Propostion I.27

Speaker: Sean Johnson
Scribe: Matt MacDonald
Artist: Daniel Moresco

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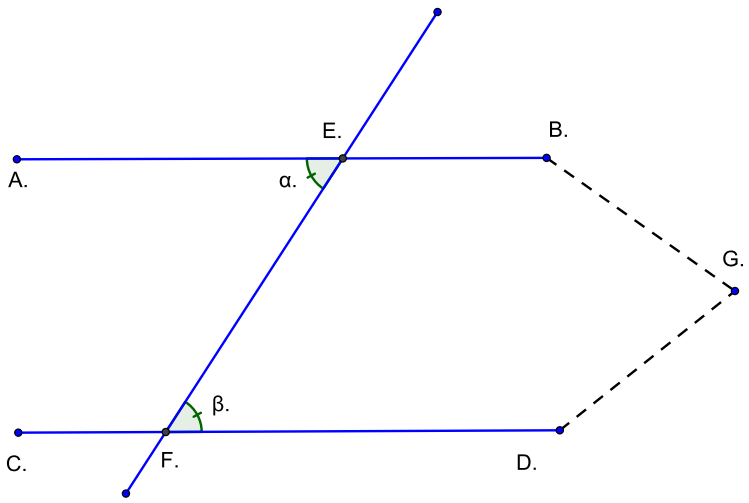
Proposition I.27

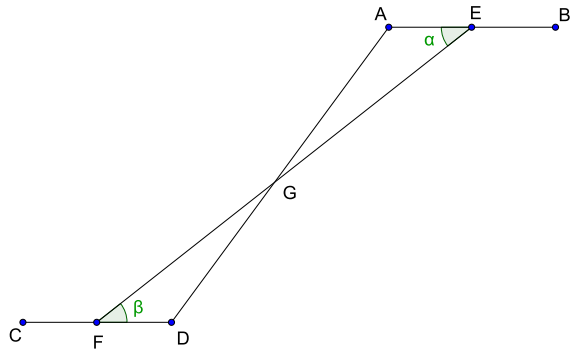
If a straight line falling on two straight lines makes the alternate angles equal, the two straight lines are parallel.



Let us have our two straight lines AB and CD, with EF falling on them, such that $\alpha = \beta$.
We claim that $CD \parallel AB$.
Suppose $CD \nparallel AB$.
Therefore, AB and CD must intersect in some point G, forming $\triangle EGF$.
But α is an exterior angle of this triangle, and α must be greater than either of the

two opposite and interior angles. (I.16)
This leads to a contradiction, as we were given that $\alpha = \beta$, and β is one of the interior and opposite angles of $\triangle EGF$.
Therefore, $CD \parallel AB$.
Q.E.F.

Comments: 1. A case that is not considered in this proof is if the two straight lines meet at a point as shown here:



If this were the case, the original proof would not hold. It is possible that we may require a definition stating that lines can only meet in one point in order to prove this case.