MT 453 Elements

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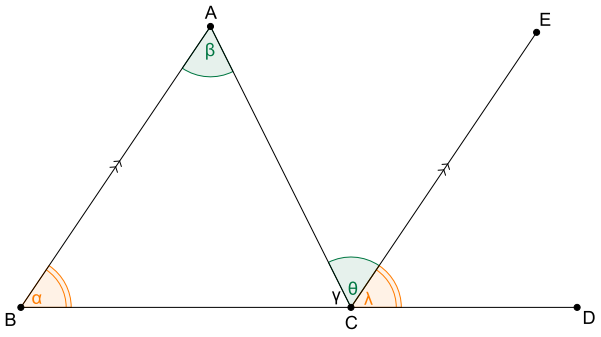
Proposition I.32

An exterior angle of a triangle equals the sum of the two opposite interior angles. The sum of all the angles in a triangle equals two right angles.

Draw triangle ABC with base BC. Extend BC through C to point D.



Draw line CE so that CE is parallel to BA.
We know that $\beta = \theta$ because they are alternate interior angles. [Proposition I.31]
We also know that $\alpha = \lambda$. [Propostion I.31]
Thus $\alpha + \beta = \theta + \lambda$ by c.n.2. Since $\theta + \lambda = \angle ACD$, $\angle ACD = \alpha + \beta$.
Therefore, the exterior angle ($\angle ACD$) is equal to the sum of the two opposite interior angles ($\alpha + \beta$).

We also know that $\gamma + \theta + \lambda =$ ⌟⌟. [Propostion I.13]
Therefore, $\gamma + \alpha + \beta =$ ⌟⌟
Therefore, the sum of all the angles in a triangle equals two right angles. QED

Comments:

- Note that these results are only true when working in the plane. If we were to look at shapes in spherical or hyperbolic geometry, Propostion I.32 would not hold.
- In spherical geometry, the angles in a triangle have sum that is greater than two right angles. Moreover, the exterior angles do not equal the sum of the opposite interior angles.
- In hyperbolic geometry, the angles in a triangle have sum that is less than two right angles.