Linear Algebra Notes

Chapter 3

SOLUTIONS TO EXERCISES

Exercise 3.1. *Prove or disprove:*

(a) $\operatorname{tr}(AB) = \operatorname{tr}(A)\operatorname{tr}(B)$.
(b) $\operatorname{tr}(AB) = \operatorname{tr}(BA)$.
(c) $\operatorname{tr}(A+B) = \operatorname{tr}(A) + \operatorname{tr}(B)$.
(d) $\operatorname{tr}(A^{-1}) = \frac{1}{\operatorname{tr}(A)}$.
(e) $\operatorname{tr}(xA) = x\operatorname{tr}(A)$. *(Here x is a scalar.)*
(f) $\det(xA) = x\det(A)$.
(g) $\det(A+B) = \det(A) + \det(B)$.
(h) $\det(xI - A) = x^2 - \operatorname{tr}(A)x + \det(A)$.

Answers:

a) False. Take any two scalar matrices, for example.

b) True. $\operatorname{tr}(\begin{bmatrix} a & b \\ e & d \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix}) = ax + bz + cy + dw$, which is unchanged if you switch the matrices.

c) True. $\operatorname{tr}(\begin{bmatrix} a & b \\ e & d \end{bmatrix} + \begin{bmatrix} x & y \\ z & w \end{bmatrix}) = a+x+d+w$, which is unchanged if you switch the matrices.

d) False, even for the identity matrix.

e) True. $\operatorname{tr}(t\begin{bmatrix} a & b \\ e & d \end{bmatrix}) = ta + td = t(a+d) = t\operatorname{tr}(\begin{bmatrix} a & b \\ e & d \end{bmatrix})$.

f) False, even for the identity matrix. What is true: $\det(xA) = x^2\det(A)$.

g) False. Take, for example, $A = I$, $B = -I$.

h) True. Just compute it.