MT 453 Elements Day 5
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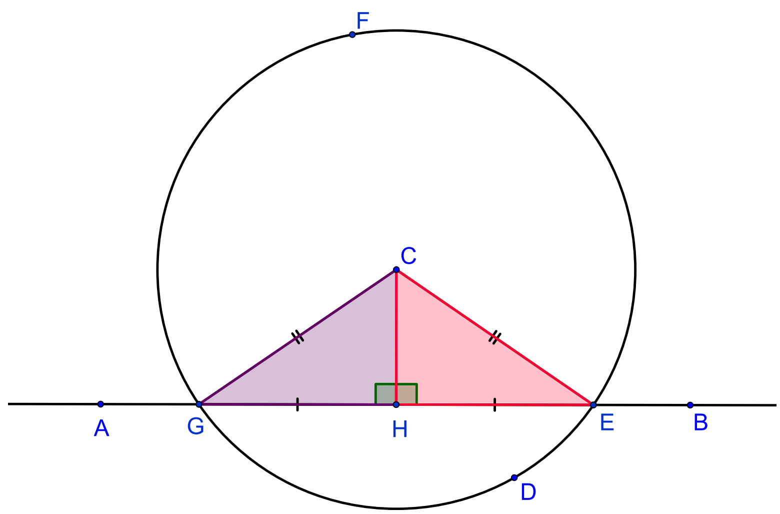
Proposition I.12

To draw a perpendicular to a given infinite line from a given point not on the line.



Let AB be the given infinite straight line and let C be a point not on AB.
We want to draw a perpendicular line to AB through C.
Choose any point D on the side of AB opposite to C.
Draw the circle DFG with center C and radius CD. (post. 3)
This circle cuts AB at two points G and E. (no justification)
Bisect EG at H. (prop. I.10)
Draw the line segments CG, CH, CE. (post. 1)
We claim that CH is perpendicular to AB.
By construction, we have $GH = EH$.
We have $CG = CE$ since these are radii of the circle DFG. (def. 15)
The side CH is common to triangles $\triangle CHG$ and $\triangle CHE$.
Since these triangles have three sides equal, they are congruent: $\triangle CHG \simeq \triangle CHE$. (prop. I.10)
The corresponding angles are equal: $\angle CHG = \angle CHE$.

Since the line CH set up on AB makes adjacent angles equal, we have $\angle CHG = \angle CHE = \lrcorner$. (def. 10)
Q.E.F.

Comments: 1. The existence of the two points G, E is not justified. We need a proposition saying that if two points on either side of a line are given and a circle centered at one of the points passes through the other point then the line cuts the circle in two points. This is another continuity result, similar to what was omitted in I.1.

2. We don't actually need two points G, E, we just need H. If we could construct the circle centered at C which is tangent to AB then H would be the point of tangency. Later on in Book III we almost have this: Prop. III.17 is the construction of a tangent to a given circle. Unfortunately, here we need to construct a circle with a given tangent!