MT 453 Elements Day 3

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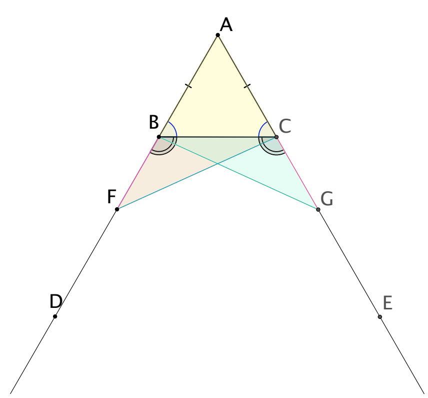
Proposition I.5

Given an isosceles triangle, find that the angles at the base are equal to one another. If the two equal sides are extended, the angles below the base are equal to one another.

Let $\triangle ABC$ be an isosceles triangle, with side AB equal to side AC.



Extend AB to create AD, and extend AC to create AE. (Post. 2)
Pick a point on BD at random, and call it F.
Let the point G on AE be the same distance from A as F is on AD from A. (I.3)
Then $AF = AG$. Draw the straight lines between F and C and between B and G. (Post. 1)
The new triangles $\triangle AFC$ and $\triangle AGB$ have common angle $\angle FAG$.
We know that $AF = AG$ and $AB = AC$. Therefore the base FC equals the base GB. (I.4)
Therefore, $\triangle AFC$ is congruent to $\triangle AGB$, and so $\angle AFC = \angle AGB$ and $\angle ACF = \angle ABG$. (I.4)
Now $AF - AB = BF$ and $AG - AC = CG$, so $BF = CG$. (C. N. 3)
We now know $BF = CG$, $FC = GB$, and $\angle BFC = \angle CGB$, so $\triangle BFC$ is congruent to $\triangle CGB$. (I.4)
Therefore, $\angle CBG = \angle BCF$ and $\angle CBF = \angle BCG$, which proves the second part of the Proposition.
But also, $\angle ABG - \angle CBG = \angle ABC$ and $\angle ACF - \angle BCF = \angle ACB$.
Therefore, $\angle ABC = \angle ACB$. (C. N. 3)

Q.E.D.

Comments:

1. Why can the point F be "chosen at random"? That lines are made up of points is not a postulate.

2. When showing that $\triangle BFC$ is congruent to $\triangle CGB$, Euclid notes that they share a common base. But that is not needed to apply I.4, since he has two sides and the included angle equal.

3. The proof is interesting in that the original triangle is deformed, creating new triangles, which are then used to prove the result about the original.