Math 202 Multivariable Calculus
Exam 2 SOLUTIONS

The first two questions are worth 10 points each, and rest are worth 20 points.

1. Calculate the line integral $\int_{\mathbf{c}} y^2\, dx + x\, dy$, where $\mathbf{c}$ is the line segment from $(1,0)$ to $(2,3)$.

SOLUTION: $c(t) = (1+t, 3t),\ 0 \le t \le 1$. answer: $15/2$

2. Calculate the line integral $\int_{\mathbf{c}} x\, dx + y\, dy$, where $\mathbf{c}$ is the following path:

SOLUTION: $Q_x - P_y = 0$, so $\mathbf{F}$ is conservative. You could replace the squiggly path by a line, or find the potential: $f(x,y) = (1/2)(x^2+y^2)$, and the line integral is $f(2,3) - f(1,1) = 11/2$.

3. Calculate the flux $\int_{\mathbf{c}} \mathbf{F} \cdot \mathbf{N}\, ds$ where $\mathbf{F} = (x, 0)$, $\mathbf{c}$ is the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

SOLUTION:

$$\int_{\mathbf{c}} Q\, dx - P\, dy = \int_{\mathbf{c}} -x\, dy = -6\pi.$$

4. Calculate the integral $\iint_R x^2 y^2\, dR$, where R is the unit disk centered at $(0,0)$.

SOLUTION:

$$\iint_R x^2 y^2\, dR = \int_0^{2\pi} \int_0^1 (r\cos\theta)^2 (r\sin\theta)^2 r\, dr d\theta = \pi/24.$$

5. Calculate the integral $\iint_R y^2\, dR$, where R is the parallelogram with vertices $(0,0)$, $(2,0)$, $(2,1)$, $(4,1)$. (Use a linear mapping.)

SOLUTION: $R(u,v) = (2u + 2v, v)$, Jacobian $= 2$,

$$\iint_R y^2\, dR = 2\int_0^1 \int_0^1 v^2\, du dv = \frac{2}{3}.$$

6. Find the area of the four-sided polygon with vertices $(0,0)$, $(1,0)$, $(2,4)$, $(3,3)$, by computing a certain line integral.

SOLUTION: We integrate $x\, dy$ over the four sides. The side from $(0,0)$ to $(1,0)$ gives zero since $dy = 0$. The other sides are $\mathbf{c}_2(t) = (1+2t, 3t)$, $\mathbf{c}_3(t) = (3-t, 3+t)$, $\mathbf{c}_4(t) = (2-2t, 4-4t)$, all $0 \le t \le 1$. The answer is $\frac{9}{2}$.

MANY OF YOU DID NOT PARAMETRIZE THE SEGMENTS CORRECTLY.
LEARN THIS, PLEASE.
OTHERWISE, THE EXAM WENT VERY WELL.