MT 453 Elements

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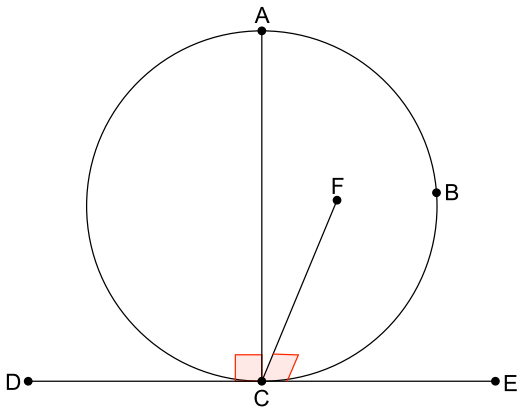
Proposition III.19

If a tangent of a circle meets a chord at a right angle, the chord is a diameter



Given circle ABC and let DE be a tangent of the circle at point C
Let AC be a chord of circle ABC and let $AC \perp DE$
Claim: The center of the circle ABC is on AC
Suppose however that the center is not on AC and that F is the center
Draw FC
Since FC is a radius that meets the tangent DE, $\angle FCE$ is a right angle (Prop III.18)
But $\angle ACE$ is also a right angle
Therefore $\angle FCE = \angle FCE$
But $\angle FCE$ is a part of $\angle ACE$, and a whole can not equal a part
A contradiction, therefore F is not the center of the circle

Therefore the center of the circle must lie on AC
AC is the diameter
QEF

Comments: The last two lines of Euclid's proof just say that this proof will work for any point F that is not on AC. However this seems redundant seeing as we chose F specifically so that it does not lie on AC at the beginning of the proof.

If F is chosen to lie on the other side of AC, different from our picture, we would simply just compare $\angle FCD$ and $\angle ACD$