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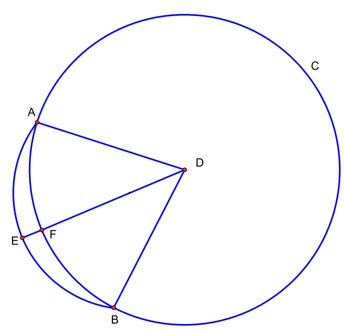
Proposition III.2

Given a circle and two points on a circle, the straight line connecting them falls in the circle.



Find the center D of circle ABC (Prop III.1)
We are going to use proof by contradiction, so suppose line AB falls outside the circle.
Draw AD, BD, EFD.
$\angle EAD = \angle EBD$ (Prop I.5)
$\angle DEB > \angle EAD$ because $\angle DEB$ is the exterior angle of $\triangle ADE$ (Prop I.16)
$\angle DEB > \angle EBD$ (by invisible common notion)
$\angle DEB$ subtends the greater side of $\triangle DBE$, so $DB > DE$
But $DB = DF$ because they are radii of the same circle, so $DF > DE$, which is a contradiction because the part (DF) cannot be greater than the whole (DE)
Q.E.D.

Notes:
1. You can use this same contradiction if the straight line falls on the circle, so if it can't be outside the circle, and it can't be on the circle, the line must be in the circle.
2. This proof shows that the circle is convex.