

MT 453 Elements Day 16

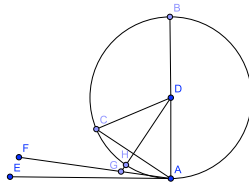
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Proposition III.16

The line drawn at a right angle to the diameter of a circle at its extremity will fall outside the circle, and into the space between the straight line and the circumference, another straight line cannot be interposed. Further the angle of the circumference is greater, and the remaining angle, less than any acute rectilinear angle.

Let ABC be a circle and AB the diameter. Let AE be drawn tangent to the circle at A.



Case 1:

Assume AE does not fall outside, and fall inside as AC

Draw DC .

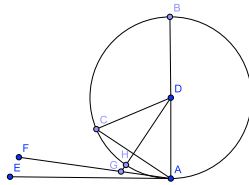
$DC = DA$.

Then $\angle DCA = \angle DAC$. Prop. I.5

$\angle DAC = \sphericalangle = \angle DCA$.

In a \triangle , any 2 angles $\neq \sphericalangle \sphericalangle$, $\Rightarrow \Leftarrow$.

Therefore, AE must fall outside the circle.



Case 2:

Assume there is another line AF that is completely outside the circle.

Draw $DG \perp AF$ from D .

$\angle DGA = \perp$, therefore DAF is less than \perp .

Then DA is greater than DG .

So $DA = DH$.

Then since DA is subtended by the larger angle, DH is greater than DG .

But the whole is greater than the part, $\Rightarrow \Leftarrow$.

Therefore, another line cannot be interposed.

Case 3:

This case is complicated and unnecessary.

However, it is related to an epsilon-delta proof.

Porism: A straight line drawn at right angles to the extremity of a diameter will touch the circle.