

MT 453 Elements Day 6

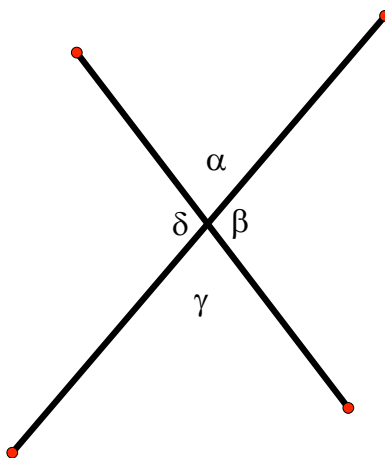
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January 30, 2009

Proposition I.15

If two straight lines cut one another, the vertical angles are equal to one another.



Proof:

Label the angles α , β , γ , and δ .

We know that $\alpha + \beta = \text{right angle}$. (prop. I.13)

But we also know that $\beta + \gamma = \text{right angle}$. (prop. I.13)

Thus, $\alpha = \text{right angle} - \beta$ and $\gamma = \text{right angle} - \beta$. (c.n. 3)

Therefore, $\alpha = \gamma$. (c.n. 1)

Similarly, $\beta = \delta$.

Q.E.F.

Comments:

1. This is the same proof that Euclid gives in the Elements, it just uses different notation which makes it easier to understand.
2. After this proof there is a Porism, which is a consequence of the proof. The Porism was not in the original Elements written by Euclid, but is believed to be included in editions written around 100-200 B.C.

Porism:

If two lines cut one another, the angles at the point of section equal four right angles.

This Porism leads to the fact that if you want to surround a point with regular polygons, there are only three ways to do this: with a square, with six equilateral triangles, or with three hexagons. This is a fundamental fact about two-dimensional Euclidean geometry.