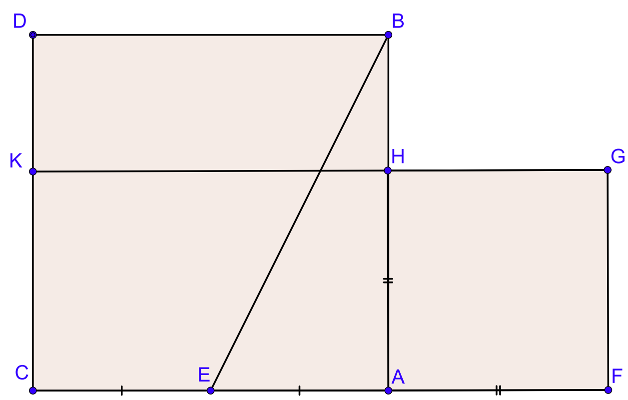
MT 453 Elements

Speaker: Kaitlyn Valente
Scribe: Vincent Embser
Artist: Sean Johnson

March 9, 2009

Proposition II.11

To cut a line AB at a point H such that $(AB)^2 = (AB)(HB)$.



Let AB be a straight line.
Draw square $ABCD$. (I.46)
Bisect AC at E (Prop. I.11)
Draw BE.
Extend CA to F so that $BE = EF$.
Draw square $AFGH$.
Claim: $(AB)(HB) = (AB)^2$
Proof:
Extend GH to K.
$(CF)(AF) + (EA)^2 = (EF)^2$ (Prop. II.6)

Since $EF = EB$, $(CF)(AF) + (EA)^2 = (EB)^2$. (C.N.1)
Now $(EB)^2 = (EA)^2 + (AB^2$.
$(CF)(AF) + (EA)^2 = (EA)^2 + (AB)^2$.
So $(CF)(AF) = (AB)^2$.
So $CFGK = ABDC$.
Subtract $AHCK$ from both sides, which leaves $(AH)^2 = HBDK$.
$(AH)^2 = (HB)(BD)$.
But $BD = AB$, so $(AH)^2 = (HB)(AB)$.
Q.E.F.

Comments:
1. Euclid refers to this as the ”Extreme and Mean ratio”.

2. Another way of writing this is $AB/AH = AH/AB$. This is called three magnitudes in continued proportion, and means we have a golden rectangle.

3. If we have a line AB cut in this way such that $AH = x$ and $HB = 1$, then $x = \frac{(1+\sqrt{5})}{2}$, which is called the Golden Ratio, and is referred to as τ, from the Greek τome, ”to cut”.