Linear Algebra
Chapter 8
Eigenvalues and Eigenvectors
solutions to exercises

Exercise 8.1. *Find the eigenvalues and eigenvectors of the following matrices.*

a)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Solution: $\lambda = 1, \mu = -1, \mathbf{u} = (1, 1), \mathbf{v} = (1, -1)$.

b)

$$A = \begin{bmatrix} 3 & 4 \\ 2 & -3 \end{bmatrix}$$

Solution: $\lambda = \sqrt{17}, \mu = -\sqrt{17}, \mathbf{u} = (4, \sqrt{17} - 3), \mathbf{v} = (4, -\sqrt{17} - 3)$.

c)

$$A = \begin{bmatrix} 3 & 0 \\ 2 & -3 \end{bmatrix}$$

Solution: $\lambda = 3, \mu = -3, \mathbf{u} = (3, 1), \mathbf{v} = (0, 1)$.

d)

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Solution: $\lambda = \frac{1}{2}[3 + \sqrt{5}], \mu = \frac{1}{2}[3 - \sqrt{5}], \mathbf{u} = (1, \frac{1}{2}[1 - \sqrt{5}]), \mathbf{v} = (1, \frac{1}{2}[1 + \sqrt{5}])$.

e)

$$A = \begin{bmatrix} 15 & -10 \\ 21 & -14 \end{bmatrix}$$

Solution: $\lambda = 1, \mu = 0, \mathbf{u} = (5, 7), \mathbf{v} = (2, 3)$.

Exercise 8.2. *Suppose A is a reflection matrix*

$$A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}, \quad a^2 + b^2 = 1.$$

a) *Find the eigenvalues and eigenvectors of A.*
b) *Find a vector on the reflecting line of A.*
c) *Find a vector perpendicular to the reflecting line of A.*
All of your answers will involve a and b.

Solution: $\lambda = 1, \mu = -1, \mathbf{u} = (b, 1-a)$ is on the reflecting line, and $\mathbf{v} = (b, -1-a)$ is perpendicular to the reflecting line.

Exercise 8.3. *Suppose A is a matrix whose entries you do not know, but you do know that its eigenvalues are $\lambda = 2$ and $\mu = 3$, with corresponding eigenvectors* $\mathbf{u} = (1, 3),\ \mathbf{v} = (6, -1)$.

a) Find A.

Solution:

$$A = \frac{1}{19} \begin{bmatrix} 56 & -6 \\ -3 & 39 \end{bmatrix}.$$

b) Find the eigenvalues and eigenvectors of A^{100}.

Solution: λ^{100}, μ^{100}, *same eigenvectors as A.*

Exercise 8.4. *Let*

$$A = \begin{bmatrix} 16 & -10 \\ 21 & -13 \end{bmatrix}.$$

Find an explicit formula for A^n.

Solution:

$$A^n = \begin{bmatrix} 15 \cdot 2^n - 14 & 10(1 - 2^n) \\ 21(2^n - 1) & 15 - 14 \cdot 2^n \end{bmatrix}.$$