

## MT 453/01 Euclid's Elements, Spring 2009

**Schedule:** MWF 11, Cushing 335

**Instructor:** Prof. Mark Reeder, Carney 322, reederma@bc.edu

**Office hours:** MWF 1-2.

**Course website:** <http://www2.bc.edu/~reederma/Euclid.html>

**Text:** *The Elements* by Euclid, translated by Heath. The required edition is by Green Lion Press, and can be ordered online from [www.greenlion.com](http://www.greenlion.com).

Hardcover: ISBN 1888009-18-7, \$38. (Preferable, for durability.)

Softcover, ISBN 1888009-19-5, \$24.95.

**Software:** All writing that is to be viewed by others will be typed in  $\text{\LaTeX}$  (free online, see below). You will also need some software for putting diagrams into pdf files, such as GeoGebra (free at [www.geogebra.org/cms/](http://www.geogebra.org/cms/)) or Geometers Sketchpad is (student edition \$39.95 at [www.dynamicgeometry.com](http://www.dynamicgeometry.com)).

**COURSEWORK:** The work in this course consists of five parts.

**1. Homework:** It will be assigned each week and collected the following week.

**2. Presentations:** Class time will consist of student presentations of Propositions and their Proofs from *The Elements*, and discussion of these. The presenter's task is to make the statements and arguments of Euclid clear and understandable, in down-to-earth terms, while retaining logical precision. Presentations should also promote some form of class participation (see below).

**3. Scribal Duties:** Each presentation is assigned a pair of scribes, who will together produce a written version of the presentation, including any improvements that may arise from the discussion, as well as important points in the discussion itself. This written version will be typed in  $\text{\LaTeX}$  and put on the course website.

**4. Class Participation:** This is crucial. Beside the presenter and scribes, everyone in the class will be involved in each presentation in some way. We will seek creative ways to encourage this. For example, the presentation could consist of guiding the entire class to work out the proof themselves. We also must remember that *The Elements* is one long argument, intended to convince a Skeptical Foe of the truth of its assertions. Each of us is to be that Skeptical Foe, who must be compelled to believe, and who therefore asks many questions. Disputation is the order of the day.

**5. Final Paper:** In place of the final exam there will be a take-home paper, due the day of the final exam Friday May 8.

**Grading:** Grades will be based on the five items of coursework listed above, with

each item weighted equally. There will be no exams.

**COURSE DESCRIPTION:** *The Elements* are a connected chain of logical deductions, beginning with five basic axioms, occupying thirteen Books. This is far too much material for one semester, so we will aim for certain major results and prove just those Propositions needed to get to our goal. The first goal is the Pythagorean Theorem and its Converse (I.47, I.48) which covers most of Book I. On the way, most of basic High School geometry is derived from the five axioms. This will occupy us until close to Spring Break.

Other goals include the construction of the Pentagon, the existence of Pi, and the Euclidean Algorithm, and others. These will be adjusted as we go along, according to our pace.

**AIMS OF THE COURSE:** There are several, beyond the basic outline above.

**1. Mathematical reasoning:** The idea of proof, and even the need for it, is a serious difficulty for most math majors, even at the advanced undergraduate level. *The Elements* contains direct and accessible mathematical proofs, expressed in ordinary language, without need for mathematical preliminaries. It is a *tabula rasa* in which mathematical structure is built from scratch, using axioms and deductions. This is the prototype for many other forms of critical thinking.

**2. Context in the undergraduate curriculum:** Much of undergraduate mathematics, even that which is called “modern”, is foreshadowed in *The Elements*, the study of which can provide context for the more advanced courses. For example, at the beginning of MT320 students are surprised to find that they don’t know what a real number is. Euclid himself struggles with this; geometric “magnitudes” are used as an unwieldy substitute for a definition of real numbers. Two thousand years later, Richard Dedekind found a construction of the real numbers, arising from his contemplation of *The Elements*.

Another example: *The Elements* uses “ $\epsilon$ -arguments” in at least two contexts (Euclidean Algorithm and Volumes by Exhaustion) where Euclid (speaking for Eudoxus) shows that two things are equal if their difference is arbitrarily small. Archimedes offers this argument as well, for those who mistrust his Lever.

**3. Historical context:** It is an interesting question why the Greeks themselves felt the need for mathematical proof. One theory is that the axiomatic method of proof is a consequence of Democracy. The need for irrefutable argument, in the face of a skeptical opponent, could have arisen in the disputatious political cauldron of 4th century Athens.

Whether or not this is actually true, the model of reasoning contained in *The*

*Elements* is part of the foundation of Western Civilization. At one time, every decent education included the *Elements*. A middle-aged Abraham Lincoln, after his one-term congressional career and finding his education deficient, embarked on a thorough study of the first six books of *The Elements*, to improve his skills as a lawyer and orator.

One aim of the course is to paint this larger picture of the importance of Euclid to future (or practicing) teachers of secondary mathematics.

**4. Communication:** Even in Heath's beautiful English translation, the language of the *The Elements* takes some work to understand. The difficulties are not stylistic. There is no "art" in the *The Elements* and it is certainly not entertaining in the usual sense. The difficulty is one of translation between the languages of Geometry and English. All language is translation of some kind. I hope that students will enhance their linguistic perception, precision, and patience by seeking the geometric meaning in Heath's English, and learning to communicate this to others in spoken and written forms.

**REMARKS ON L<sup>A</sup>T<sub>E</sub>X:** This is a version of T<sub>E</sub>X, designed by Donald Knuth at Stanford, in the 1970s, which has become the industry standard for mathematical typesetting. Every mathematician uses some version of T<sub>E</sub>X to write their papers. In our class, we won't need too many fancy mathematical symbols. But I insist on L<sup>A</sup>T<sub>E</sub>X for uniformity on the website and because it is useful in any other math course you may take. This syllabus was typed with L<sup>A</sup>T<sub>E</sub>X, as is everything you may find on my webpage. Instructions and templates are on the course website.

Any pdf file may be inserted into a L<sup>A</sup>T<sub>E</sub>X document. This is how you will make the diagrams. There are many options for software that produces pictures in pdf: GeoGebra is free. Geometer's Sketchpad costs money, but is a common tool in the high school curriculum.