

MT 453 Elements Day 12

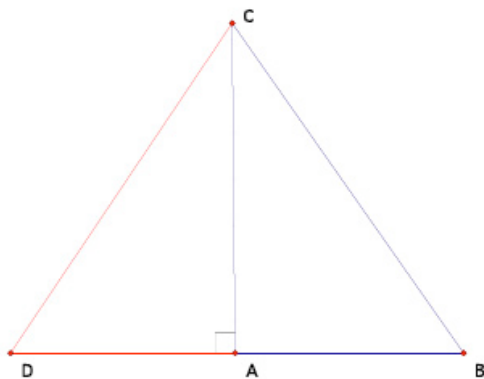
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Proposition I.48

If in a triangle the square on one side equals the squares on the other 2 sides then the angle between those 2 sides is right.



Let ABC be our given triangle were $\text{sq}.BC = \text{sq}.AC + \text{sq}.AB$.

Draw AD so that $AD = AB$ (post I.3) and $\angle DAC$ is right (prop. I.11). DB is not necessarily a straight line.

Connect DC (post 1)

$\text{sq}.AD = \text{sq}.AB$ since $AD=AB$

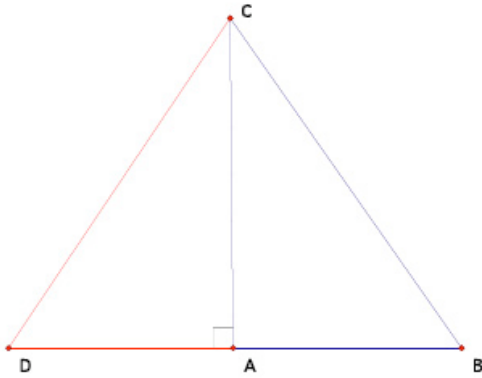
$\text{sq}.AD + \text{sq}.AC = \text{sq}.AB + \text{sq}.AC$ (c.n. 2)

$\text{sq}.CD = \text{sq}.AD + \text{sq}.AC$

$\text{sq}.CD = \text{sq}.AB + \text{sq}.AC$ (prop I.47)

$\text{sq}.BC = \text{sq}.AB + \text{sq}.AC$

Therefore, $\text{sq}.CD = \text{sq}.BC$ (c.n. 1)



Therefore, $CD = BC$ (c.n. 1)

We know $CD = BC$, $AD = AB$, and AC is common.

Therefore, $\triangle CAD \cong \triangle CAB$ (prop I.8)

Therefore, $\angle CAD = \angle CAB$ (prop I.8)

Therefore, $\angle CAB$ is right. (c.n.4)

Q.E.D.

Comments:

1. Interesting that this Euclid uses I.47 to prove I.48 (the converse).
2. It does not matter what the figures are on the sides of triangles as long as they are similar and similarly described.