MT 453 Elements

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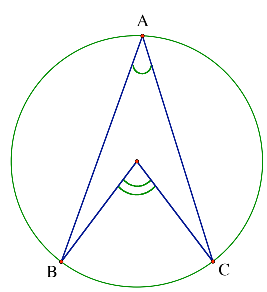
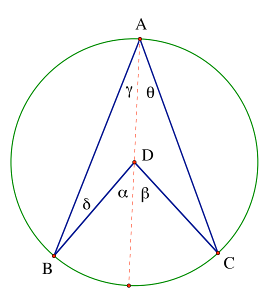
Proposition III.20

The angle at the center of a circle is twice the angle at the circumference given that both share a common circumference as a base.

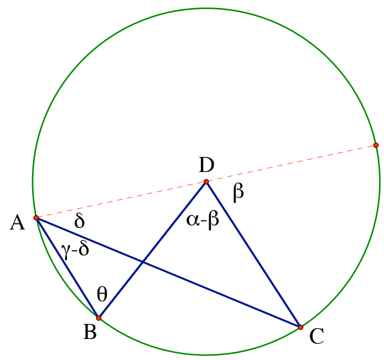


Case 1: B and the center are on the same side of AC
Draw circle BAC and $\angle BAC$ at the circumference and $\angle BDC$ at the center of the circle so that the angles share common circumference BC.



Claim that $\angle BDC = 2\angle BAC$.
Draw AD and extend it in the direction of D. [Post. I.2]
$AD = BD$. Therefore, $\gamma = \delta$. [Prop I.5]
So $\gamma + \gamma = \delta + \gamma$ [c.n.2]
Thus $2\gamma = \delta + \gamma$
Also $\alpha = \delta + \gamma$ [Prop I.32] and thus $\alpha = 2\gamma$ [c.n.1]
Similarly $\beta = 2\theta$
So $\alpha + \beta = 2\gamma + 2\theta = 2(\gamma + \theta)$
Thus $\angle BDC = 2\angle BAC$

Case 2: B and the center are not on the same side of AC.



$\gamma = \theta$
$2\gamma = \theta + \gamma$, and it is known that $\alpha = 2\gamma$
$\beta = 2\delta$
So by c.n.3, $\alpha - \beta = 2(\gamma - \delta)$
Thus $\angle BDC = 2\angle BAC$
QED

NOTE: All of the reasoning except for the definition of circumference is from Book I.