Math 216
Homework 1
Due Wed. Jan. 23

1.1 Prove that if a and b are positive integers such that $a \mid b$ and $a \mid b-1$ then $a = 1$.

Some remarks: If you take any integer n and divide it by 4, you get one of four possible remainders $r = 0, 1, 2, 3$. We say $n \equiv r \mod 4$. For example, $17 \equiv 1 \mod 4$ and $7 \equiv 3 \mod 4$. If $n \equiv r \mod 4$, then you can express n as $n = 4k + r$ for some integer k. Now back to the exercises...

1.2. Let m and n be integers such that $m \equiv 1 \mod 4$ and $n \equiv 1 \mod 4$. Prove that $mn \equiv 1 \mod 4$.

More remarks: The primes are divided into three classes, according to whether $p \equiv 1, 2, 3 \mod 4$, thus:

$$\begin{aligned} &5, 13, 17, 29, \ldots \\ &2 \\ &3, 7, 11, 13, \ldots \end{aligned}$$

(Why can't we have $p \equiv 0 \mod 4$?)

1.3. Prove that there are infinitely many primes p of the form $p = 4k+3$. Follow this outline: Assume there are only finitely many such primes, listed as $p_1 = 3, p_2 = 7, \ldots, p_n$, and consider the integer $N = 4p_1 \cdots p_n - 1$. First show that $N \equiv 3 \mod 4$. Then use problem 1.2 to show there is a prime q dividing N, where $q \equiv 3 \mod 4$. Finally, use problem 1.1 to arrive at a contradiction.

1.4. Can you use the same method to prove there are infinitely many primes of the form $4k + 1$? Why or why not?

1.5. Determine if each of the numbers 7919, 8051 is prime, and factor into primes if it is not prime. Explain how you did it.