Math 216
Practice B For Exam 1

B1. Write out proofs 1,2,3,6 of the infinitude of primes, without looking at any notes or the book. If you can't do this, then do it first with the Book/notes at hand, and then later without this help.

B2. Prove there are infinitely many primes of the form $4k+1, 4k+3, 6k+5, k2^n+1$.

B3. Calculate orders of elements in $\mathbb{Z}_p^\times$ for small primes like 17,19,23,...

B4. Find $2^{1000} \mod 29$ and do other problems of this kind (make up your own).

B5. Find factors of $7^{1000} - 1$ and do other problems of this kind.

B6. Decide if $2^{11} - 1$, and $2^{13} - 1$ are prime (without a calculator).

B7. Calculate the sums

a) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

b) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + - \cdots$

c) $x + 2x + 3x^2 + 4x^3 + \cdots$ (where $|x| < 1$).

B8. Let S be the set of positive integers having exactly one digit equal to zero. Show that $\sum_{k \in S} \frac{1}{k} < \infty$ and give an upper bound on the sum.

B9. Calculate the sum $\sum_{k \in S} \frac{1}{k}$ where S is the set of positive integers whose prime divisors are among $\{2, 3, 5, 7, 11\}$. What happens to this sum when the set of primes gets larger and larger?

B9. Calculate the sum $\sum_{k \in S} \frac{1}{k^2}$ where S is the set of positive integers whose prime divisors are among $\{2, 3, 5, 7, 11\}$. What happens to this sum as the set of primes gets larger and larger?

B10. Study the estimate of N_s in the proof of $\sum \frac{1}{p} = \infty$. Now, for a positive integer N, let $\pi(N)$ be the number of primes $\leq N$. Using the idea of the N_s estimate, prove that

$$\pi(N) \geq \frac{\log N}{\log 4}.$$

(Hint: Let $k = \pi(N)$, so that $\{p_1, \ldots, p_k\}$ are the primes $\leq N$. Count possibilities for a and b.)

B11. Redo all the problems from the previous homeworks and practice sheets.