MT 453 Elements Day 13
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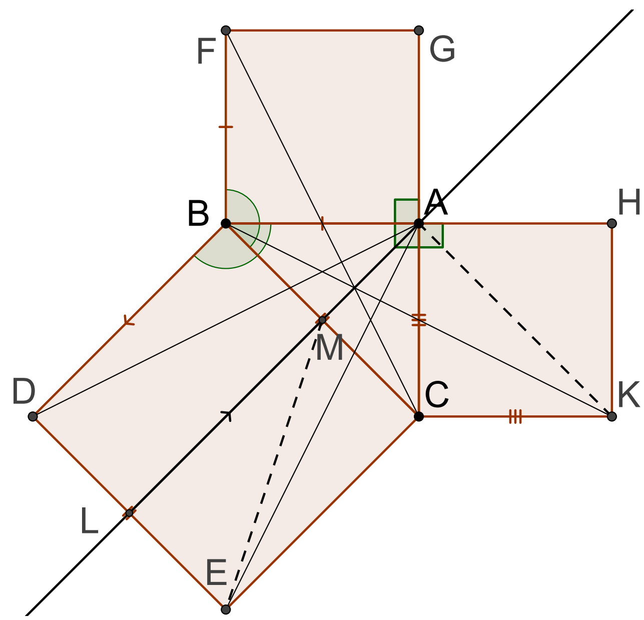
Proposition I.47

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.



Let ABC be a right triangle.
Construct the squares $ABFG, ACKH, BCED$ on the sides AB, AC, BC. [I.46]
Draw the parallel to BD through A, cutting DE at L. [I.31]
Since $\angle BAG = \lrcorner = \angle BAC$, the line CA is in a straight line with AG. [I.14]
Likewise, BA is in a straight line with AH. [I.14]
The two right angles $\angle DBC$ and $\angle FBA$ are equal. [post. 4]
Adding $\angle BAC$ to each of these gives $\angle DBA = \angle FBC$. [c.n.2]
Sides $BA = BF, BD = BC$, so $\triangle ABD \simeq \triangle FBC$. [I.4]
The rectangle BL has twice the area of $\triangle ABD$, since both have the same base BD and parallel AL. [I.41]
The square GB has twice the area of $\triangle FBC$, since both have the same base FB and parallel GC. [I.41]
Since doubles of equals are equal, the rectangle BL equals the square on AB.
Drawing AE and BK, we can similarly show that the rectangle CL equals the square on AC.
But the square on BC is the sum of the rectangles BL and CL
Therefore the square on BC is the sum of the squares on AB and AC.
Q.E.D.

Comments: One can see this proof dynamically. Start with $\triangle FBA$, which is half the square on AB. Sliding the vertex at A to C parallel to the base FB, we have $\triangle FBA = \triangle FBC$. Rotating about the vertex B, we get $\triangle FBC = \triangle ABD$. Moving A to M along the parallel AL, we get $\triangle ABD = \triangle BMD$, which is half the rectangle BL. Therefore half the square on AB is half the rectangle BL.