Linear Algebra
Study problems for Exam 1

Note: These problems are not in any particular order.

1. a) Suppose the matrix A has eigenvalues 2 and 3. What is the characteristic polynomial of A^{-1}?

b) Suppose $A^2 = I$. What are the possible characteristic polynomials of A? (There are three possibilities.)

c) Suppose A has eigenvalues 2 and 3. What are the eigenvalues of A^T?

2. Find the matrix of the linear map L which sends $(1,1)$ to $(-1,-1)$, and $(1,0)$ to $(1,0)$.

3. Suppose that A is a rotation matrix and B is a reflection matrix. Show that $BAB^{-1} = A^{-1}$. (Hint: What kind of matrix is BA?)

4. Find the matrix of the reflection about the line $y = 3x$. Your answer should have rational entries. (One method: You know that $A_L = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$, with $a^2 + b^2 = 1$. You also know the eigenvectors of A_L. Another method: $\theta = \arctan 3$; use double angle identities.)

5.

a) Suppose A has $\lambda = 0$ as an eigenvalue. What is $\det A$?

b) Suppose both eigenvalues of A are zero. What is $\operatorname{tr} A$?

c) Find a matrix other than $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, both of whose eigenvalues are 0.

6. Express the vector $\mathbf{w} = (3,7)$ as a linear combination of $\mathbf{u} = (1,2)$ and $\mathbf{v} = (1,3)$. (Make up new vectors and do more of these.)

7. Two lines ℓ and ℓ' through the origin make an angle of $\pi/4$, going counterclockwise from ℓ to ℓ'. Let A and A' be the corresponding reflection matrices. Find the matrix AA'.

8. Consider the following sequence of numbers:

$$x_0 = 0,\ x_1 = 1,\ x_2 = 1,\ x_3 = 3,\ x_4 = 5,\ x_5 = 11,\ x_6 = 21, \ldots .$$

Find a formula for x_n that does not involve x_k for $k < n$.

9. Suppose $P = 400$ students. Each month 80% of those in Dorms move to Apts and 20% of those in Apts move to Dorms.

a) Find the migration matrix M, its eigenvalues and eigenvalues

b) If the initial distribution is $(200, 200)$, give a formula for the population distribution after n-months. (Do it from scratch, without relying on the formulas from chapter 7.)

c) What is the stable equilibrium point? What initial distribution will take longest to reach the stable equilibrium point?

10. For each of the following matrices, find a matrix B such that $B^{-1}AB$ is diagonal.
$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A = \begin{bmatrix} 0 & b \\ b^{-1} & 0 \end{bmatrix}$, $b \neq 0$.