MT 453 Elements Day 14

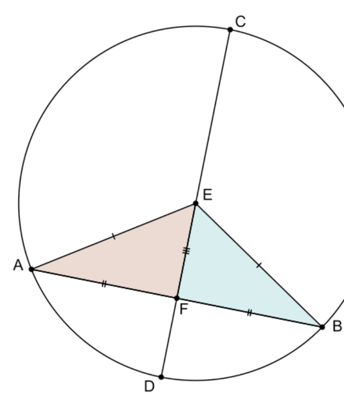
Speaker: Kerry Fitzmaurice
Scribes: Bill Keane, Tracy Maciolek

February 18, 2009

Proposition III.3

Given a circle ABC with center E, diameter CD, and a straight line AB not through the center but intersecting CD at F:

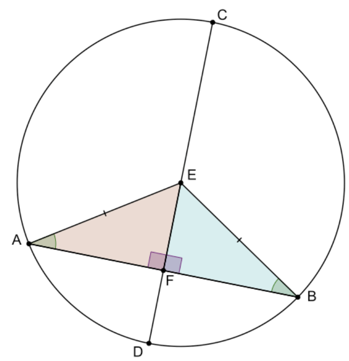
1. *If CD bisects AB, then $\angle AFC = \angle BFC = \lrcorner$.*
2. *If $\angle AFC = \angle BFC = \lrcorner$, then CD bisects AB at F.*



Assume first that CD bisects AB. Let E be the center (Prop. III.1), and draw EA and EB (Post. 1). Then EA=EB because they are radii, AF=FB by assumption, and EF is common, so $\triangle EAF \cong \triangle EBF$ (Prop. I.8). Therefore $\angle EFA = \angle EFB$, and hence $\angle EFA = \angle EFB = \lrcorner$ (Def. I.10).

Secondly, assume $\angle EFA = \angle EFB = \lrcorner$. Then $\triangle EAB$ is isosceles, so $\angle EAF = \angle EBF$ (Prop. I.5). Since $\angle EFA = \angle EFB$ and $EA = EB$, we have $\triangle EAF \cong \triangle EFB$ (Prop. I.26). Therefore $AF = FB$.

Q.E.D.



Comment:
What happens if AB goes through the center E, so that we no longer have a triangle?
Then (1) is true (the diameter bisects the chord), but (2) need not be (the angles need not be right.)