Math 216
Practice C For Exam 2

C1. The golden ratio is the number $\tau = 2\cos(\frac{2\pi}{5}) = \frac{1+\sqrt{5}}{2}$, which you studied in HW 5. You found that $\tau^2 - \tau - 1 = 0$. Use this equation to prove by induction that in the continued fraction expansion of τ (see 6.2) we have $a_n = 1$ for every n.

C2. Use the formula $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ to find the 5 solutions of the equation $x^5 = 1$.

C3. Compute the sum $\sum \frac{1}{n^2}$ where n runs over the positive integers not divisible by 5. (There are two ways: Imitating 5.5, and the product formula for $\zeta(s)$.)

C4(OMIT). Compute $\zeta(6)$. (Use and imitate the solution to HW 5.7.)

C5. Prove that $\sqrt{\pi}$ is irrational, using what we have proved about π.

C6. Prove from scratch that e is irrational.

C7. Suppose you have an infinite set $S \subset \mathbb{N}$ of positive integers. Define a bijection $f : \mathbb{N} \longrightarrow S$.

C8. Prove that if S is uncountable, and there is an injection $f : S \longrightarrow T$, then T is uncountable.

C9. Prove that the set T of all bijections $f : \mathbb{N} \longrightarrow \mathbb{N}$ is uncountable. (Hint: Let S be the set of 0,1 sequences, and define an injection $f : S \longrightarrow T$.

C10. Suppose $f : S \longrightarrow T$ is any function, and A, B are two subsets of S. Prove that $f(A \cup B) = f(A) \cup f(B)$.

C11. Suppose $f : S \longrightarrow T$ in the problem above is injective. Prove that $f(A \cap B) = f(A) \cap f(B)$. Give an example of a non-injective f such that $f(A \cap B) \neq f(A) \cap f(B)$.

C12. Let S be a countable set (in particular, S is infinite). Prove that the set of subsets of S is uncountable. (Imitate the Cantor argument. See the second paragraph on p. 95, if you are stuck.)

C13. Write down, several times, the definitions of bijection, injection, surjection, countable, uncountable, rational number, irrational number, algebraic number, transcendental number.