

MT 453 Elements Day 9

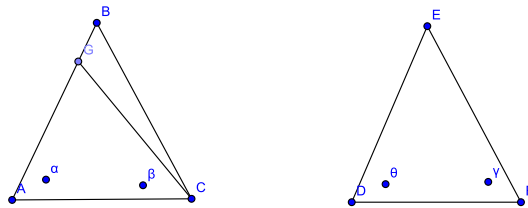
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Scribes: Sarah, Andrew

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Proposition I.26

Given 2 triangles with 2 equal angles and one equal side, either adjoining the angles or opposite one of the equal angles, then the remaining sides and angles are equal.

Let $\triangle ABC$ and $\triangle DEF$ be the given triangles with angles $\gamma, \beta, \theta, \nu$ labeled.
Let $\gamma = \theta$ and $\beta = \nu$.



Case 1: $AC = DF$.

If $AB \neq DE$, let AB be the larger side.

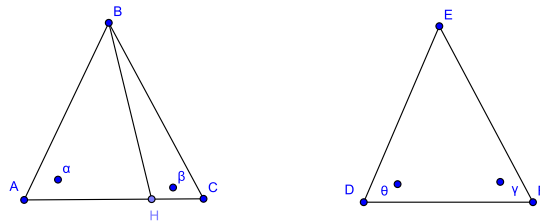
Choose G on AB such that $AG = DE$ and then connect GC .

Then $\triangle AGC \cong \triangle DEF$. (Prop. I.4)

Therefore, $\angle ACG = \nu$.

But $\nu = \beta$, which means $\angle ACG = \beta$.

But this is impossible. Thus, $AC = DF$.



Case 2: $AB = DE$

Assume $AC \neq DF$ and let AC be the greater side.

Choose H on AC such that $AH = DF$ and connect BH . (Prop. I.3)

Then $\triangle ABH \cong \triangle DEF$. (Prop. I.4)

Therefore, $\angle BHA = \nu$ and $\beta = \angle BHA$.

And $\angle BHA$ is the exterior angle of $\triangle BHC$.

Therefore, $\angle BHA > \beta$. (Prop. I.16)

But this is impossible. Therefore, $AC = DF$.

Then since $AB = DE$, $AC = DF$ and $\angle \gamma = \angle \theta$, $\triangle BAC \cong \triangle EDF$.