MT 453 Elements Day 18

Speaker: Thomas Quan
Scribes: Kaitlyn Valente, Richard Embser

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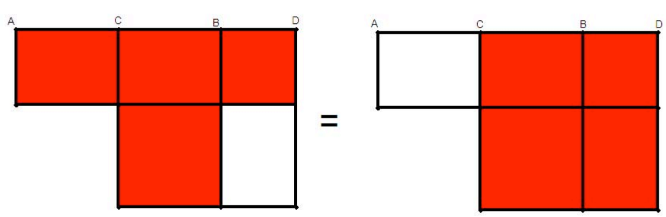
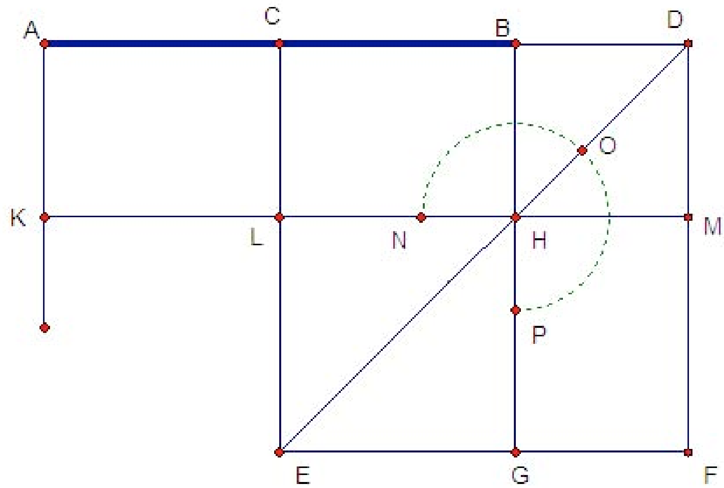
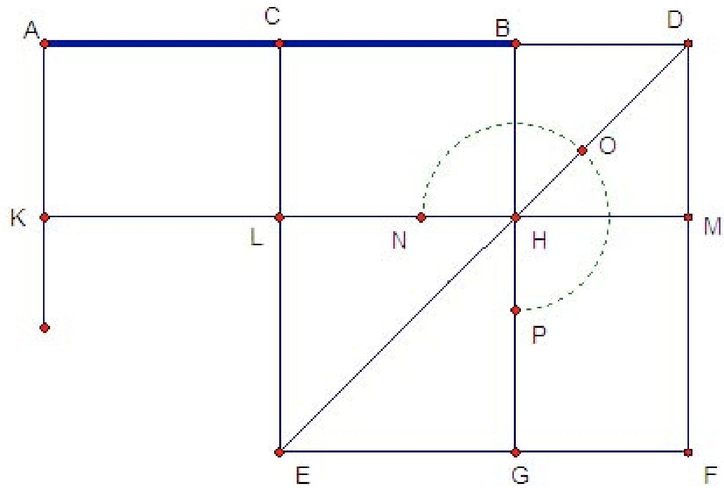
Proposition II.6

If a straight line is bisected and a straight line is added to it in a straight line, then the rectangle contained by the whole with the added straight line and the added straight line together with the square on the half equals the square on the straight line made up of the half and the added straight line.





Proof:
Let AB be bisected at C. (Prop. I.10)
Let BD be added to AB in a straight line. (Post. 2)
Claim: $(AD)(BD) + CB^2 = CD^2$.
Draw square $CEFD$ on CD. (Prop. I.46)
Connect DE. (Post. 1)
Through B draw BG parallel to CE or DF. (Prop. I.31)
Let H be the point where BG and DE intersect.
Through H draw KM parallel to AD or EF. (Prop. I.31)
Let L be the point where CE and KM intersect.
Through A draw AK parallel to CL or DM. (Prop. I.31)
Since we bisected AB at C, we know that $AC = CB$.
We know that $[AL] = [CH]$ because they are parallelograms on equal bases and in the same parallels. (Prop. I.36)
Since, in any parallelogram the complements of the parallelograms about the diameter equal one another, we know that $[CH] = [HF]$. (Prop. I.43)
Therefore, $[AL] = [HF]$. (c.n. 1)
Adding $[CM]$ to both, we get $[AL] + [CM] = [HF] + [CM]$. (c.n. 2)
Simplifying we have $[AM]$ = gnomon NOP.
We know that $[BM]$ and $[LG]$ are squares. (See proof of II.4)
Thus, $BD = DM$. (def. I.22)
Therefore, we can say $[AM] = (AD)(BD)$.



CB = LH because they are opposite sides of a parallelogram. (Prop. I.34)
CB^2 = LH^2 = $[LG]$.
Adding $[LG]$ to each side we get $[AM]$ + $[LG]$ = gnomon NOP + $[LG]$. (c.n. 2)
Simplifying we have that $(AD)(BD)$ + CB^2 = CD^2, which is what we were trying to prove.
Q.E.D.

Comments:
1. We need definition II.2 for this proof, which states "And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon."
2. This proof can be expressed algebraically by letting AD = x, BD = y, AB = b, and y = $x - b$.
Therefore, $(AD)(BD)$ + CB^2 = CD^2 becomes xy + $(\frac{b}{2})^2$ = $(\frac{b}{2} + y)^2$
Subbing $x - b$ in for y we get $x(x - b)$ + $(\frac{b}{2})^2$ = $(x - \frac{b}{2})^2$
If we know that $x - y$ = b and that xy = c^2, we can rewrite the equation as c^2 + $(\frac{b}{2})^2$ = $(x - \frac{b}{2})^2$.
We can plug this into Proposition I.47 and we can solve for x.