MT 453 Elements Day 5

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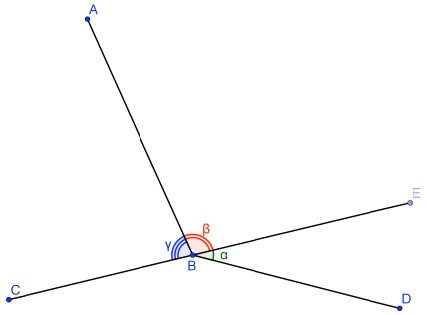
Propsition.I.14

Given a straight line, if two lines are drawn not lying on the same side, and make the adjacent angles equal to two right angles, the two straight lines must fall upon each other.



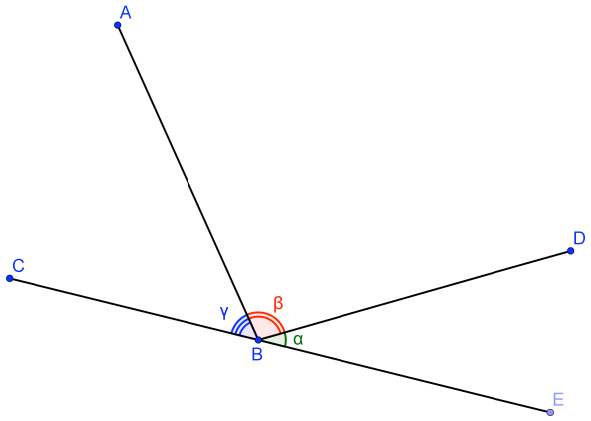
Given straight line AB, draw two straight lines CB and BD that are not lying on the same side of AB. Let $\gamma + (\alpha + \beta) = \text{⌟⌟}$

Claim: BC is in a straight line with BD.
Suppose BC is not straight with BD.
Extend CB and draw point E (Post 2).
$\gamma + \beta = \text{⌟⌟}$ (I.13).
$\gamma + \beta = \gamma + (\beta + \alpha) = \text{⌟⌟}$ (c.n.1).
Subract γ from both. (c.n.3), so $\beta = \beta + \alpha$, but this is a contradiction because the whole is greater than the part (c.n.5). $\therefore \beta \neq \beta + \alpha$.
Therefore, BC must be straight with BD.

Q.E.D.

Comments: Euclid does not explicitly say in his proof in the Elements that E is an extension of CB.

Alternate Picture



In this picture, AB is the given line and the two straight lines BC and BE are set on AB and $\gamma = \alpha + \beta$. Then, draw BD so that it is in a straight line with BC. You can prove by contradiction that BE and BC are in a straight line.