on their levels. The equilibrium profits increase in s and t for sufficiently low levels of the parameters, $ts < \frac{1}{16}$. For other values of s and t , the decline in market participation is not compensated by the higher price associated with larger s and t . In other words, for $\frac{1}{16t} \leq s \leq \frac{1}{4t}$, sellers prefer to operate in search markets with lower search costs and lower product heterogeneity.

[Figure 1 HERE]

The equilibrium consumer decisions to engage in search, buy at a firm located at a distance x , or engage in sequential search are illustrated in Figure 1. Only consumers who value the product at higher than $\bar{v} = 2\sqrt{ts}$ engage in search. Consumers visiting a firm closer than $x^* = \sqrt{\frac{s}{t}}$ from her ideal position buy the product; others continue to search.

To derive total welfare in the random search equilibrium, consider a market with price p and consumers' market participation $(1 - \hat{v})$, where \hat{v} is the critical value of willingness-to-pay for market participation. The expected utility of a consumer who values the product at $v \geq \hat{v}$ is $EU(v) = v - \hat{v}$. Note that $EU(\hat{v}) = 0$ and consumer welfare is $\int_{\hat{v}}^1 EU(v)dv = \frac{1}{2}(1 - \hat{v})^2$. Defining the total welfare in a market with price p and the critical value of willingness-to-pay \hat{v} as a sum of profits and consumer welfare, $W(p, \hat{v}) = (1 - \hat{v})p + \int_{\hat{v}}^1 EU(v)dv$, we find that total welfare can be written as

$$W(p, \hat{v}) = (1 - \hat{v})p + \frac{1}{2}(1 - \hat{v})^2. \quad (8)$$

While consumer welfare depends only on market participation, total welfare depends on the price level as well. In the random search equilibrium, total welfare is $W = \frac{1}{2} - \sqrt{ts} \geq 0$ for $s \leq \frac{1}{4t}$. We conclude that consumer welfare and total welfare are higher in random search markets with low product heterogeneity and search cost.

2.2 Search with Referrals

The strategy of a firm in search markets with referrals is a uniform price and a referral policy. We assume that a firm (salesperson) can observe a customer's ideal position x from a conversation with her, but cannot observe her willingness-to-pay v . We look for a referral equilibrium in which firms choose symmetric price and referral strategies (p_R^*, x_R) . The symmetric referral rule states that if the distance between a customer's position and a firm is more than x_R , the firm gives a referral. The equilibrium strategy of firm i ($i = 1, \dots, n$) is to choose a price, p_i , and the critical distance for giving referrals, $x_R(p_i, p)$, to maximize its profits, given that other firms set a common price p and use referral rule x_R . Given our assumption of honest referrals, a firm would refer a consumer to the competitor selling the

best-matching product if and only if the consumer is definitely leaving the firm.

Figure 2 illustrates search with referrals. A consumer randomly chooses the first firm to visit; say, firm i , located at distance x from her ideal position (point C). When firm i is located at least $x_R(p_i, p)$ away from point C , the consumer is referred to the best-matching firm (firm j), and she has to decide whether to buy at firm i , follow the referral, continue random search, or leave the market. When firm i is located closer than $x_R(p_i, p)$ from the consumer's ideal position, the consumer does not receive a referral and has to either buy at firm i , continue random search, or leave the market. In the symmetric equilibrium, the consumer follows the referral and buys at firm j when firm i is located further than $x_R = x_R(p, p)$, and buys at firm i immediately otherwise.

Let us start the analysis with a second-round search. Suppose that a consumer has been assigned to firm i located at distance x from her ideal position. Thus, her utility (gross of search cost) from purchasing commodity there is $v - p - tx$, where p is the symmetric price charged by firms. In the symmetric referral equilibrium, a consumer can get a referral with a probability of $1 - 2x_R$. If she gets a referral and follows the suggestion, her utility will be $v - p$. A consumer who visited a firm with distance $x < x_R$ from her ideal position will not get a referral. Then, her choice is one of the following three: (i) make no purchase, receiving $(-s)$; (ii) purchase the product, receiving $(v - p - tx - s)$; or (iii) engage in sequential search. As we will show below, only consumers with willingness-to-pay v such that $v - p - tx_R \geq 0$ enter the market in equilibrium, and since $v - p - tx \geq 0$ for all $x < x_R$, it is better for the searcher to purchase rather than not. If she engages in further sequential search, she visits a random firm (firm $k \neq i$), which is located at distance \tilde{x} from the consumer and charges price p . At firm k , the consumer gets and follows a referral with probability $(1 - 2x_R)$, recalls

firm i 's offer, or buys firm k 's product. The expected payoff from engaging in one additional search is then

$$\begin{aligned}
& \Delta EU(x; x_R) && (9) \\
&= (1 - 2x_R)(v - p) + 2(x_R - x)(v - p - tx) + 2 \int_0^x (v - p - t\tilde{x})d\tilde{x} - s - (v - p - tx) \\
&= (1 - 2x_R)tx + tx^2 - s.
\end{aligned}$$

It is easy to see that for any $x_R \in [0, \frac{1}{2}]$ and any $x \in [0, x_R]$, we have $\partial \Delta EU(x; x_R) / \partial x = t(1 - 2x_R + 2x) \geq 0$. This means that as long as $\Delta EU(x_R; x_R) \leq 0$, consumers who have $x < x_R$ would not engage in further search.

A consumer who visited a firm located at distance x_R from her ideal position is indifferent between searching and not searching (given that all other firms are using referral rule x_R): $\Delta EU(x_R; x_R) = 0$. It follows from

$$\begin{aligned}
\Delta EU(x_R; x_R) &= (1 - 2x_R)tx_R + tx_R^2 - s && (10) \\
&= tx_R - tx_R^2 - s = 0
\end{aligned}$$

that

$$x_R = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(s/t)} \quad (11)$$

for $t \geq 4s$. This value x_R describes the symmetric equilibrium referral rule. If a consumer gets a referral and $v \geq p$, then she follows the referral. If she does not get a referral, it means that sequential search is not beneficial, and as a result, she either purchases or goes home without purchase; she purchases if and only if $v \geq p + tx$. In contrast, if $t < 4s$ then there will be no referral equilibrium.

Next, we calculate the demand function of firm i assuming that other firms choose a symmetric price p and make referrals to customers outside of x_R . We still assume that every

firm including firm i gives honest referrals and that the referrals are given as long as the firm is sure that it cannot sell its product to the customer. At the first step, we find the consumers' optimal stopping rule when she observes a price p_i at firm i that is different from p . Let x be the distance between a consumer's ideal point and the location of firm i . We look for the threshold distance for firm i , $x = x_R(p_i, p)$, such that a consumer with distance $x_R(p_i, p)$ from the ideal point is indifferent between purchasing i 's product and searching further.

Suppose that an additional search after visiting firm i matches the consumer with a product at firm $k \neq i$ with distance \tilde{x} from her ideal point. There are two cases: (i) $\tilde{x} \geq x_R$ and she gets a referral, or (ii) $\tilde{x} < x_R$ and she does not get firm k 's referral (in this case, she will not search further). A consumer prefers to buy firm i 's rather than firm k 's product if and only if $v - p_i - tx < v - p - t\tilde{x}$ (or $\tilde{x} > x + \frac{p_i - p}{t}$). One additional round of search results in consumer utility no less than $v - p - tx_R$.

First assume that the consumer utility from firm i 's product is at least as high as that under the worst realization of an additional search: $v - p_i - tx \geq v - p - tx_R$ (or $x + \frac{p_i - p}{t} \leq x_R$, or $p_i \leq p + tx_R - tx$). In this case, the consumer may recall firm i 's product under some realizations of \tilde{x} . If $\tilde{x} > x_R$, she gets firm j 's referral and obtains $(v - p)$. If $x + \frac{p_i - p}{t} \leq \tilde{x} \leq x_R$, she recalls firm i 's product. Finally, if $\tilde{x} < x + \frac{p_i - p}{t}$, she buys at firm j . Therefore, a customer's gain from engaging in an additional search when firm i charges p_i and other firms charge p is as follows:

$$\begin{aligned}
& (1 - 2x_R)(v - p) + 2 \left(x_R - x - \frac{p_i - p}{t} \right) (v - p_i - tx) + 2 \int_0^{x + \frac{p_i - p}{t}} (v - p - t\tilde{x}) d\tilde{x} \\
& - s - (v - p_i - tx) \\
& = t \left[x + \left(\frac{1}{2} \sqrt{1 - 4(s/t)} + \frac{p_i - p}{t} \right) \right]^2 - \frac{t}{4}.
\end{aligned} \tag{12}$$

By setting the above gain to be zero, we obtain the threshold distance $x_R(p_i, p)$ for firm i :

$$x_R(p_i, p) = x = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4(s/t)} - \frac{p_i - p}{t}. \tag{13}$$

Note that $x_R(p_i, p) = x_R - \frac{p_i - p}{t}$ and that the condition we assumed is satisfied for $x = x_R(p_i, p)$. Second, suppose that $v - p_i - tx < v - p - tx_R$ (or $x + \frac{p_i - p}{t} > x_R$, or $p_i > p + tx_R - tx$).

In that case, the consumer leaves firm i to search further.

Now we can present firm i 's demand function. Consider the following three cases: (i) $p_i < p$, (ii) $p_i \geq p$ and $p_i \leq p + tx_R$, and (iii) $p_i > p + tx_R$. In case (i), all consumers who visit firm i buy there since i 's offer is better than engaging in further random search.

In case (ii), consumers located at $x < x_R(p_i, p)$ buy from firm i . With the above threshold value $x_R(p_i, p)$, firm i 's *demand function per consumer engaged in search activities* when there are n firms is

$$\begin{aligned}
D_i^n(p_i, p) &= \frac{1}{n} 2x_R(p_i, p) + \frac{n-1}{n} \times \frac{1}{n-1} (1 - 2x_R) \\
&= \frac{1}{n} \left(1 - \frac{2(p_i - p)}{t} \right),
\end{aligned} \tag{14}$$

where $x_R = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4(s/t)}$ if $p_i \in [p, p + tx_R]$. The first term represents demand from consumers who are assigned to firm i in their initial search (with probability $1/n$), while the second term represents demand by referrals from other firms (consumers who visit other firms first are referred to firm i with probability $1/(n-1)$). In case (iii), we have $x_R(p_i, p) < 0$, and

no consumer who is assigned to firm i purchases the commodity. Moreover, $v - p_i < v - p - tx_R$ holds for any v , and even consumers who get referrals do not purchase at firm i . Thus, demand for firm i in this case is zero.

Given this, *firm i 's profit function per consumer engaged in search activities* (with n -replicated consumer population) is

$$\tilde{\pi}_i(p_i, p) = \lim_{n \rightarrow \infty} p_i n D_i^n(p_i, p) = p_i \left(1 - \frac{2(p_i - p)}{t} \right), \quad (15)$$

if $p_i \in [p, p + tx_R]$, $\tilde{\pi}_i(p_i, p) = p_i$ if $p_i < p$, and $\tilde{\pi}_i(p_i, p) = 0$ if $p_i > p + tx_R$.

To calculate firm i 's profit function, we need to determine who participates in the market. Since we are analyzing a symmetric equilibrium, we assume that consumers expect that every firm charges price p , and that the common referral rule is described by $x_R = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(s/t)}$. Consider a consumer whose willingness-to-pay v satisfies $v - tx_R - p \geq 0$. She has the following expected utility from the initial search (given the optimal stopping rule characterized by x_R):

$$\begin{aligned} EU_R(v, p) &= 2 \int_0^{x_R} (v - tx - p) dx + (1 - 2x_R)(v - p) - s & (16) \\ &= (v - p) - tx_R^2 - s \\ &= v - tx_R - p, \end{aligned}$$

where $tx_R - tx_R^2 - s = 0$ follows from the definition of x_R in equation (10). That is, a consumer whose willingness-to-pay v satisfies $v - tx_R - p \geq 0$ obtains a nonnegative expected utility, $EU_R(v, p) \geq 0$. On the other hand, if a consumer's willingness-to-pay v satisfies $v - tx_R - p < 0$, it is easy to see that $EU_R(v, p) < 0$ for any stopping rule. Thus, given that p is a prevailing symmetric price, a consumer engages in the initial search if and only if her willingness-to-pay v is not less than $\bar{v}_R(p) = tx_R + p$.

Firm i 's profit function when other firms are expected to set price p is, then,

$$\pi_i(p_i, p) = (1 - p - tx_R)\tilde{\pi}_i(p_i, p) = (1 - p - tx_R)p_i \left(1 - \frac{2(p_i - p)}{t}\right). \quad (17)$$

In Proposition 2 we show that there is a unique symmetric equilibrium in this model.

Proposition 2. *When $s \leq \frac{t}{4}$ and $t - 1 \leq \frac{t}{2}\sqrt{1 - 4(s/t)}$, there exists a unique symmetric referral equilibrium with price $p_R^* = \frac{t}{2}$; profits $\pi_R^* = p_R^*(1 - p_R^* - tx_R) = \frac{t}{2}(1 - t + \frac{t}{2}\sqrt{1 - 4(s/t)})$; the critical value of willingness-to-pay for market participation is $\bar{v}_R = p_R^* + tx_R = t - \frac{t}{2}\sqrt{1 - 4(s/t)}$, where $x_R = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(s/t)}$. Referral intensity $r = 1 - 2x_R = \sqrt{1 - 4(s/t)}$ decreases with consumer search costs and increases with product heterogeneity. The equilibrium price is perfectly insensitive to the value of search costs, while it increases as product heterogeneity increases. Consumers' market participation and profits decrease in search cost, s , and they can increase or decrease in product heterogeneity, t .*

Proof. We first show that there is a symmetric equilibrium when $\bar{v}_R \leq 1$. Since π_i is concave in p_i , the first-order condition evaluated at $p_i = p$ characterizes symmetric equilibrium. The first-order condition is $\partial\pi_i(p_i, p)/\partial p_i = (1 - p - tx_R)(1 - (4p_i - 2p)/t) = 0$. Thus, there is a unique symmetric equilibrium price $p_R^* = \frac{t}{2}$. The value of \bar{v}_R can be found by substituting p_R^* into $v_R(p) = p + tx_R$. We obtain $\bar{v}_R = t - \frac{t}{2}\sqrt{1 - 4(s/t)}$. Finally, $\bar{v}_R \leq 1$ if and only if $t - 1 \leq \frac{t}{2}\sqrt{1 - 4(s/t)}$, which holds for $t \leq 1$ provided $s \leq t/4$. For $t > 1$, the inequality can be written as $s \leq \frac{1}{4t}(2 - t)(3t - 2)$ or $s \leq 2 - 0.75t - 1/t$. The equilibrium price $p_R^* = \frac{t}{2}$ increases in t and does not depend on s . Consumers' market participation, measured by $(1 - \bar{v}_R)$, decreases in search cost, s , because $\partial\bar{v}_R/\partial s = 1/\sqrt{1 - 4(s/t)} > 0$. For product heterogeneity, t , $\partial\bar{v}_R/\partial t = \frac{1}{2} \left(2\sqrt{1 - 4(s/t)} - (1 - 2(s/t))\right) \frac{1}{\sqrt{1 - 4(s/t)}} > 0$ is equivalent to $2\sqrt{1 - 4(s/t)} > (1 - 2(s/t))$. Since $1 - 2(s/t) > 0$, this is equivalent to

$4(1 - 4(s/t)) > (1 - 2(s/t))^2$, or $4(s/t)^2 + 12(s/t) - 3 < 0$. The last inequality holds if and only if $s/t < (\sqrt{3} - \frac{3}{2})$. Hence, $\partial \bar{v}_R / \partial t > 0$ and consumers' market participation decreases in t for $s < (\sqrt{3} - \frac{3}{2})t \approx 0.23t$. However, there is a region of parameter values for which the opposite is true. When $(\sqrt{3} - \frac{3}{2})t \leq s \leq t/4$, then $\partial \bar{v}_R / \partial t < 0$ and consumers' market participation increases in t in the equilibrium. Equilibrium profits $\pi_R^* = \frac{t}{2}(1 - t + \frac{t}{2}\sqrt{1 - 4(s/t)})$ decrease in search cost, $\partial \pi_R^* / \partial s < 0$. The effect of t on the profits is ambiguous. Profits increase in t if and only if $t - 3s + (1 - 2t)\sqrt{1 - 4(s/t)} > 0$. For example, it suffices to require $t \leq \frac{1}{2}$. In contrast, for relatively large t , profits decrease with a further increase in product heterogeneity. For $s \rightarrow 0$, profits increase in t for $t < 1$ and decrease in t for $1 < t \leq 2$. ■

According to Proposition 2, a referral equilibrium exists whenever $s \leq \frac{t}{4}$ and $t - 1 \leq \frac{t}{2}\sqrt{1 - 4(s/t)}$. Therefore, it is necessary and sufficient for the existence of the equilibrium that either (i) $t \leq 1$ and $s \leq \frac{t}{4}$ or (ii) $t > 1$ and $s \leq 2 - \frac{3t}{4} - \frac{1}{t}$.

Figure 3 illustrates the equilibrium consumer decisions for different realizations of v and x . Only those consumers whose willingness-to-pay is higher than $\bar{v}_R = p_R^* + tx_R = t - \frac{t}{2}\sqrt{1 - 4(s/t)}$ engage in search. Consumers visiting a firm located closer than $x_R = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(s/t)}$, buy the product, while others follow the firm's referral and buy from the referred seller.

[Figure 3 HERE]

Why does search cost s not matter in the determination of price p_R^* in this case? By lowering its price, firm i can increase its sales only through an increase in the retention rate $2x_R(p_i, p) = 2x_R - 2(p_i - p)/t$ of consumers who are assigned to firm i in their initial search.

However, a change in the retention rate, which equals $-2/t$, is not affected by search cost s since sequential search never takes place in the equilibrium.¹⁰ Thus, in search markets with referrals the equilibrium price is determined only by heterogeneity parameter t .

Using the general formulation of total welfare in equation (8), total welfare in the referral equilibrium can be written as

$$\begin{aligned} W_R &= p_R^* (1 - \bar{v}_R) + \frac{1}{2} (1 - \bar{v}_R)^2 \\ &= \frac{1}{2} \left(1 + \frac{t}{2} \sqrt{1 - 4(s/t)} - t \right) \left(1 + \frac{t}{2} \sqrt{1 - 4(s/t)} \right) \end{aligned} \quad (18)$$

We will next show that total welfare decreases in s , while its relationship with t is ambiguous. Differentiating the welfare with respect to search cost, we find that $\partial W_R / \partial s < 0$ if and only if $t\sqrt{1 - 4(s/t)} - t + 2 > 0$, which holds for $t \leq 2$. For $t > 2$, condition $t - 1 \leq \frac{t}{2}\sqrt{1 - 4(s/t)}$ required for the existence of the equilibrium cannot hold, since $t - 1 > \frac{t}{2}$. Hence, as in the random search equilibrium, total welfare decreases in search cost in referral equilibrium.

Next, consider the effect of product heterogeneity on total welfare. In the proof of Proposition 2, it is established that for sufficiently large s relative to t , $(\sqrt{3} - \frac{3}{2})t \leq s \leq t/4$, market participation increases in t in the equilibrium. Hence, for these search costs, total welfare definitely increases with increasing product heterogeneity. On the other hand, for sufficiently small search costs, the effect is negative since $\partial W_R / \partial t = -\frac{1}{4}\theta(t, s) / \sqrt{t(t - 4s)}$, where $\theta(t, s) \equiv 2(s(2 - 3t) + t(t - 1)) + \sqrt{t(t - 4s)}(2 - t + 2s)$, and for $s \rightarrow 0$, $\partial W_R / \partial t \rightarrow -\frac{1}{4}t < 0$.

To summarize, the comparative statics results predict that consumer welfare and total

¹⁰Here, we can see an analogy with the Diamond paradox. In both cases, sequential search does not occur, and equilibrium price is independent of the level of search cost (as long as it is positive). However, here, there is still competition among stores trying to keep initial customers, and the monopoly price does not prevail as the equilibrium price.

welfare are higher in markets with low search cost. The effects of product heterogeneity on the equilibrium profits, consumer welfare, and total welfare are ambiguous. The results are quite intuitive. An increase in search cost does not affect the equilibrium price, but it reduces referral intensity and market participation. Therefore, search cost is detrimental from the industry, consumer, and total welfare points of view. In contrast, higher product heterogeneity can improve consumer welfare and total welfare because product heterogeneity stimulates referral activity, which can encourage consumers' market participation despite a price increase.

2.3 Comparison of Random Search and Referral Equilibria

So far, we have described random search and referral equilibria. Proposition 3 below compares the regions of their existence, while Proposition 4 compares the properties of the equilibria, and shows that unless search cost s is very low relative to heterogeneity parameter t , referral equilibrium Pareto dominates random search equilibrium.

Proposition 3. *If $t \leq 1$, then both random search and referral equilibria exist if and only if $s \leq \frac{t}{4}$. However, if $1 \leq t \leq \frac{5}{3}$ then a referral equilibrium exists for larger set of parameter values for s , whereas if $\frac{5}{3} \leq t$, then a random search equilibrium exists for larger set of parameter values for s than referral equilibrium (in particular, referral equilibrium cannot exist for $t > 2$).*

Proof. According to Proposition 1, conditions $s \leq \frac{1}{4t}$ and $s \leq \frac{t}{4}$ are necessary and sufficient for the existence of a random search equilibrium. From Proposition 2, a referral equilibrium exists if and only if $s \leq \frac{t}{4}$ and $t - 1 \leq \frac{t}{2}\sqrt{1 - 4(s/t)}$. The latter inequality holds for $t \leq 1$, and for $t > 1$ it is equivalent to $s \leq \frac{1}{4t}(2 - t)(3t - 2)$ or $s \leq 2 - 0.75t - 1/t$. When $t \leq 1$, both

equilibria exist for $s \leq \frac{1}{4t}$. For $t > 1$, $s \leq \frac{1}{4t}$ always implies $s \leq \frac{t}{4}$, and $\frac{1}{4t} < 2 - 0.75t - 1/t$ if and only if $1 < t < \frac{5}{3}$. This completes the proof. ■

Figure 4 illustrates Proposition 3 by showing the regions of parameter values of t and s for which random search and referral equilibria exist. Random search equilibrium can exist for any level of product heterogeneity provided search cost is sufficiently low. In contrast, referral equilibrium may exist only for a sufficiently low heterogeneity parameter, $t \leq 2$. As product heterogeneity approaches the critical level of 2, the market collapses because the equilibrium price $p_R^* = \frac{t}{2}$ approaches unity and thus becomes prohibitively high for consumers.

[Figure 4 HERE]

Intuitively, when the product is highly heterogeneous, firms have greater market power to set high prices in random search and referral equilibria, which discourages consumers' market participation. Even for a very high t , a random search market would not close down for a sufficiently low search cost since the low search cost imposes competitive pressure on firms in the market. In referral equilibrium, prices are not affected by search costs, and the search market with referrals does not open for $t > 2$. The condition $s \leq \frac{t}{4}$ is necessary for both equilibria to exist. Readers may wonder what happens when $s > t/4$. From the discussion on random search equilibrium, we know that if $s > \frac{t}{4}$, consumers have no incentive to search at random beyond the first firm. Since referrals are given only to consumers who would otherwise engage in random search, consumers do not get referrals. However, in the appendix it is shown that there is no symmetric pure-strategy equilibrium in which consumers search once and for all. Hence, a symmetric equilibrium with positive consumers'

participation and profits does not exist when $s > t/4$.¹¹

Next we compare random search and referral equilibria in the region of parameter values for which both of them exist.

Proposition 4. *Whenever both random search and referral equilibria exist, we have $p_R^* \geq p^*$ and $x_R^* \leq x^*$. Consumers are better off (thus, market participation is larger or $\bar{v} \geq \bar{v}_R$) in referral equilibrium if and only if $s \geq 0.09t$, i.e. search cost is relatively high. Thus, $s \geq 0.09t$ guarantees that firms earn more profits in referral equilibrium, thus referral equilibrium Pareto-dominates random search equilibrium.*

Proof. Referral equilibrium price is higher than random search equilibrium price if and only if $p^* = \sqrt{ts} \leq \frac{t}{2} = p_R^*$. This is equivalent to $s \leq \frac{t}{4}$. Thus, if both types of equilibria exist, then $p^* \leq p_R^*$. Now, $x_R^* = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(s/t)}$ and $x^* = \sqrt{s/t}$, thus, $x_R^* \leq x^*$ if and only if $2\sqrt{\frac{s}{t}} - 1 \geq \sqrt{1 - \frac{4s}{t}}$, or $4\frac{s}{t} - 4\sqrt{\frac{s}{t}} + 1 \geq 1 - \frac{4s}{t}$. This is equivalent to $2\sqrt{\frac{s}{t}} - 1 \geq 0$, or $s \leq t/4$, which is necessary for the existence of either type of equilibria.

Next, we compare consumers who are indifferent between participating or not under two equilibria. This determines consumer welfare. In a random search equilibrium, $\bar{v} = 2\sqrt{ts}$ is a threshold value, while in a referral equilibrium, $\bar{v}_R = t - \frac{t}{2}\sqrt{1 - 4(s/t)}$. Consumers are better off in a referral equilibrium if and only if $\bar{v}_R \leq \bar{v}$, or $\sqrt{t} - 2\sqrt{s} \leq \frac{1}{2}\sqrt{t - 4s}$. This is equivalent to $(\sqrt{t} - 2\sqrt{s})^2 \leq \frac{1}{4}(\sqrt{t} - 2\sqrt{s})(\sqrt{t} + 2\sqrt{s})$, or $3\sqrt{t} - 10\sqrt{s} \leq 0$, or $s \geq 0.09t$. ■

¹¹This is a version of so-called the Wernerfelt paradox. Wernerfelt (1994) considered a monopolist's price setting problem with $t = 0$ (homogeneous goods) and $s > 0$, and proved that due to self-selection of consumers (according to v), the lowest willingness-to-pay of market participants always exceeds an expected price by s , and there is no equilibrium with positive demand.

These results are not surprising. If search cost s is very low, consumers surely prefer a random search equilibrium, since the random search equilibrium price p^* is low while the referral equilibrium price p_R^* is insensitive to s . If search cost is not very low, a referral equilibrium is favored by consumers, and since the equilibrium price is higher in a referral equilibrium than in a random search equilibrium, firms favor this equilibrium as well. A referral equilibrium Pareto-dominates a random search equilibrium whenever the gain in efficiency due to higher matching quality and lower search costs is higher than the loss due to increased price. Referrals handle the information problem of matching buyers and sellers but worsen the monopoly problem of higher prices. In the referral equilibrium, a fraction $r = 1 - 2x_R = \sqrt{1 - 4(s/t)}$ of consumers are perfectly matched. Not surprisingly, the firm faces a more inelastic demand. Recall that firm i 's demand per consumer engaged in search activities is $D_i(p_i, p) = \left(1 - \frac{2(p_i - p)}{t}\right)$ and $D_i(p_i, p) = \left(\frac{p - p_i}{\sqrt{ts}} + 1\right)$ in search markets with and without referrals, respectively. It is easy to confirm that the elasticity of demand a firm faces, $E_{D_i} = \frac{\partial D_i(p_i, p)}{\partial p_i} \frac{p_i}{D_i(p_i, p)}$, is $\left(-\frac{p_i}{\frac{t}{2} - (p_i - p)}\right)$ and $\left(-\frac{p_i}{\sqrt{ts} - (p_i - p)}\right)$ in search markets with and without referrals. Since $\sqrt{ts} \leq \frac{t}{2}$ holds for $s \leq \frac{t}{4}$, firm i 's demand elasticity in a search market with referrals is lower in absolute value.

3 Extensions

In this section, we study the referral equilibrium in the presence of referral fees and third-party brokers.

1. Caps on Pure (“Naked”) Referral Fees

Consider pure referral fees – fees paid to the referring firm based on the referral alone. It could be argued that a cap on referral fees would protect consumers. We will analyze how

the equilibrium price and market participation are affected by referral fees in the presence of a binding cap.

Let $c > 0$ be a (specific) cap on referral fees, and assume that firms making referrals act honestly as long as they are offered the same referral fees. If referral fees are decided by firms, then the firm that offers the highest referral fee gets all referrals from other firms. Thus, by the standard Bertrand competition, the equilibrium referral fee must be the same as an exogenously determined cap whenever it is lower than the equilibrium price. That is, all firms offer a referral fee c as long as $p_R^*(c) > c$. Again, we assume that firms cannot price-discriminate between referred consumers and those engaged in random search, and we look for a symmetric equilibrium. We have the following proposition.

Proposition 5. *Suppose that referral fees are allowed, and that there is an exogenous cap $c > 0$ for referral fees. If $s \leq \frac{t}{4}$ and c is small,¹² then there exists a unique symmetric referral equilibrium with caps, in which $p_R^*(c) = \frac{t}{2} + c$, $\bar{v}_R(c) = p_R^*(c) + tx_R$, $\pi_R^*(c) = (1 - p_R^*(c) - tx_R)p_R^*(c)$, and $x_R = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(s/t)}$. Referral intensity $r = 1 - 2x_R = \sqrt{1 - 4(s/t)}$ is not affected by referral fees. The equilibrium price increases in c and market participation decreases in c . An increase in c can increase or decrease firms' equilibrium profits. The profit-maximizing c is attained by $c^* = \frac{1}{2} - \frac{3t}{4} + \frac{t}{4}\sqrt{1 - 4(s/t)}$.*

Proof. With a fixed referral fee c , firm i 's profit function is written as the sum of profits from consumers buying from firm i on their first visit, on their visit by referral, and the payments from other firms for the referrals the firm makes. A measure $(1 - p - tx_R)$ of

¹²If $\bar{v}_R(c) = p_R^*(c) + tx_R \leq 1$, i.e., if $c \leq 1 - \frac{t}{2} - tx_R$ is satisfied then there exists a referral equilibrium, where $x_R = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(s/t)}$ and $s \leq t/4$.

consumers participates in the market, since only consumers with values exceeding $p + tx_R$ initiate search. Firm i sells to $2x_R(p_i, p)$ of the searchers who visit firm i first and collects referral fee c for the rest of consumers it refers. A measure $(1 - 2x_R)$ of consumers who first visit another firm gets referrals to firm i , and firm i receives $(p_i - c)$ for each of the referral customers. Therefore, the profits can be written as

$$\begin{aligned}\pi_i(p_i, p; c) &= (1 - p - tx_R) (2p_i x_R(p_i, p) + (1 - 2x_R)(p_i - c) + c(1 - 2x_R(p_i, p))) \quad (19) \\ &= (1 - p - tx_R) \left(p_i \left(1 - \frac{2(p_i - p)}{t} \right) + c \times \frac{2(p_i - p)}{t} \right).\end{aligned}$$

if $p_i \leq p + tx_R$, and $\pi_i(p_i, p) = 0$ otherwise. By taking the first-order condition, we have

$$\frac{\partial \pi_i}{\partial p_i} = (1 - p - tx_R) \left(1 - \frac{4p_i - 2p}{t} + \frac{2c}{t} \right). \quad (20)$$

Thus, the equilibrium price, given a referral fee c , is $p_R^*(c) = \frac{t}{2} + c$. Since a consumer who is indifferent about participating in the market has a willingness-to-pay $\bar{v}_R(c) = p_R^*(c) + tx_R$, an increase in c reduces market participation and consumers' expected utility.

The equilibrium profit is described by

$$\pi_R^*(c) = (1 - p_R^*(c) - tx_R)p_R^*(c), \quad (21)$$

where $x_R = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(s/t)}$. This implies that a marginal increase in c improves firms' profits if and only if $p_R^*(c) < \frac{1 - tx_R}{2}$ (the monopoly price given tx_R), or $c < \frac{1}{2} - \frac{3t}{4} + \frac{t}{4}\sqrt{1 - 4(s/t)}$. For the equilibrium to exist, some consumers have to engage in search: $\bar{v}_R(c) = c + t - \frac{1}{2}t\sqrt{1 - 4\frac{s}{t}} \leq 1$, or $c \leq 1 - t + \frac{1}{2}t\sqrt{1 - 4(s/t)}$. If $t \leq 2$, then $1 - t \geq \frac{1}{2} - \frac{3t}{4}$, and $1 - t + \frac{1}{2}t\sqrt{1 - 4(s/t)} \geq \frac{1}{2} - \frac{3t}{4} + \frac{t}{4}\sqrt{1 - 4(s/t)}$. Therefore, for $c \in \left(0, \frac{1}{2} - \frac{3t}{4} + \frac{t}{4}\sqrt{1 - 4(s/t)}\right)$, profits increase with the cap on referral fees, while profits decrease with c for $\frac{1}{2} - \frac{3t}{4} + \frac{t}{4}\sqrt{1 - 4(s/t)} \leq c \leq 1 - t + \frac{1}{2}t\sqrt{1 - 4(s/t)}$. ■

This proposition is of some interest, since it says that referral costs are borne 100% by consumers. That is, consumers are clearly worse off by having a high referral fee c . The equilibrium price is higher by c , implying $\bar{v}_R(c) > \bar{v}_R$, and the number of market participants shrinks.¹³ This result is intuitive since the cost of raising the price is reduced by the presence of referral fees. That is, even if a firm cannot sell a commodity to a customer it can still obtain a referral fee c .

Next, consider a referral fee in the form of a commission (an ad valorem referral fee). Let $\gamma > 0$ be a cap on the commission, and assume that firms making referrals act honestly as long as they are offered the same referral payments. As a result of referral competition, all firms will offer the same commission γ as long as it does not exceed unity ($1 \geq \gamma$). As before, we assume that firms cannot price-discriminate between consumers and look for a symmetric equilibrium. We have a qualitatively similar result with a slight difference.

Proposition 6. *Suppose that referral fees are allowed, and that there is an exogenous cap on commission $\gamma \in (0, 1)$ for referrals. If $s \leq \frac{t}{4}$ and γ is small,¹⁴ then there exists a unique symmetric referral equilibrium with caps, in which $p_R^*(\gamma) = \frac{t}{2} \left(\frac{1-\gamma\sqrt{1-4(s/t)}}{1-\gamma} \right)$, $\bar{v}_R(\gamma) = p_R^*(\gamma) + tx_R$, and $\pi_R^*(\gamma) = (1 - p_R^*(\gamma) - tx_R)p_R^*(\gamma)$. The equilibrium price increases in γ and s , and it increases in t whenever s is sufficiently small relative to t . An increase in γ can increase or decrease firms' equilibrium profits. The profit-maximizing commission is attained by $\gamma^* = \left(\frac{2/t + \sqrt{1-4(s/t)} - 3}{2/t - \sqrt{1-4(s/t)} - 1} \right) \in (0, 1)$ if $t < 2/3$ or $s < (1 - 2t) \left(1 - \frac{1}{t}\right)$.*

¹³Note simultaneously that mild referral fees give stores strictly positive incentives to make referrals, unlike our basic model.

¹⁴If $\bar{v}_R(\gamma) = p_R^*(\gamma) + tx_R \leq 1$, the existence of equilibrium is guaranteed. The inequality holds if and only if $t < 2$, $s \leq \frac{t}{4}$, and $\gamma \leq 1 - \frac{1}{2/t-1} \left(1 - \sqrt{1-4(s/t)}\right)$.

Proof. Due to referral competition, firms set the same referral fee $\gamma_i = \gamma$. With a commission γ , firm i 's profit function is a sum of profits from consumers buying from firm i on their first visit, on their visit by referral, and the payments from other firms for the referrals the firm makes. A measure $(1 - p - tx_R)$ of consumers participates in the market since only consumers with values exceeding $p + tx_R$ initiate search. Firm i sells to $2x_R(p_i, p)$ of the searchers who visit firm i first and collects referral payments γp for the rest of consumers who visit firm i first. A measure $(1 - 2x_R)$ of consumers who first visit another firm gets referrals to firm i , and firm i receives $(1 - \gamma)p_i$ for each of the referral customers. Therefore, the profits can be written as

$$\begin{aligned}
\pi_i(p_i, p; \gamma) &= (1 - p - tx_R) (2p_i x_R(p_i, p) + \gamma p (1 - 2x_R(p_i, p))) + (1 - 2x_R)(1 - \gamma)p_i & (22) \\
&= (1 - p - tx_R) \left(2p_i \left(x_R - \frac{p_i - p}{t} \right) + \gamma p \left(1 - 2x_R + \frac{2(p_i - p)}{t} \right) \right) + (1 - 2x_R)(1 - \gamma)p_i \\
&= (1 - p - tx_R) \left(p_i - 2p_i \frac{p_i - p}{t} + \gamma p \left(1 - 2x_R + \frac{2(p_i - p)}{t} \right) - \gamma(1 - 2x_R)p_i \right) \\
&= (1 - p - tx_R) \left(p_i - \frac{2(p_i - p)}{t} (p_i - p\gamma) - \gamma(p_i - p)(1 - 2x_R) \right)
\end{aligned}$$

if $p_i \leq p + tx_R$, and $\pi_i(p_i, p) = 0$ otherwise. The first-order condition is then

$$\frac{\partial \pi_i}{\partial p_i} = (1 - p - tx_R) (1 - (1 - 2x_R)\gamma - 2(p_i - p)/t - (2p_i - 2\gamma p)/t) = 0. \quad (23)$$

Thus, the equilibrium price, given commission γ , is $p_R^*(\gamma) = \frac{t}{2(1-\gamma)}(1 - (1 - 2x_R)\gamma) = \frac{t}{2} \left(\frac{1-\gamma\sqrt{1-4(s/t)}}{1-\gamma} \right)$. It is easy to see that $dp_R^*(\gamma)/d\gamma > 0$ and $\partial p_R^*(\gamma)/\partial s < 0$. The sign of $\partial p_R^*(\gamma)/\partial t$ is the same as the sign of $\sqrt{1 - 4(s/t)} + (2(s/t) - 1)\gamma$. If $2(s/t) - 1 > 0$, then $\partial p_R^*(\gamma)/\partial t > 0$. If $2(s/t) - 1 \leq 0$, then $\sqrt{1 - 4(s/t)} - (1 - 2(s/t))\gamma > \sqrt{1 - 4(s/t)} - 1 + 2(s/t) \geq 0$. Solving the inequality, we find that $\partial p_R^*(\gamma)/\partial t > 0$ if and only if $(s/t) < \frac{1}{2\gamma^2} \sqrt{1 - \gamma^2} (1 - \sqrt{1 - \gamma^2})$. The critical level for search cost relative to product heterogene-

ity decreases in γ , varying from $\frac{1}{4}$ for $\gamma \rightarrow 0$ to 0 for $\gamma \rightarrow 1$.

Since $p_R^*(\gamma)$ is monotonically increasing in γ , if $p_R^*(\gamma) = \frac{1-tx_R}{2}$ can be attained for some γ then the maximum profit is achieved. Resolving $p_R^*(\gamma) = \frac{1-tx_R}{2}$ in terms of γ , we obtain $\gamma^* = \left(2/t + \sqrt{1 - 4(s/t)} - 3\right) / \left(2/t - \sqrt{1 - 4(s/t)} - 1\right)$, where $2/t - \sqrt{1 - 4(s/t)} - 1 \neq 0$ is assumed. Consider first the numerator $a = 2/t + \sqrt{1 - 4(s/t)} - 3$. It is positive whenever $\sqrt{1 - 4(s/t)} > 3 - 2/t$. This inequality always holds for $t < 2/3$. For $t \geq 2/3$, it is equivalent to $1 - 4(s/t) > (3 - 2/t)^2$, or $s < (1 - 2t) \left(1 - \frac{1}{t}\right)$. For the latter inequality to hold it is necessary that $t < 1$. Next, consider the denominator $b = 2/t - \sqrt{1 - 4(s/t)} - 1$. It is positive whenever $2/t - 1 > \sqrt{1 - 4(s/t)}$. Since $t \leq 2$, this is equivalent to $(2/t - 1)^2 > 1 - 4(s/t)$, or $s > 1 - \frac{1}{t}$. This always holds for $t \leq 1$. Hence, $\gamma^* > 0$ when $t < 2/3$, or $s < (1 - 2t) \left(1 - \frac{1}{t}\right)$, or $t > 1$ and $s < 1 - \frac{1}{t}$. We next show that $\gamma^* < 1$ if and only if $b > 0$. For $b > 0$, $\gamma^* < 1$ is equivalent to $2/t + \sqrt{1 - 4(s/t)} - 3 < 2/t - \sqrt{1 - 4(s/t)} - 1$, or $\sqrt{1 - 4(s/t)} < 1$, which is always true. For $b < 0$, the inequality is reversed and therefore $\gamma^* < 1$ can never hold. Hence, an interior optimal γ^* exists whenever $a > 0$ and $b > 0$. Note that $a > 0$ implies $b > 0$. Summing up the results, $\gamma^* \in (0, 1)$ if and only if $a > 0$, i.e. for $t < 2/3$ or $s < (1 - 2t) \left(1 - \frac{1}{t}\right)$. ■

2. Third-Party Referrals

Let us now introduce third-party referral agents (brokers), who do not sell products but specialize in referral services, taking c or γ as referral fees in dollars or in fractions (as a commission), respectively. Suppose that there are referral agents of measure $\alpha > 0$. Thus, now the total measure of firms and brokers is $1 + \alpha$, and consumers visit them randomly. To maintain symmetry of search costs, we assume that the first visit costs $s > 0$, even if a

consumer visits a broker.¹⁵ If a consumer visits a broker, she gets an honest referral with probability one, whereas if she visits a firm then she does not get a referral with probability $2x_R$ (and she purchases a product at the firm). Note that the broker is a passive player in this model. She makes honest referrals to consumers who visit her, and charges referral fees c to firms. In contrast, a firm maximizes the following profit function for cases of caps in dollars and fractions, respectively:

$$\begin{aligned}\pi_i(p_i, p; c, \alpha) &= \frac{1 - p - tx_R(\alpha)}{1 + \alpha} [2x_{RT}(p_i, p)p_i + \alpha(p_i - c) + (1 - 2x_R(\alpha))(p_i - c)] \quad (24) \\ &= \frac{1 - p - tx_R(\alpha)}{1 + \alpha} \left[p_i \left(1 - \frac{2(p_i - p)}{t} \right) + c \times \frac{2(p_i - p)}{t} + \alpha(p_i - c) \right]\end{aligned}$$

and

$$\pi_i(p_i, p; \gamma, \alpha) = \frac{1 - p - tx_R(\alpha)}{1 + \alpha} \left[p_i - (1 - 2x_R(\alpha))\gamma(p_i - p) - \frac{2(p_i - \gamma p)(p_i - p)}{t} + \alpha(1 - \gamma)p_i \right]. \quad (25)$$

The following proposition characterizes the referral equilibrium in the presence of referral payments and third-party brokers.

Proposition 7. *Suppose that referral fees are allowed, there are referral brokers with measure α , and there is an exogenous cap $c > 0$ on referral fees (or there is an exogenous cap on commission rates $\gamma \in (0, 1)$). If the cap is small and $s \leq \frac{(1+\alpha)t}{4}$, then $p_{RT}^*(c, \alpha) = \frac{t(1+\alpha)}{2} + c$ ($p_{RT}^*(\gamma, \alpha) = \frac{t}{2} \left(\frac{1-\gamma\sqrt{1-4s/t}}{1-\gamma} + \alpha \right)$), $x_R(\alpha) = \frac{1+\alpha}{2} - \frac{1+\alpha}{2} \sqrt{1 - \frac{4s}{(1+\alpha)t}}$, and $\bar{v}_{RT}(c, \alpha) = p_{RT}^*(c, \alpha) + tx_R(\alpha)$ ($\bar{v}_{RT}(\gamma, \alpha) = p_{RT}^*(\gamma, \alpha) + tx_R(\alpha)$). The critical referral distance $x_R(\alpha)$ decreases in the measure of brokers, α , while the equilibrium price $p_{RT}^*(c, \alpha)$ ($p_{RT}^*(\gamma, \alpha)$)*

¹⁵A consumer needs to incur a search cost s in order to find the location of a store or a broker anyway.

increases in α . The critical reservation utility values for market participation increases (thus market participation decreases) in α in most cases. An increase in α can increase or decrease firms' equilibrium profits.

Proof. In the case of a flat referral fee c , the first-order condition is $\frac{\partial \pi_i}{\partial p_i} = \left(\frac{1-p-tx_R}{1+\alpha}\right) \left(1 - \frac{4p_i-2p}{t} + \frac{2c}{t} + \alpha\right) = 0$. Thus, the equilibrium price given a referral fee c is $p_{RT}^*(c) = \frac{t(1+\alpha)}{2} + c$. Similarly, the first-order condition in the case of a percentage referral fee γ includes an additional marginal benefit $\alpha(1-\gamma)$, and the price is $\alpha\frac{t}{2}$ higher than $p_R^*(\gamma)$ of Proposition 6. With the third-party brokers, the value of x_R is affected, unlike the case with c or γ only. Equation (9), which shows the expected payoff from engaging in one additional search, is modified as

$$\begin{aligned} & \Delta EU(x; x_R; \alpha) \tag{26} \\ &= \frac{\alpha}{1+\alpha}(v-p) + \frac{1}{1+\alpha} \left[(1-2x_R)(v-p) + 2(x_R-x)(v-p-tx) + 2 \int_0^x (v-p-tx') dx' \right] \\ & \quad -s - (v-p-tx) \\ &= \left(1 - \frac{1}{1+\alpha} 2x_R\right)tx + \frac{1}{1+\alpha}tx^2 - s. \end{aligned}$$

It is easy to see that for any $x_R \in [0, \frac{1+\alpha}{2}]$ and any $x \in [0, x_R]$, we have $\partial \Delta EU(x; x_R) / \partial x = t(1 - 2x_R/(1+\alpha) + 2x/(1+\alpha)) \geq 0$. This means that as long as $\Delta EU(x_R; x_R) \leq 0$, every consumer who has $x < x_R$ would not engage in an additional search.

Now, let $\Delta EU(x_R; x_R; \alpha) = 0$: that is, a consumer who visited a firm x_R apart from her ideal position is indifferent about searching (given that all other firms are choosing referral

rule x_R):

$$\begin{aligned}\Delta EU(x_R; x_R; \alpha) &= \left(1 - \frac{1}{1+\alpha} 2x_R\right)tx_R + \frac{1}{1+\alpha}tx_R^2 - s \\ &= tx_R - \frac{1}{1+\alpha}tx_R^2 - s = 0,\end{aligned}\tag{27}$$

or

$$x_R(\alpha) = \frac{1+\alpha}{2} - \frac{1+\alpha}{2}\sqrt{1 - \frac{4s}{(1+\alpha)t}}\tag{28}$$

for $(1+\alpha)t \geq 4s$. This value $x_R(\alpha)$ describes the symmetric equilibrium referral rule. Now,

we calculate

$$\begin{aligned}EU_R(v, p; \alpha) &= \frac{\alpha}{1+\alpha}(v-p) + \frac{1}{1+\alpha}\left[2\int_0^{x_R}(v-tx-p)dx + (1-2x_R(\alpha))(v-p)\right] - s \\ &= \frac{\alpha}{1+\alpha}(v-p) + \frac{1}{1+\alpha}\left[(v-p) - t(x_R(\alpha))^2\right] - s \\ &= v-p - \frac{t(x_R(\alpha))^2}{1+\alpha} - s \\ &= v-p - tx_R(\alpha)\end{aligned}\tag{29}$$

Thus, the critical value for market participation is $\bar{v}_R(c; \alpha) = p_R(c; \alpha) + tx_R(\alpha)$ and $\bar{v}_R(\gamma; \alpha) = p_R(\gamma; \alpha) + tx_R(\alpha)$. To determine how $\bar{v}_R(c; \alpha)$ and $\bar{v}_R(\gamma; \alpha)$ are affected by α , we first examine how $x_R(\alpha)$ responds to an increase in α . Calculations reveal that

$$\frac{dx_R}{d\alpha} = \frac{1}{2}\left[1 - \frac{\sqrt{(1+\alpha)^2t^2 - 4s(1+\alpha)t + 4s^2}}{\sqrt{(1+\alpha)^2t^2 - 4s(1+\alpha)t}}\right] < 0.\tag{30}$$

Finally, we check the effects on $\bar{v}_R(c; \alpha)$ and $\bar{v}_R(\gamma; \alpha)$. Since $\frac{dp_{RT}^*(c, \alpha)}{d\alpha} = \frac{dp_{RT}^*(c, \gamma)}{d\alpha} = \frac{t}{2}$, we have

$$\frac{d\bar{v}_R(c; \alpha)}{d\alpha} = \frac{d\bar{v}_R(\gamma; \alpha)}{d\alpha} = \frac{t}{2}\left[2 - \frac{\sqrt{(1+\alpha)^2t^2 - 4s(1+\alpha)t + 4s^2}}{\sqrt{(1+\alpha)^2t^2 - 4s(1+\alpha)t}}\right].\tag{31}$$

Thus, $\frac{d\bar{v}_R(c; \alpha)}{d\alpha} = \frac{d\bar{v}_R(\gamma; \alpha)}{d\alpha} \gtrless 0$ if and only if $4((1+\alpha)^2t^2 - 4s(1+\alpha)t) - ((1+\alpha)^2t^2 - 4s(1+\alpha)t) \gtrless 0$.

$\alpha)t + 4s^2) \geq 0$. Straightforward calculations yield:

$$\begin{aligned}
& 4((1 + \alpha)^2 t^2 - 4s(1 + \alpha)t) - ((1 + \alpha)^2 t^2 - 4s(1 + \alpha)t + 4s^2) \tag{32} \\
&= 4((1 + \alpha)t - 2s)^2 - 16s^2 - ((1 + \alpha)t - 2s)^2 \\
&= 3 \left[((1 + \alpha)t - 2s)^2 - \frac{16}{3}s^2 \right] \\
&= 3((1 + \alpha)t - 2s - \frac{4}{\sqrt{3}}s)((1 + \alpha)t - 2s + \frac{4}{\sqrt{3}}s).
\end{aligned}$$

Since $(1 + \alpha)t - 4s \geq 0$ by equilibrium condition, $\frac{d\bar{v}_R(c;\alpha)}{d\alpha} = \frac{d\bar{v}_R(\gamma;\alpha)}{d\alpha} \geq 0$ if and only if $(1 + \alpha)t - (2 + \frac{4\sqrt{3}}{3})s \geq 0$. Thus, if $s < \frac{(1 + \alpha)t}{2 + \frac{4\sqrt{3}}{3}} = (\sqrt{3} - \frac{3}{2})(1 + \alpha)t$, then the critical value goes up and market participation goes down with α . However, if $s \in ((\sqrt{3} - \frac{3}{2})(1 + \alpha)t, \frac{(1 + \alpha)t}{4}] \approx (0.23205t(1 + \alpha), 0.25t(1 + \alpha)]$, market participation increases. ■

Proposition 7 shows that equilibrium price increases even further with third-party referrals. The reason is that the demand curve a firm faces becomes steeper, since the firm has more perfect-match consumers for whom it does not compete.¹⁶ Note that this result follows even if $c = 0$ or $\gamma = 0$. Although there are cost-saving gains for consumers from having third-party referrals, an increase in α usually hurts consumers through equilibrium price increases unless α is small and search cost is relatively high. The market size, $1 - p_{RT}^*(c) - tx_R$, may shrink a lot as a result of a high equilibrium price. The presence of brokers affects the referral intensity in an intuitive way: all else equal, brokers render consumer search more attractive, prompting firms to refer more consumers to other firms.

From total welfare point of view, there is a trade-off between moral hazard in referral activity and a hold-up problem in pricing. Third-party referrals ameliorate the moral-hazard problem in referral activity since the third-party brokers always immediately match their

¹⁶This effect is first observed in Anderson and Renault (2000).

customers with firms that sell their ideal commodities, while the ordinary firms may try to sell their commodities to consumers even if their commodities are not perfect matches. At the same time, third-party referrals worsen the hold-up problem present in search markets. Since firms cannot precommit to prices before consumers incur search costs, they are tempted to charge a high price. Consumers anticipate that event in the equilibrium and thus do not initiate search in the first place. For very high search costs, consumers favor markets with brokers because in such markets the gains in referral efficiency outweigh the losses of a price increase.

In a number of industries, members of professional organizations resist paying referral fees to non members, arguing that industry profits will be siphoned by outsiders. According to Proposition 7, however, the existence of brokers can increase the market price; the effect on profit of each (non broker) firm is unknown, since the market size is likely to be reduced by an increase in α , but the equilibrium price (profit margin) is increased. Thus, as long as c or γ is not too high, each firm's profit can in fact increase with the presence of brokers.

4 Conclusion

We consider an environment where consumers need to conduct a costly search to gain information on products or services offered in a horizontally differentiated market. Examples range from real estates and other professional services to an antique market or high-tech consumer products. In the framework of the search market, we study the practice of referring consumers to competing sellers. In the basic model, there are no referral fees, and thus it seems unlikely that a firm could benefit from referring a consumer to its competitor. A firm would give a referral to a consumer only if the firm cannot sell its product to her. Perhaps

surprisingly, we find that the custom of giving honest referrals raises equilibrium prices and thus can, in fact, be favored by sellers and trade organizations.

Although given set prices, referrals benefit consumers because they improve the product-match quality and save buyers' search costs, in the equilibrium the referral practice can actually hurt consumers when search costs are very low relative to product heterogeneity. In the search market without referrals, the equilibrium price becomes trivial for very low search costs, whereas in referral equilibrium prices are insensitive to search costs. The possibility that referrals benefit firms and hurt consumers is somewhat paradoxical, and it is only realized for very low search costs. We find that referrals are usually beneficial for consumers. Referrals increase profits and improve total welfare even under broader conditions – as long as the market does not contract too much in response to higher prices.

Referral fees give sellers strong incentives to refer a consumer to other sellers when they cannot sell to the consumer, but the equilibrium price increases with the fees. Common referral fees (achieved, for example, as an outcome of a referral fee competition in markets with caps on referral fees) do not affect the referral intensity. Thus consumers and consumer protection agencies would like to see referral fees banned, provided the no-fee referral equilibrium exists and the entry of third-party brokers is not discouraged. (For example, if there is a cost to giving referrals, a cap on referral fees would allow firms to recover that cost.) The same result holds for referral payments in the form of commissions. Finally, we look at the role of third-party brokers who do not sell the product but are simply in the business of matching buyers and sellers in exchange for a referral payment. Brokers further improve the match quality because they always provide consumers with the best match, and their presence further increases the equilibrium price. Hence, there is a welfare trade-off between

an efficiency gain due to lessened moral hazard in referrals and an efficiency loss due to deterioration in the hold-up problem of high prices in search markets.

The analysis sheds light on a number of policy-relevant questions. For example, why would professional lawyers, accountants, physicians, and real estate agents favor a prohibition of referral fees? One explanation could be that these professionals are afraid that referral fees may lead to a conflict of interest and loss of consumer trust.¹⁷ The argument is that this cannot be to the benefit of consumers and, in the end, is not in the interest of the service provider. This paper shows that when referrals are not trustworthy and thus consumers rationally discount them, in the resulting random search equilibrium, prices and profits are lower. Therefore, firms favor the equilibrium with truthful referrals, given the relaxing effect referrals have on price competition. Although referrals may or may not benefit consumers, the referral equilibrium is a Pareto improvement over the random search equilibrium as long as consumer search costs are not too low relative to product heterogeneity.

¹⁷Another explanation is that tight caps on referral fees discourage entry of brokers, which could benefit the industry. The argument is similar to that in Colwell and Kahn (2001). The authors look at third-party referrals made by a middleman and argue that the prohibition on referral fees discourages the entry of intermediaries. They also find that consumers may prefer non disclosure of referral fees to full disclosure.

Appendix

Here we show that there is no symmetric equilibrium in which consumers search once and for all. Suppose, to the contrary, there exists an equilibrium with a symmetric price p in which consumers search once at most. If there is a consumer with value $v \geq p + \frac{t}{2}$ who participates in the initial search, then the consumer's willingness-to-pay is at least $p + t/4 + s$, because her expected payoff from searching is

$$2 \int_0^{\frac{1}{2}} (v - p - tx) dx - s = v - p - \frac{t}{4} - s, \quad (\text{A1})$$

and hence she engages in the initial search if and only if $v \geq p + \frac{t}{4} + s$.

If there is a consumer with value $v < p + \frac{t}{2}$ who participates in the initial search, then the consumer's willingness-to-pay is at least $p + \sqrt{ts}$ because her expected utility from search is

$$2 \int_0^{\frac{v-p}{t}} (v - p - tx) dx - s = \frac{(v - p)^2}{t} - s, \quad (\text{A2})$$

and hence she engages in the initial search if and only if $v \geq p + \sqrt{st}$.

This leads to a version of the Wernerfelt paradox (Wernerfelt, 1994). Let $\beta = \min(t/2, \sqrt{ts}) > 0$. Given that consumers expect price p , as long as the charged price (actual price) at a visited firm is less than or equal to $p + \beta$ then the participating consumers who visited the firm all purchase the product. Thus, the demand curve is vertical at least between p and $p + \beta$, and all firms have incentive to raise their prices from p (as long as $p \leq 1$). Therefore, there is no equilibrium price in which consumers engage in search once and for all. ■

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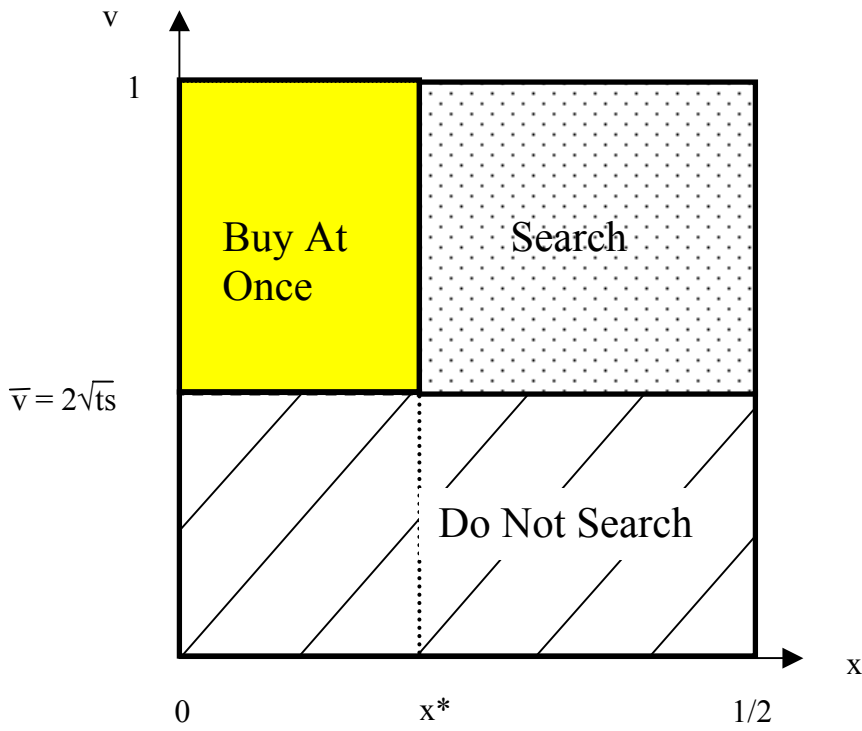
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Technical Details (Not for Publication)

Equation (30)

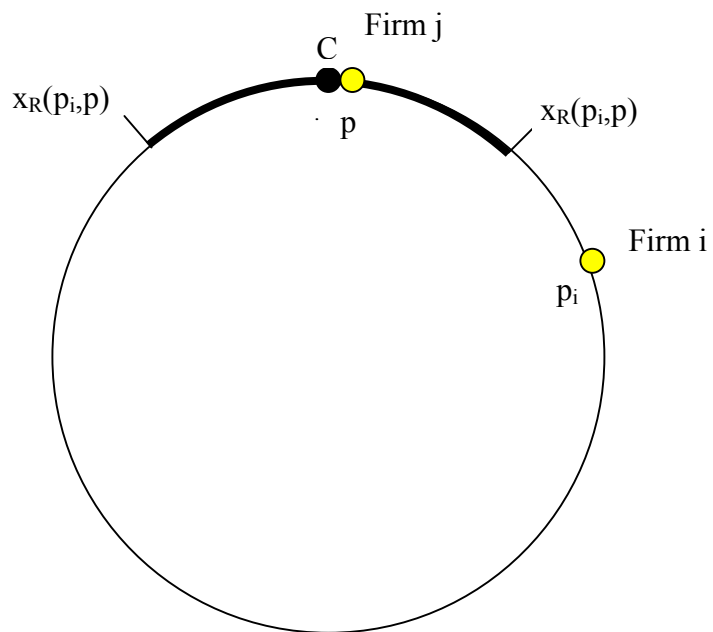
$$\begin{aligned}
 \frac{dx_R}{d\alpha} &= \frac{1}{2} \left(1 - \sqrt{1 - \frac{4s}{(1+\alpha)t}} \right) + \frac{1+\alpha}{4} \left(1 - \frac{4s}{(1+\alpha)t} \right)^{-\frac{1}{2}} \frac{4s}{(1+\alpha)^2 t} & (30) \\
 &= \frac{1}{2} \left[1 - \sqrt{1 - \frac{4s}{(1+\alpha)t}} - \frac{1}{2} \left(1 - \frac{4s}{(1+\alpha)t} \right)^{-\frac{1}{2}} \frac{4s}{(1+\alpha)t} \right] \\
 &= \frac{1}{2} \left[1 - \sqrt{1 - \frac{4s}{(1+\alpha)t}} \left(1 + \frac{1}{2} \left(1 - \frac{4s}{(1+\alpha)t} \right)^{-1} \frac{4s}{(1+\alpha)t} \right) \right] \\
 &= \frac{1}{2} \left[1 - \sqrt{1 - \frac{4s}{(1+\alpha)t}} \left(\frac{1 - \frac{4s}{(1+\alpha)t} + \frac{2s}{(1+\alpha)t}}{1 - \frac{4s}{(1+\alpha)t}} \right) \right] \\
 &= \frac{1}{2} \left[1 - \sqrt{\frac{(1+\alpha)t - 4s}{(1+\alpha)t}} \left(\frac{(1+\alpha)t - 2s}{(1+\alpha)t - 4s} \right) \right] \\
 &= \frac{1}{2} \left[1 - \frac{(1+\alpha)t - 2s}{\sqrt{(1+\alpha)t} \sqrt{(1+\alpha)t - 4s}} \right] \\
 &= \frac{1}{2} \left[1 - \frac{\sqrt{(1+\alpha)^2 t^2 - 4s(1+\alpha)t + 4s^2}}{\sqrt{(1+\alpha)^2 t^2 - 4s(1+\alpha)t}} \right] < 0.
 \end{aligned}$$

Figure 1. Consumer Decisions in Random Search Equilibrium



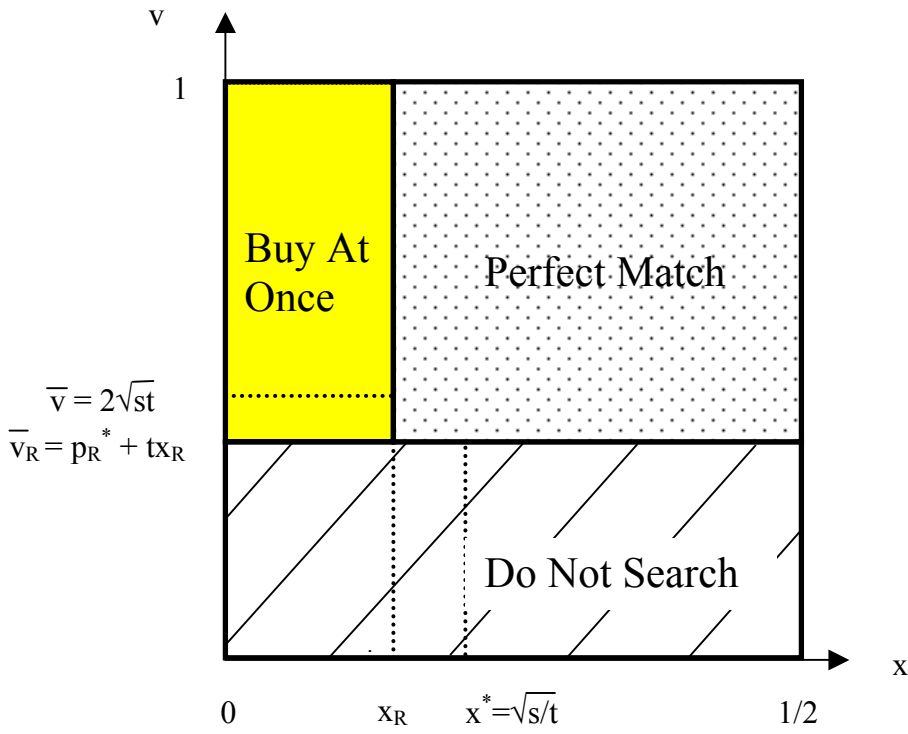
Notes: In the region labeled "Buy At Once" consumers receive utility (gross of search costs) $v - tx - p^*$. In the region labeled "Search" consumers receive utility $v - \sqrt{ts} - p^*$. In the region labeled "Do Not Search" consumers receive zero utility.

Figure 2. Search with Referrals



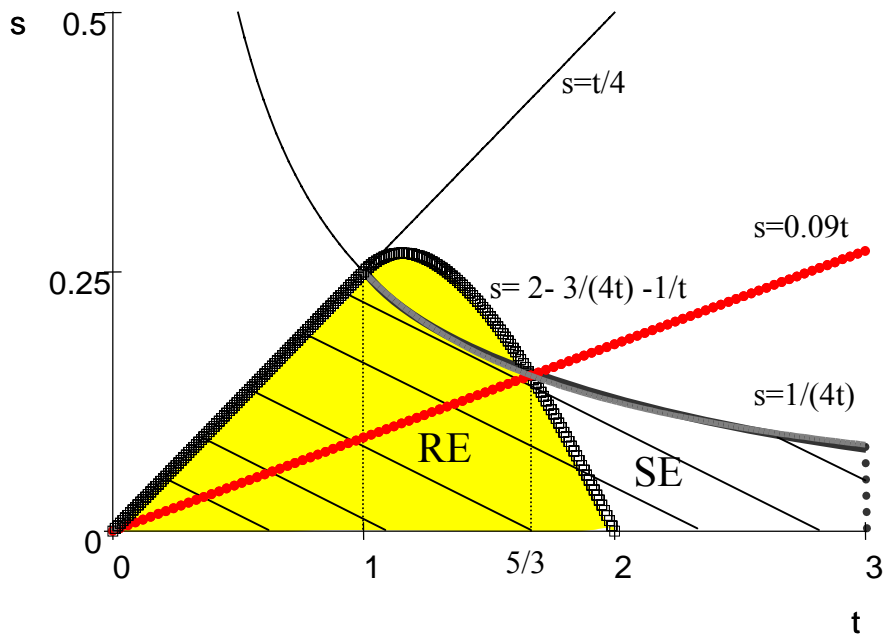
Notes: A consumer located at point C is assigned to firm i , and has to decide whether to follow the referral (if it is given), buy at firm i , continue random search, or leave the market. Firm i refers the consumer to firm j (perfect match) if and only if the consumer is located at least $x_R(p_i, p)$ away.

Figure 3. Consumer Decisions in Referral Equilibrium.



Notes: In the region labeled “Buy At Once” consumers receive utility (gross of search costs) $v - p_R^* - tx$. In the region labeled “Perfect Match” consumers receive utility $v - p_R^*$. In the region labeled “Do Not Search” consumers receive zero utility.

Figure 4. Existence of Random Search and Referral Equilibria



Notes: Random search equilibrium (SE) exists for $s \leq \min\{1/(4t), t/4\}$, while referral equilibrium (RE) exists whenever i) $t \leq 1$ and $s \leq t/4$ or ii) $t > 1$ and $s \leq 2 - 3/(4t) - 1/t$. When the two equilibria exist, referrals lead to a Pareto improvement for $s \geq 0.09t$.