Side - Angle - Side

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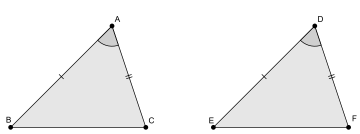
Proposition I.4

If two sides on one triangle are equal (respectively) to two sides on a second triangle and the angles formed by these two sets of lines are equal (respectively) then the bases are equal to eachother, the triangles are equal and the remaining angles are equal.



Take two traingles ABC and DEF.
Let $AB = DE$ and $AC = DF$ and angle BAC equal angle EDF.
Place point A on point D and place AB on DE.
Since $AB = DE$, B coincides with E.
Because angle BAC equals angle EDF, AC coincides with DF.
Since $AC = DF$, C coincides with F.
Since B coincides with E and C coincides with F, BC coincides with EF
$BC = EF$ (c.n. 4)
Since all three sides are equal, the triangle ABC coincides with triangle DEF
Triangle ABC is equal to triangle DEF. (c.n. 4)
Angle ABC coincides with angle DEF, so angle ABC is equal to angle DEF. (c.n. 4)
Angle ACB coincides with angle DFE, so angle ACD is equal to angle DFE. (c.n. 4)
Q.E.D

Comments: 1. Nothing is used in this proposition except common notion 4, which states that 'things which coincide with one another are equal to one another.' He never explains what he means by 'coincide,' but essentially, he wants to superposition one object onto another, a somewhat shaky proposal.

2. People were actually so unhappy with this proof that a modern mathemetician actually turned it into an axiom.

3. In his actual wording, Euclid says 'If one triangle is applied to the other'... it seems to instruct you to put one on top of the other, but that only works if they are oriented in the same direction. If they aren't, then you have to flip one triangle, meaning we'd be in 3 dimensions and out of the plane.