

MT 453 Elements

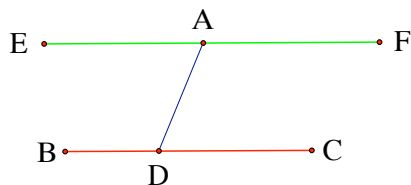
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Proposition I.31

Through a given point to draw a straight line parallel to a given straight line



Let BC be the given straight line and A the given point

Choose a point on BC called D

Connect AD

Construct $\angle DAE$ such that $\angle DAE = \angle ADC$ (Prop I.32)

Draw AF to form straight line EAF

Since $\angle EAD$ and $\angle ADC$ are alternate interior angles and are equal, EF is parallel to BC (Prop I.27)

QEF

Comments: Euclid does not say which side of A to put the point E on. If the angles are on the same side, or not alternate, the proof as described would not work.

Prop I.31 does not depend on Postulate 5 in its proof

Alternate Proof:

Let BC be the given straight line and A the given point

Construct a perpendicular straight line from A to BC such that $AD \perp BC$ (Prop I.12)

Using AD and a point F not on AD draw a perpendicular line such that $AD \perp AF$ (Prop I.12)

By definition of perpendicular, $\angle FAD$ and $\angle CDA$ are right angles

Extend AF in a straight line to point E (Post 2)

Because $\angle FAD$ is a right angle, $\angle EAD$ will also be a right angle (Prop I.13)

Since both $\angle EAD$ and $\angle ADC$ are right angles; $\angle EAD = \angle ADC$

$\angle EAD$ and $\angle ADC$ are also alternate interior angles

Therefore EF is parallel to BC (Prop I.27)

QEF

Comments:

It would have been easier to use a transitive property at the end of this proof. Seeing that since EF and BC are both perpendicular to the same line, they would therefore be parallel to each other. However this could not be found in Euclid, so the last few steps were necessary to the proof once we had constructed the two parallel lines.