MT 453 Elements Day 9

Speaker: Professor Keane
Scribes: Tracy Maciolek, Kevin O'Neil

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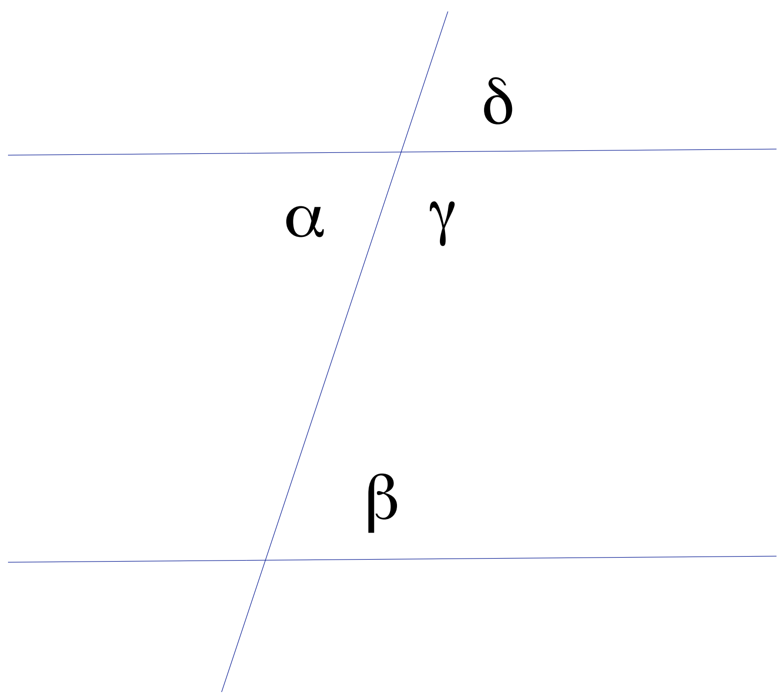
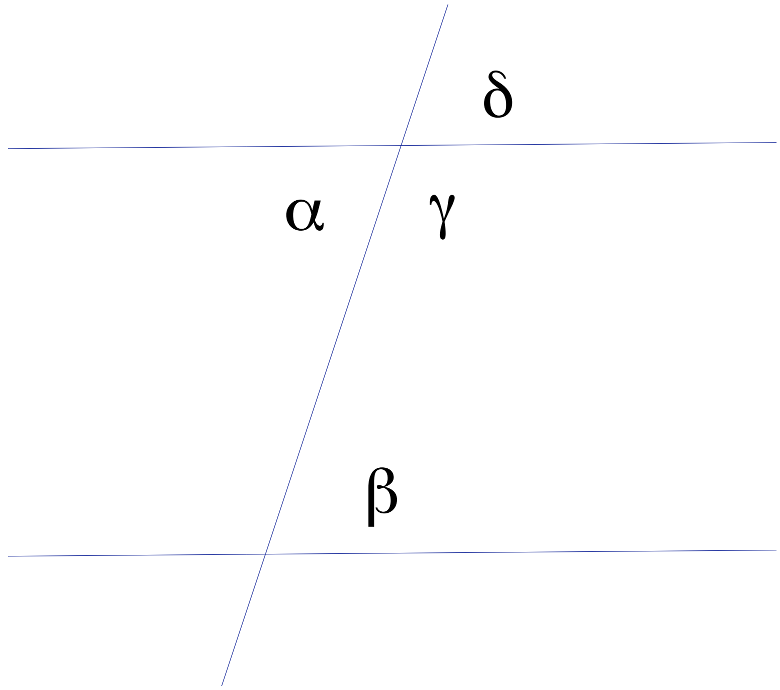
Proposition I.29

If a straight line falls on two parallel lines, then:

1. *The alternate angles are equal* ($\alpha = \beta$)
2. *The exterior angle equals the interior opposite angle* ($\delta = \beta$)
3. *The interior angles on the same side equal* ⌟ ⌟ ($\gamma + \beta =$ ⌟ ⌟).





Proof:

1. Claim: The alternate angles are equal ($\alpha = \beta$).
   Suppose the alternate angles are not equal; say $\alpha > \beta$.
   Then $\alpha + \gamma > \beta + \gamma$.
   But $\alpha + \gamma = \lrcorner\,\lrcorner$. (prop. I.13)
   So $\beta + \gamma < \lrcorner\,\lrcorner$.
   Then by Postulate 5, the two lines meet and are not parallel.
   Therefore, $\alpha = \beta$.

2. Claim: The exterior angle equals the interior opposite angle ($\delta = \beta$).
   We have that $\delta = \alpha$. (prop. I.15)
   But we have established in part 1, that $\alpha = \beta$.
   Therefore, $\delta = \beta$. (c.n.1)

3. Claim: The interior angles on the same side equal $\lrcorner\,\lrcorner$ ($\gamma + \beta = \lrcorner\,\lrcorner$).
   We have that $\delta + \gamma = \lrcorner\,\lrcorner$. (prop. I.13)
   So substituting $\delta = \beta$ (from part 2), we have,
   $\gamma + \beta = \lrcorner\,\lrcorner$.

Q.E.D.

Comments: 1. This is the first time that Postulate 5 is used the in *The Elements*. Because of the wording of Postulate 5, there is speculation that it was written specifically for use in proving Proposition I.29.

2. In Part 1 of the proof, when we suppose that $\alpha \neq \beta$, we instead suppose, without loss of generality, that $\alpha > \beta$. The same argument can apply if we suppose $\beta > \alpha$.