

MT 453 Elements Day 11

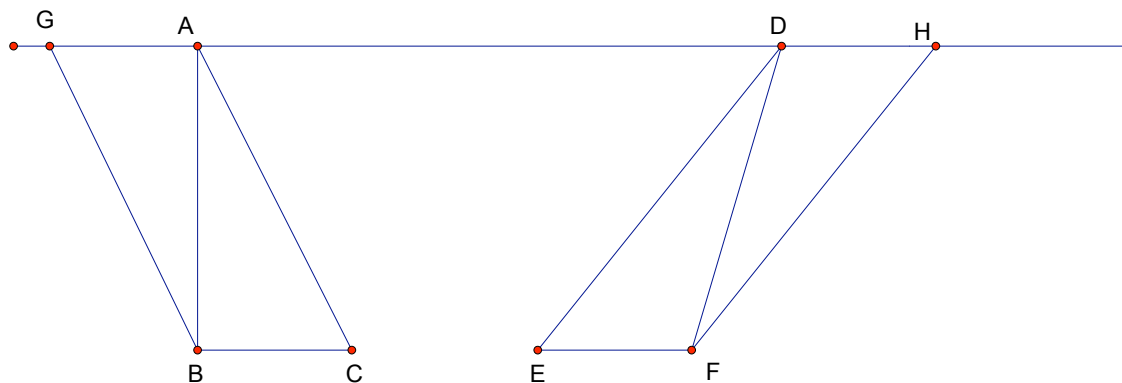
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Proposition I.38

Triangles on equal bases and in the same parallels are equal.



Proof:

Let ABC and DEF be triangles on equal bases, BC and EF , and in the same parallels, AD and BF .

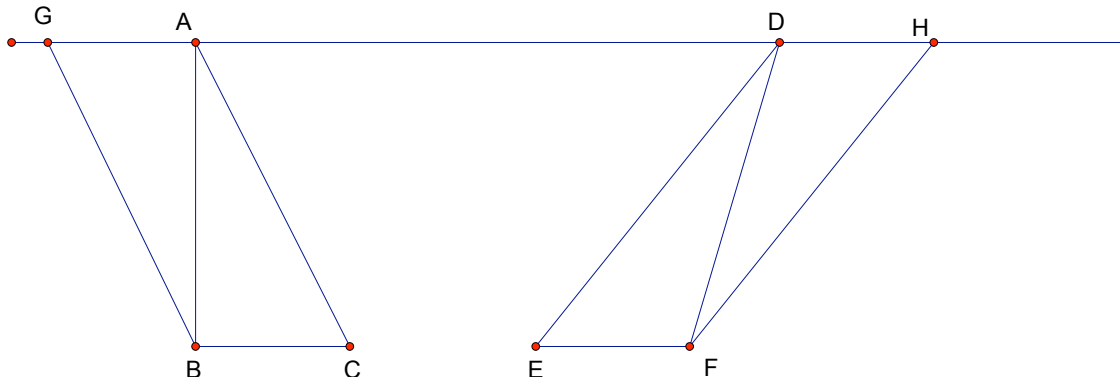
Claim:

$\triangle ABC = \triangle DEF$.

Extend AD to point G on the left and to point H on the right. (post. 2)

Let GB be drawn parallel to AC . (prop. I.31)

Let FH be drawn parallel to ED . (prop. I.31)



$\diamond GBCA = \diamond DEFH$ because they have equal bases, BC and EF , and they are in the same parallels, BF and AD . (prop. I.36)

$\diamond GBCA$ is bisected by diameter AB and $\diamond DEFH$ is bisected by diameter DF . Therefore, $\triangle ABC = \frac{1}{2} \diamond GBCA = \frac{1}{2} \diamond DEFH = \triangle DEF$. (c.n. 1)

Q.E.D.

Comments:

1. You could start this proof with bases DH and BC , so the bases could be on separate parallels, and the proof would still work because the four triangles created are all equal.

2. Within this proof is the formula for the area of a triangle. We can deform the triangle until it is a triangle that we understand, namely a right triangle. A right triangle is half of a rectangle, and since the formula for the area of rectangle is base x height, the area for a triangle is $\frac{1}{2}$ base x height.

3. After this proposition, Heron of Alexandria inserted the following proposition:

”If you have two triangles and the triangles have two sides equal to each other and the angles between the two equal sides sum to two right angles, then the triangles have the same area.”