

Math 216
Homework 8
Due Fri. April 26

8.1. Prove Euler's formula using induction on the number of edges. (You can use Euler's formula for trees, which we already proved by induction.)

8.2 Work through the Book proof of Euler's formula when G is the dodecahedron graph. (Find a spanning tree T , find G^* , find T^* , and verify that $e_T + e_{T^*} = e$.)

8.3. Draw the icosahedral graph, and show that parts A and B of the Proposition on p. 61 are true for this graph with "at most" replaced by "exactly".

8.4. Let G_1 be a hexagon with vertices $V_1 = \{1, 2, 3, 4, 5, 6\}$ labelled consecutively, and additional edges $\{1, 4\}$, $\{2, 6\}$, $\{3, 5\}$. Let G_2 be a pair of concentric triangles with exterior vertices a, b, c and interior vertices d, e, f , connected by additional edges $\{a, f\}$, $\{b, d\}$, $\{c, e\}$. Find an isomorphism from G_1 to G_2 .

8.5. Find two non-isomorphic graphs, each with 6 vertices, all of degree 3.

8.6. An **automorphism** of a graph G is an isomorphism from G to itself. The set of automorphisms of G form a group $Aut(G)$. Determine the order of $Aut(G)$ for G a triangle, and G a square.

8.7. A computer thinks of a graph G in terms of its **adjacency matrix** A_G . If G has vertices $\{v_1, \dots, v_n\}$ then the ij entry (that is, in row i , column j) of A_G is 1 if v_i is adjacent to v_j , zero otherwise. Prove that the ij entry in the squared matrix A_G^2 is the number of two-step paths from v_i to v_j .

8.8. We want to color the vertices of a graph G with two colors, say red and blue, in such a way that adjacent vertices have different colors. Try to do this for a triangle and a square. Grow a tree, and try it for your tree. Prove by induction that any tree can be colored in this way. If you want a more challenging, but doable problem, prove that G can be colored thus, if and only if G has no odd cycles. This last result tells you which maps can be colored with only two colors. A very hard theorem, proved about 30 years ago, asserts that any map can be colored with four colors.