MT 453 Elements Day 17

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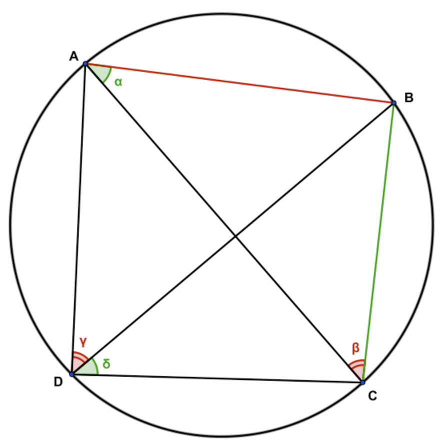
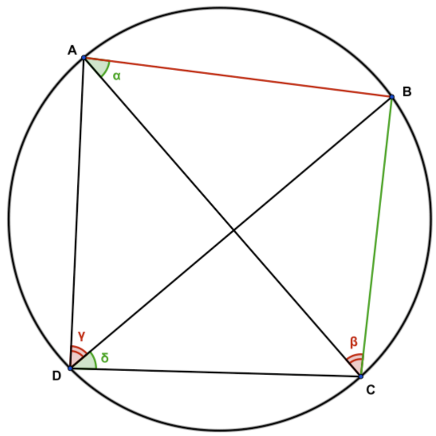
Proposition III.22

The opposite angles of a quadrilateral in a circle are equal to two right angles.



Claim: $\angle ADC + \angle ABC = \lrcorner\lrcorner$ and $\angle DAB + \angle DCB = \lrcorner\lrcorner$
Draw AC
In $\triangle ABC$,
  $\alpha + \beta + \angle ABC = \lrcorner\lrcorner$. [I.32]
$\alpha = \delta$ [III.21]
$\gamma = \beta$ [III.21]
$\angle ADC = \delta + \gamma$
$\angle ADC = \alpha + \beta$
$\angle ADC + \angle ABC = \alpha + \beta + \angle ABC$
$\angle ADC + \angle ABC = \lrcorner\lrcorner$.



Similarly, $\angle DAB + \angle DCB = \lrcorner\lrcorner$.
Q.E.D.

Comments:
This proposition does not hold for quadrilaterals not in a circle.

In a three point figure (i.e. a triangle) on a plane, the sum of the angles is always equal to two right angles. This is because for any three noncollinear points there exists a circle, which contains these three points on its circumference.

For a four point figure, however, all four points do not always fall on a circle. If all four points are on the circle, then this quadrilateral is called a **cyclic quadrilateral** and the above proposition proves that the opposite angles will be right angles.