MT 453 Elements Day 16

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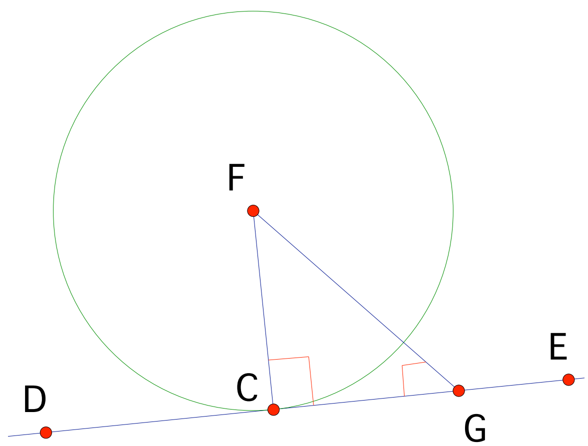
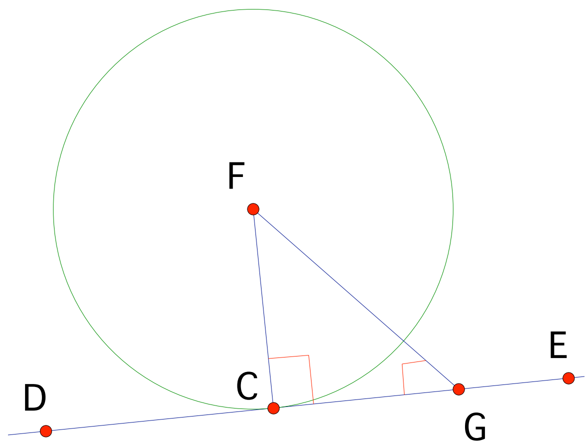


Proposition III.18

If a line touches a circle, then the radius of the circle to the point of contact is perpendicular to the line.

Proof:
Say F is the center of the circle, DE is the line, and C is the point of contact.
Draw a line perpendicular from F to DE, hitting at G.
If $G \neq C$, then G is outside the circle. (Prop. III.16)
Consider $\triangle FCG$. We have that $\angle FCG < \lrcorner$ (Prop. I.17).
Then $\angle FCG$ subtends a shorter side than $\angle FGC$. That is $FC > FG$ (Prop. I.19).
But if FG cuts the circle at B, then $FB = FC$.



Then $FB > FG$ (c.n.1), so the part is greater than the whole.
This is a contradiction, since the whole is always greater than the part (c.n.5).
So $G = C$, and so the radius is perpendicular to the line at the point of contact.
Q.E.D.

Comments: 1. At the end of the proof, Euclid says that we can similarly show that no other straight line besides FC will be perpendicular to DE. But this is redundant. We showed that, if we take any point besides C and the resulting straight line it forms with F, we will reach a contradiction. Our point G was arbitrary, and therefore, this endnote is redundant.