

MT 453 Elements Day 3

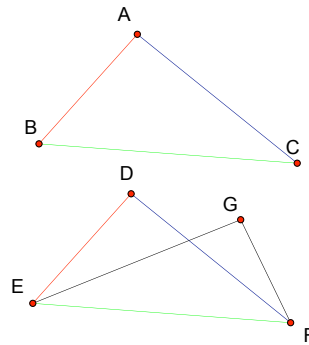
Speaker: Professor Keane

Scribes: Tracy Maciolek, Kevin O'Neil

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Proposition I.8

If two triangles have two sides equal, and the bases are equal, then the angles contained by equal sides are equal.



Let triangle ABC and triangle DEF be the triangles with $AB = DE$, $AC = DF$, and bases $BC = EF$.

Apply triangle ABC to triangle DEF so that B coincides with E , and BC falls on EF .

Then C falls on F .

Claim: AC falls on DF and BA falls on ED .

For if not, they fall beside on EG and GF .

But then 2 equal lengths (ED , EG) and 2 equal lengths (DF , GF) construct at the extremities of EF .

This would contradict Prop. I.7.

So BA falls on ED and AC falls on DF , and the angles also coincide.
Therefore, the angles are equal. (c.n. 4)
Q.E.D.

Comments: 1. In the conclusion of this proof in the text, Euclid only mentions one set of angles that coincide, and ends the proof with ‘If therefore etc,’ thereby leaving us to finish it. We can similarly conclude that the remaining angles also coincide. Therefore, this proposition states that if the sides are coinciding and equal, then the angles are coinciding and equal. Therefore, the triangles coincide, and are what we call ‘congruent.’

2. By Common Notion 4, coinciding triangles mean the triangles are equal. What do ‘equal’ triangles mean? Does it mean the sides and angles are coinciding and equal, or does it mean the areas are equal? Euclid has yet to discuss area, so we can conclude that equal triangles mean the coinciding sides and angles are equal.