Math 216
Homework 7
Due Fri. April 19

7.1. Suppose $f : S \longrightarrow T$, and $g : T \longrightarrow S$ are functions such that $g(f(s)) = s$ for all $s \in S$. Prove that g is surjective.

7.2 Let $S = \{1, 2\}$ and $T = \{1, 2, 3\}$. Find functions $f : S \longrightarrow T$ and $g : T \longrightarrow S$ satisfying the hypotheses of 7.1. Verify that f is not surjective and g is not injective.

7.3. Suppose $f : \mathbb{N} \longrightarrow S$ is surjective. Prove that S is countable.

7.4. Let $f : S \longrightarrow T$ be a function. Define a relation on S by $s \sim s'$ if $f(s) = f(s')$. Prove that $\sim$ is an equivalence relation. Let $\bar{S}$ denote the set of equivalence classes. Define a bijection from $\bar{S}$ to $f(S)$.

7.5. Let $S_1, S_2, \ldots, S_n$ be countable sets. Prove that $S_1 \times S_2 \times \cdots \times S_n$ is countable, for $n \geq 2$. Use induction on n.

7.6. Prove that the set A of algebraic numbers is countable. (Hint: Let A_n be the set of roots of polynomials of the form $x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, all $a_i \in \mathbb{Q}$. Then $A = \bigcup_{n=1}^{\infty} A_n$.)