Chapter 4

SOLUTIONS TO EXERCISES

Exercise 4.1.

(a) *Find the matrix sends* $(1,0)$ *to* $(1,2)$ *and sends* $(0,1)$ *to* $(2,1)$.
(b) *Find the matrix that sends* $(1,0)$ *to* $(2,1)$ *and sends* $(0,1)$ *to* $(1,1)$.
(c) *Find the matrix that sends* $(2,1)$ *to* $(1,2)$ *and sends* $(1,1)$ *to* $(2,1)$. *Hint for (c): Use (a) and the inverse of the matrix in (b).*

(a) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$,

(b) $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & 0 \end{bmatrix}$.

Exercise 4.2. *A octagon has eight vertices, starting at* $(1,0)$ *and rotating by multiples of* $\pi/4$. *Compute the matrix* A *that does this rotation, and then compute* $A,\ A^2, \ldots, A^7$. *Plot the first columns of these matrices. You should get to find the coordinates of the remaining seven vertices of the octagon.*

$A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$,

$A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$,

$A^3 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$,

$A^4 = -I,\ A^5 = -A,\ A^6 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$,

$A^7 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$.

Exercise 4.3. *Find the matrix that rotates by* $\pi/4$ *and then reflects about the line* $y = x$.

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

Exercise 4.4. *Find the matrix that rotates by* $3\pi/4$ *and then reflects about the line* $y = x$.

$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$.

Exercise 4.5. *If you take all the points on a line through the origin, and multiply them by a matrix, you will get another line. Let* $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. *Describe how* A

moves the lines through the origin. (Hint: Start with the lines through $\mathbf{e}_1$ *and* $\mathbf{e}_2$*, and then consider a line with slope* $m \neq 0, \infty$*.)*

A sends the line with slope m to the line with slope $m+1$, and it fixes the y-axis (infinite slope).