MT 453 Elements

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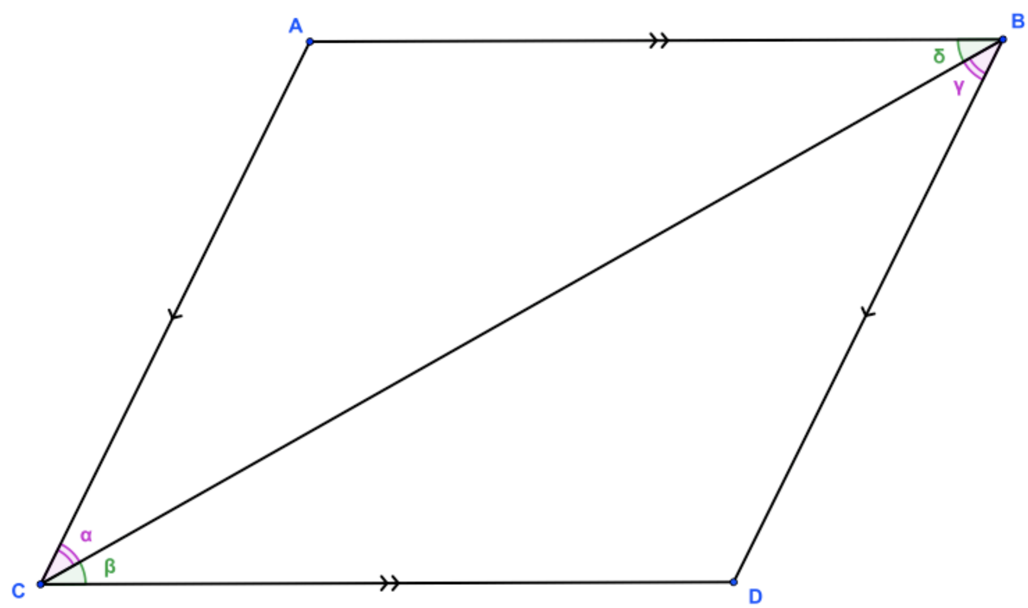
Proposition I.34

In a parallelogram the opposite sides and angles are equal and a diameter bisects the area.



Let $ABDC$ be a parallelogram.
We must show that $AB = CD, AC = BD, \angle CAB = \angle BDC$ and $\angle ACD = \angle ABD$.
Since $AC \parallel BD$ and $AB \parallel CD$ we have $\alpha = \gamma$ and $\delta = \beta$ by [I.29].
Hence $\triangle ABC \simeq \triangle DCB$, by [I.26].
So we have equality of corresponding sides: $AB = CD, AC = BD$,
as well as the remaining angles: $\angle CAB = \angle BDC$.
Finally, $\angle ACD = \alpha + \beta$ and $\angle ABD = \gamma + \delta$.
Since $\alpha = \gamma$ and $\delta = \beta$, we have $\alpha + \beta = \gamma + \delta$,
so $\angle ACD = \angle ABD$ by [c.n.2].
Q.E.D.