

MT 453 Elements Day 5

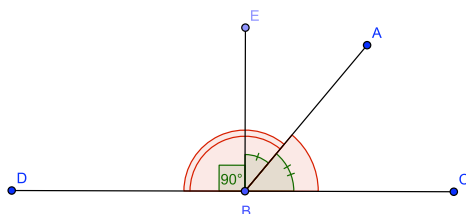
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Proposition I.13

Two adjacent angles on a straight line have measure equal to two right angles.



Draw line DC and line AB such that B lies on DC

Claim: $\angle DBA$ and $\angle ABC$ are either two right angles or have measure equal to two right angles.

There are two cases:

Case 1: If $\angle DBA = \angle ABC$, then they are right angles [def. 10]

Case 2: Suppose $\angle DBA \neq \angle ABC$

Draw BE such that BE is perpendicular to DC [I.11]

$$\angle EBC = \angle EBA + \angle ABC$$

Add $\angle DBE$ to both sides,

$$\angle EBC + \angle DBE = \angle EBA + \angle ABC + \angle DBE \text{ [C.N. 2]}$$

$$\angle DBA = \angle DBE + \angle EBA$$

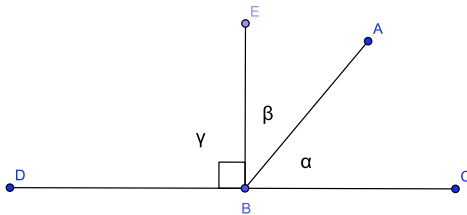
Add $\angle ABC$ to both sides,

$$\angle DBA + \angle ABC = \angle DBE + \angle EBA + \angle ABC \text{ [C.N. 2]}$$

$$\text{Thus } \angle EBC + \angle DBE = \angle DBA + \angle ABC \text{ [C.N. 1]}$$

Since $\angle EBC + \angle DBE = \text{right angle}$, $\angle DBA + \angle ABC = \text{right angle}$.
Q.E.D.

Comments:



Proposition I.13 can also be shown using associativity of addition.
Begin by labeling the angles $\alpha = \angle ABC$, $\beta = \angle EBA$, $\gamma = \angle DBE$.
We are given $\gamma + (\beta + \alpha) = \text{right angle}$.
Since addition is associative, $(\gamma + \beta) + \alpha = \text{right angle}$